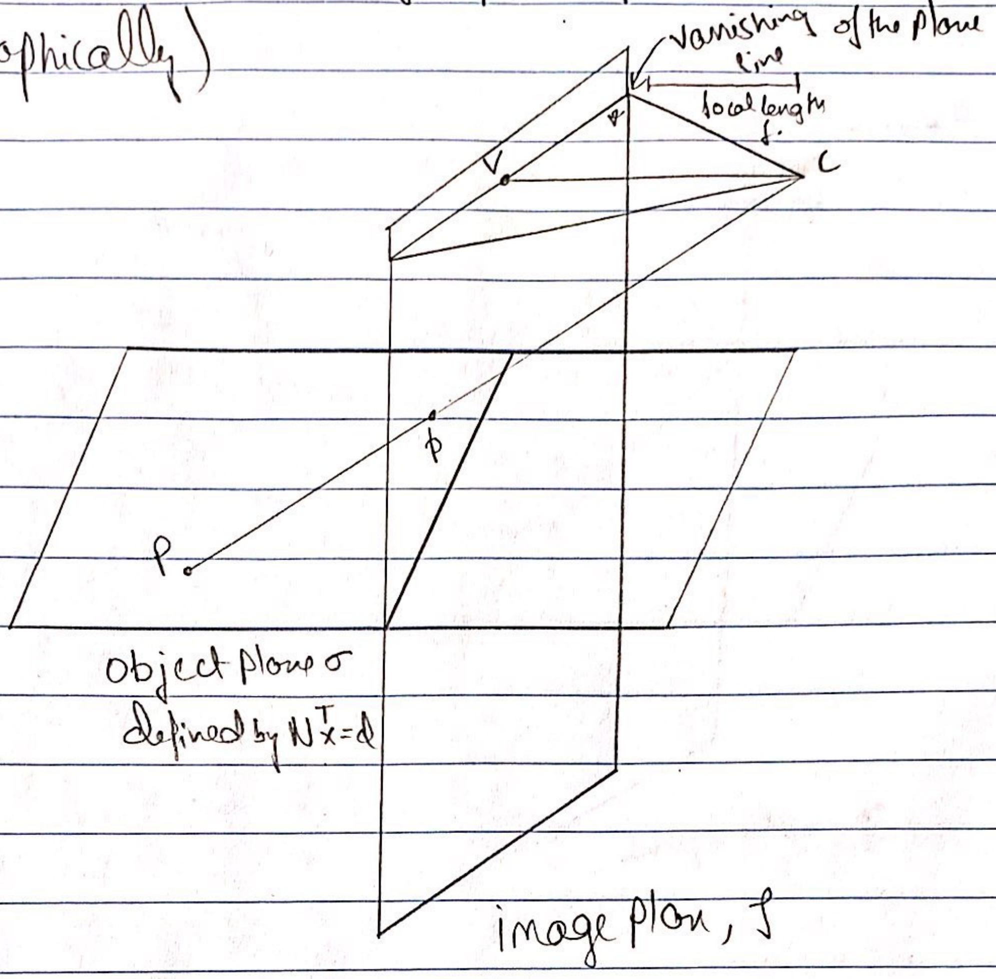
This Question is proved by 2 methods Graphically & Analytically.

Q1 (Graphically)



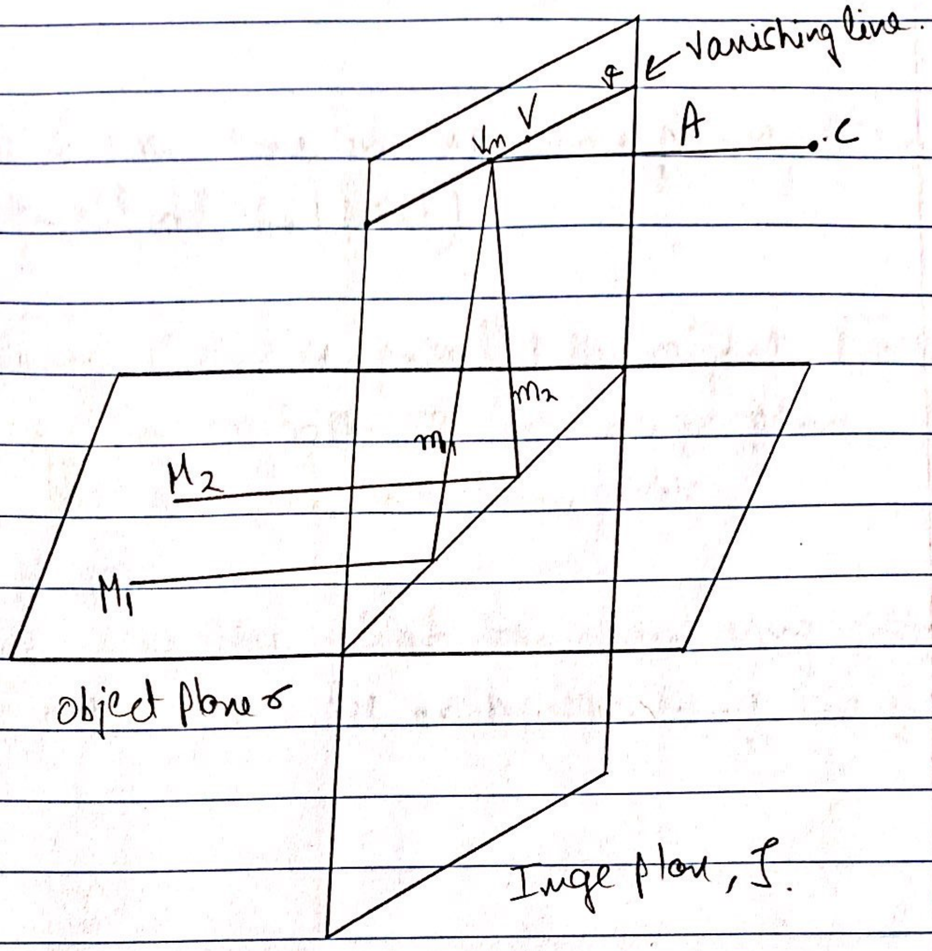
Lets say we have a plane σ (object plane) defined by $N^T X = d$. going under properspective projection which is perpendicular to image plane J.

To each point P on σ we have point p on J corresponding to the intersection of the line CP with the plane J.

C is called centre of projection & CV is the focal length f.

→ The point of intersection of the line through C, that is ⊥ to image plane (and so parallel to the object planes) is called principal vanishing point V.

→ The line v_e, which is intersection of the picture plane & through P ∥ to the object plane is called horizontal line.



Let M_1 & M_2 be lines on object plane.

- The image of M_i is the line m_i where the plane π, containing C & M_i intersects the image plane.

There is line CV_m which is // to M_i & so this line is also on π.

Since this line is Horizontal, it also line in plane determined by C and v', the vanishing line.

And V line & CVm line in the same plane, then they must intersect at Vm (point)

This plane of CVm is parallel to object plane. This places Vm on both π & image plane, so it must lie on the intersection mi

Hence we can say that the vanishing points of lines on a plane lie on the vanishing line of the plane

Q1. Analytically

For the plane

$$N^T X = d$$

$$\Rightarrow \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = d.$$

we have $x = f\frac{x}{z}$, $y = f\left(\frac{y}{z}\right)$, $z = f$.

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} \begin{bmatrix} f\frac{x}{z} & f\frac{y}{z} & f \end{bmatrix} = f\frac{d}{z}$$

for vanishing plane $z \to \infty$.

$$\therefore N_x + N_y + N_z f = 0 \quad \text{——} \; (1)$$

for Line

$$L_1(\lambda) = A + \lambda D$$

$$= [A_x + \lambda D_x , A_y + \lambda D_y , A_z + \lambda D_z]$$

Scanned with CamScanner

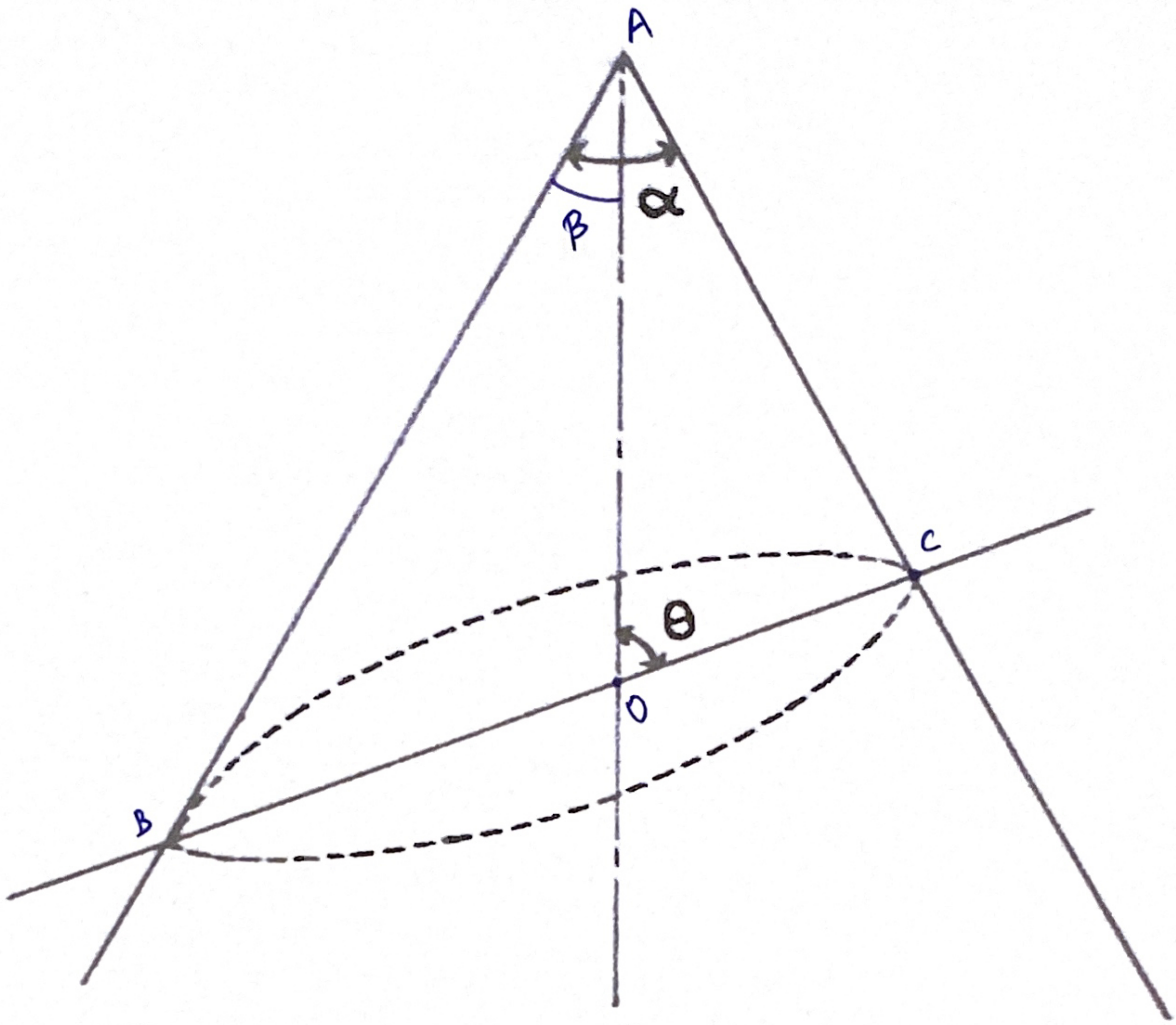
$$= \left(f \frac{A_x + \lambda D_x}{A_z + \lambda D_z} , f \frac{A_y + \lambda D_y}{A_z + \lambda D_z} \right)$$

for Vanishing line $\lambda \Rightarrow \infty$

$$= \left(f \frac{D_x}{D_z} , f \frac{D_y}{D_z} \right)$$

$$\Rightarrow N_x D_x + N_y D_y + N_z D_z = 0 \quad \text{— } 2$$

Looking at 1 & 2 we can say that.
Vanishing points of lines lie on vanishing line of plane.



Q2 a)

Let the point where the plane of ellipse intersects the axis of the cone called O, ~~&~~ let the vertex of the cone be denoted by A. . Let OA be unit of our measurement so that OA = 1. Let other vertex of the cone (triangular section) be called B, C (with B on the left of O.

let

$AB = c$

$BC = a$

$AC = b$

$\alpha = 2\beta \Rightarrow \beta = \alpha/2$

Clearly in ΔAOC,

$$\frac{OA}{\sin\angle OCA} = \frac{AC}{\sin\angle AOC}$$

So that

$$b = AC = \frac{\sin\theta}{\sin(\theta+\beta)}$$

Similarly

$$c = AB = \frac{\sin\theta}{\sin(\theta-\beta)}$$

&

$$a = BC = \frac{b\sin 2\beta}{\sin(\theta-\beta)} = \frac{\sin\theta\sin 2\beta}{\sin(\theta+\beta)\sin(\theta-\beta)}$$

let the incircle of ΔABC touch BC on point D, so that D is the focus of ellipse. BC is the major axis of the ellipse & if we know the distance of CD we can find the eccentricity of the ellipse. If 'r' is the Inradius of ΔABC then we know

$$\tan\frac{C}{2} = \frac{r}{CD}$$

So that

$$CD = \frac{r}{\tan(C/2)} = \frac{r}{\tan\left(\frac{\pi-\theta-\beta}{2}\right)} = r\tan\left(\frac{\theta+\beta}{2}\right)$$

Note further that the inradius r is given by :→

$$r = \frac{\Delta}{s}$$

$$= \frac{bc \sin 2\beta}{a+b+c}$$

$$= \frac{\dfrac{\sin^2\theta \sin 2\beta}{\sin(\theta+\beta)\sin(\theta-\beta)}}{\dfrac{\sin\theta(\sin(\theta+\beta) + \sin(\theta-\beta) + \sin 2\beta)}{\sin(\theta+\beta)\sin(\theta-\beta)}}.$$

$$= \frac{\sin\theta \sin 2\beta}{2\sin\theta\cos\beta + 2\sin\beta\cos\beta}$$

$$= \frac{2\sin\theta\sin\beta\cos\beta}{2\cos\beta(\sin\theta + \sin\beta)}$$

$$= \frac{\sin\theta\sin\beta}{\sin\theta + \sin\beta}$$

if e is the eccentricity

$$e = \frac{\text{distance of focus from center}}{\text{length of semi major axis}} = \frac{(a/2) - CD}{a/2}$$

$$= \frac{a - 2CD}{a}.$$

$$= \frac{a - 2r\tan\left(\frac{\theta+\beta}{2}\right)}{a}.$$

$$= 1 - \frac{2\sin\theta\sin\beta}{\sin\theta + \sin\beta} \cdot \frac{\sin^2\theta - \sin^2\beta}{2\sin\theta\sin\beta \cancel{\cos}\beta} \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{\sin\theta - \sin\beta}{\cos\beta} \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{2\cos\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2}}{\cos\beta} \cdot \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{2\sin\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2}}{\cos\beta}$$

$$= 1 - \frac{\cos\beta - \cos\theta}{\cos\beta}$$

$$= \frac{\cos\theta}{\cos\beta} = \frac{\cos\theta}{\cos(\alpha/2)}$$

$$\therefore \quad e = \frac{\cos\theta}{\cos\beta}$$

for $\beta \Rightarrow \sin\beta = \frac{r}{\sqrt{x^2+z^2}}$

$$\cos\beta = \frac{\sqrt{x^2+z^2-r^2}}{\sqrt{x^2+z^2}}$$

The angle θ is the same as the angle made by the cone axis with XY plane. The axis of the cone is the line joining the optical centre with the center of sphere $(X, 0, Z)$.

$$\cos\theta = \frac{X}{\sqrt{X^2 + Z^2}}$$

$$\therefore \text{eccentricity } (e) = \frac{\cos\theta}{\cos\beta}$$

$$= \frac{X}{\sqrt{X^2 + Z^2 - r^2}}.$$

Q2b,

The eccentricity of the hyperbola is greater than 1, while on the other hand the eccentricity of a parabola is equal to 1.

For eccentricity equal to 1,

$$Z^2 - r^2 = 0 \Rightarrow r = Z.$$

For eccentricity greater than 1,

$$Z < r.$$

Q3-Shape from Shading

Introduction-

For the purpose of this homework we were given a set of images of face with different light sources i.e. images of the same face under different lighting conditions were given.
The direction for the light source were also specified and we have to generate the height map in order to produce a 3D model of the images.

Q3 (a)
For part a we were given the LoadFaceImages function as a part of the skeleton code. This function as the name suggests reads or loads the image dataset of 64 different light source direction images and an ambient image wherein there was no light source present when the image was taken.

Q3 (b)
For the purpose of preprocessing, I used the NumPy broadcasting method and subtracted the ambient image from the other images without the use of looping.
The values were also checked for a negative value, and if we encounter a negative value after subtraction it was replaced by a 0 and the other positive values were rescaled between 0 and 1.

Q3 (c)
For this part we had to implement the function photometric_stereo. This function takes in the stack of images and produces the albedo for the image and the estimates of the surface normal.

The result with albedo, surface normal and the mean value for the dataset are as follows-

1. B01-

```
The mean= 0.1827564389346267
```

```
plot_albedo_and_surface_normals(albedo_image, surface_normals)
```

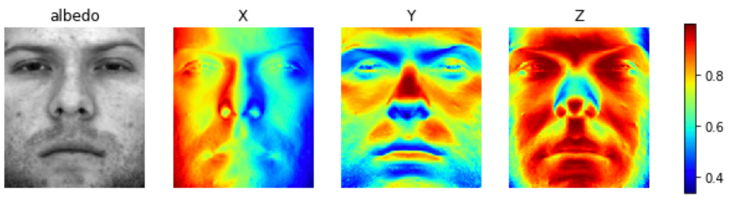


Figure 1 Albedo and Surface Normals for Subject B01

2. B02-

```
The mean= 0.1981905184270746
```

```
plot_albedo_and_surface_normals(albedo_image, surface_normals)
```

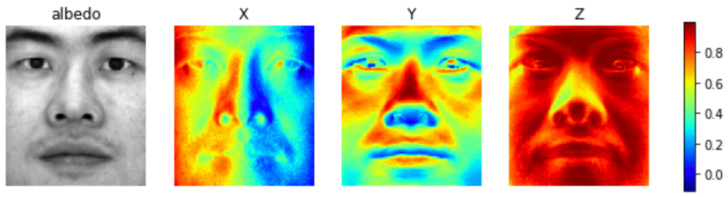


Figure 2 Albedo and Surface Normals for Subject B02

3. B05-

```
The mean= 0.13836687442617926
```

```
plot_albedo_and_surface_normals(albedo_image, surface_normals)
```

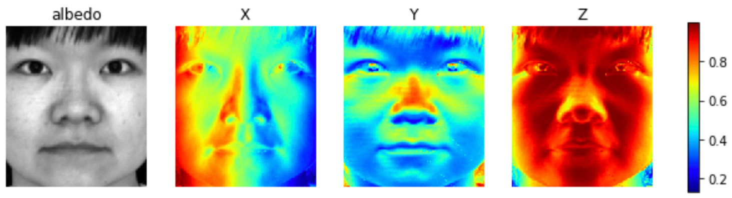


Figure 3 Albedo and Surface Normals for Subject B05

4. B07-

```
The mean= 0.14859763573997892
```

```
plot_albedo_and_surface_normals(albedo_image, surface_normals)
```

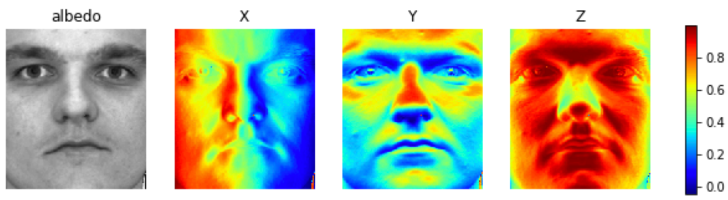


Figure 4 Albedo and Surface Normals for Subject B07

Q3 (d) (i)

For this part we implemented the get_surface function that takes the surface_normals and the type pf integration method as input and produces the height map. The function is implemented for 4 types of integration methods namely Row, Column, Average and Random.

For the column integration method, the, integration was done along the columns, which means that the results have centerline which is smooth horizontally. However, there was some reconstruction losses when producing the integration results vertically. These losses can be seen from the image results of subject B01 in the bottom half of the Images. -

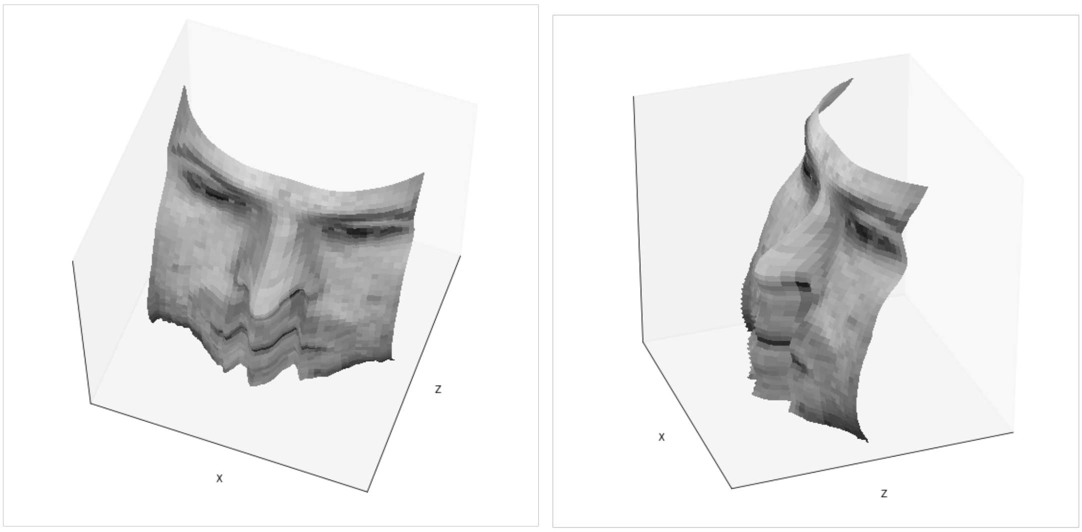


Figure 5 Subject B01 losses in Column Integration.

Similarly, for the row integration, integration was done along the rows, which means that the results have centerline which is smooth vertically. However, there was some reconstruction losses when producing the integration results horizontally. These losses can be seen from the image results of subject B01 in the middle half (nose section) of the Images. –

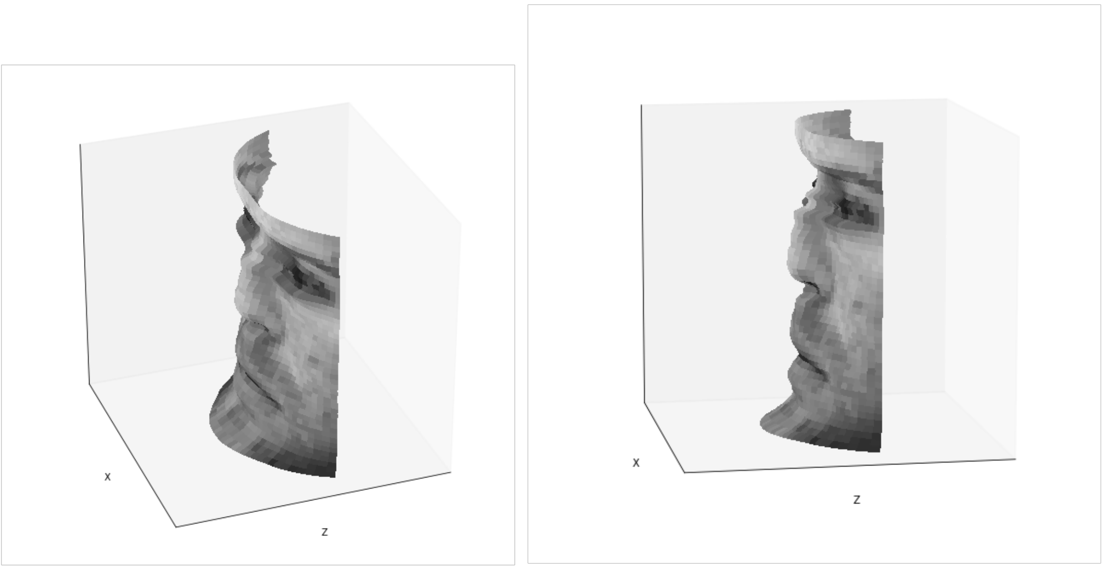


Figure 6 Subject B01 losses in Row Integration.

The Average method here is a combination of both the row and the column methods. Because this method implements both the row and column implementation produces better results than one single method. Here are the same images which had losses in the row and the column method.

Figure 7 Images for the subject B01 showing that the results for the average method are better than column and row method

The random method produces the results by adding more integration paths than just rows and columns method. And this is the reason that this method produces the images with the least amount of losses. As clearly seen from the images below-

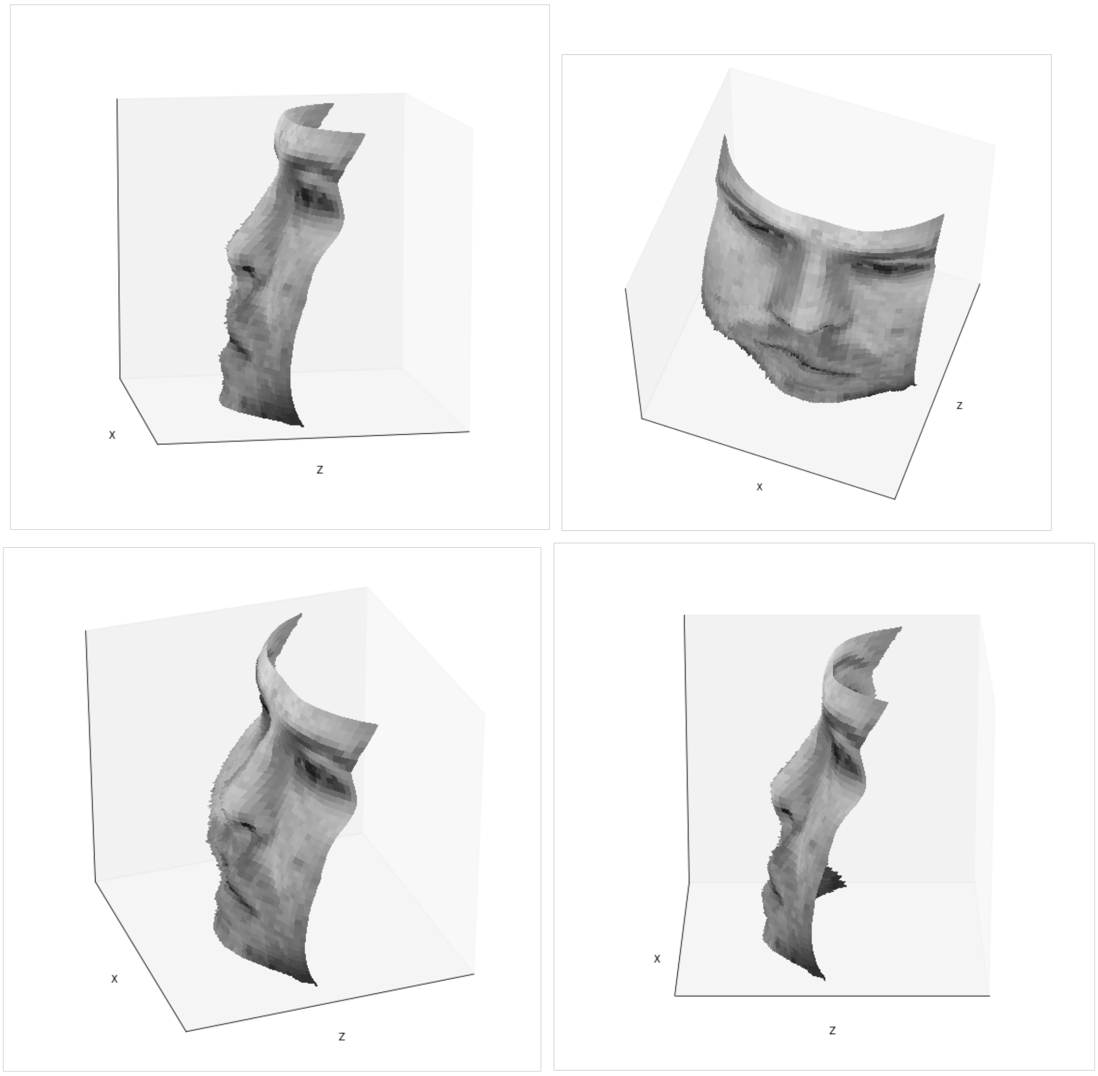


Figure 8 Images for the subject B01 showing that the results for the random method for the same viewpoint as the average method

Time Comparison in Seconds

```
Time Taken for the Various methods
Row      0.001277923583984375
Column   0.000804901123046875
Average  0.0035676956176757812
Random   68.87232613563538
```

Although the Random Integration method takes more time than the others, but it also produces the better reconstruction of the images.

The other views for the random integration method for the subject B01 are as follows-

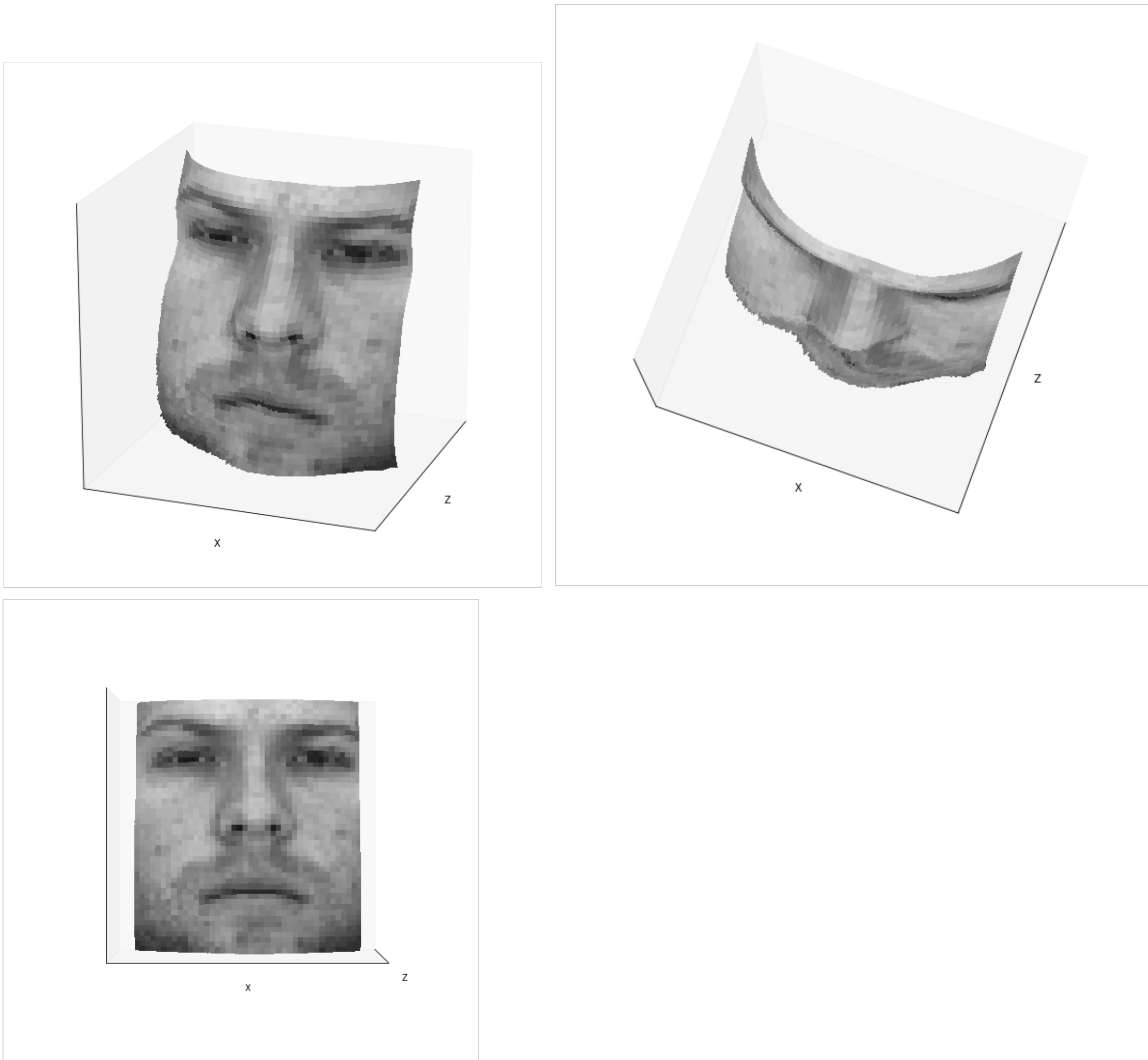


Figure 9 Other viewpoints for the Random method For subject B01

Q3 (d)(ii)
The Best viewpoints are shown for different subjects and random integration method-

Subject: B02-

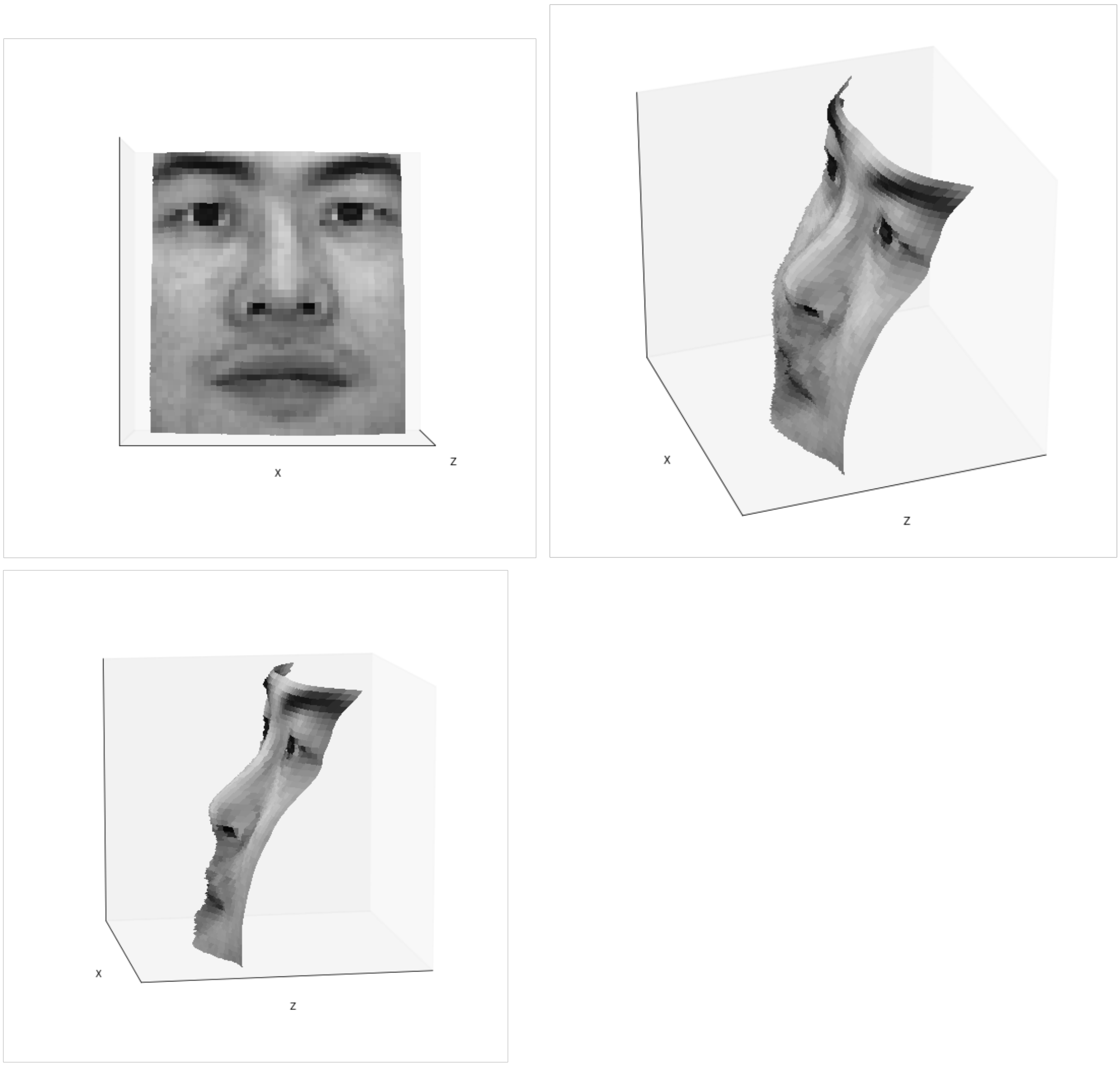


Figure 10 3D images for the Random method for subject B02

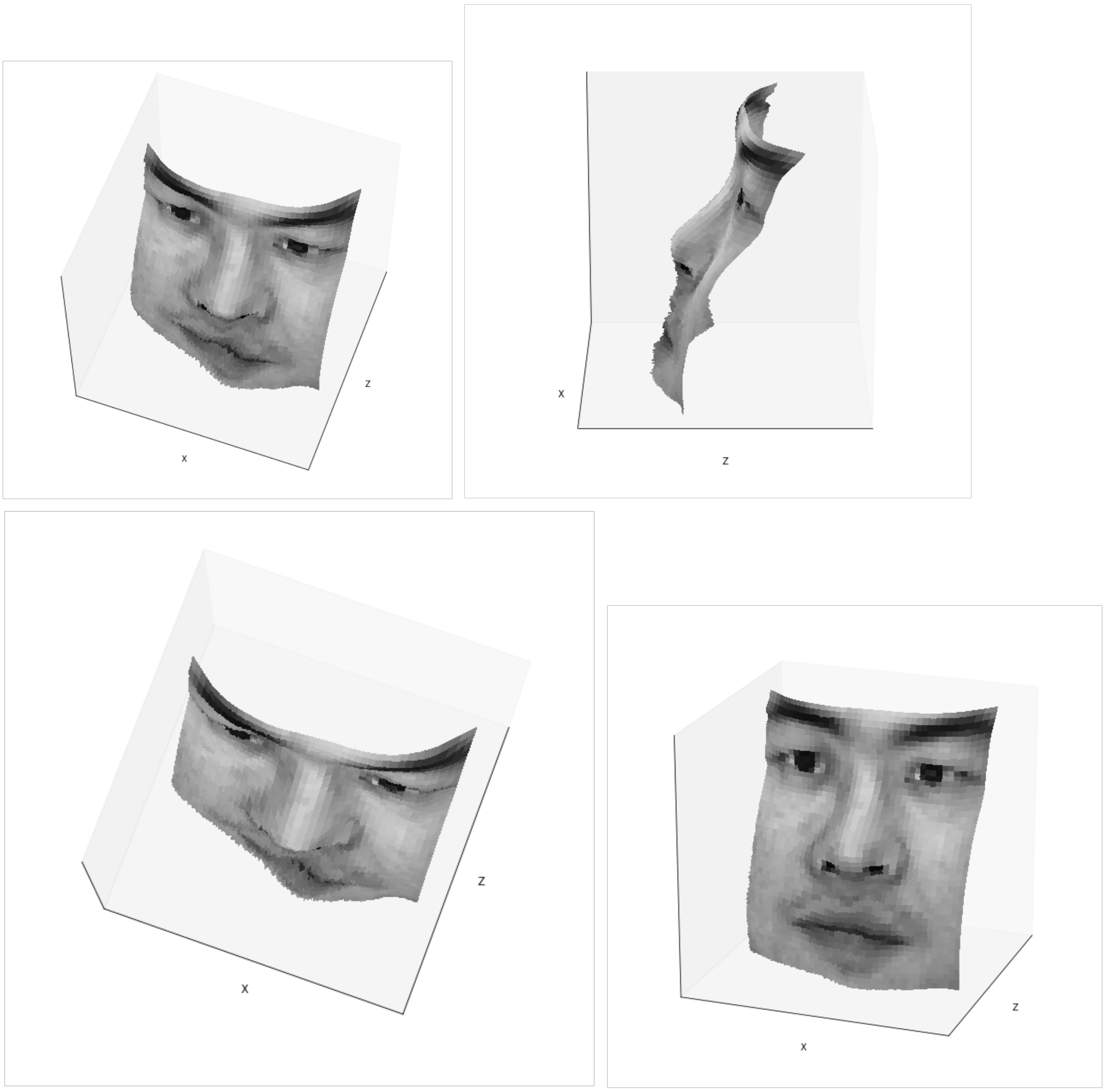


Figure 11 3D images for the Random method for subject B02

Computation Time for Subject B02-

```
Time Taken for the Various methods
Row      0.0007162094116210938
Column   0.0008032321929931641
Average  0.004538774490356445
Random   72.45055103302002
```

Subject: B05-

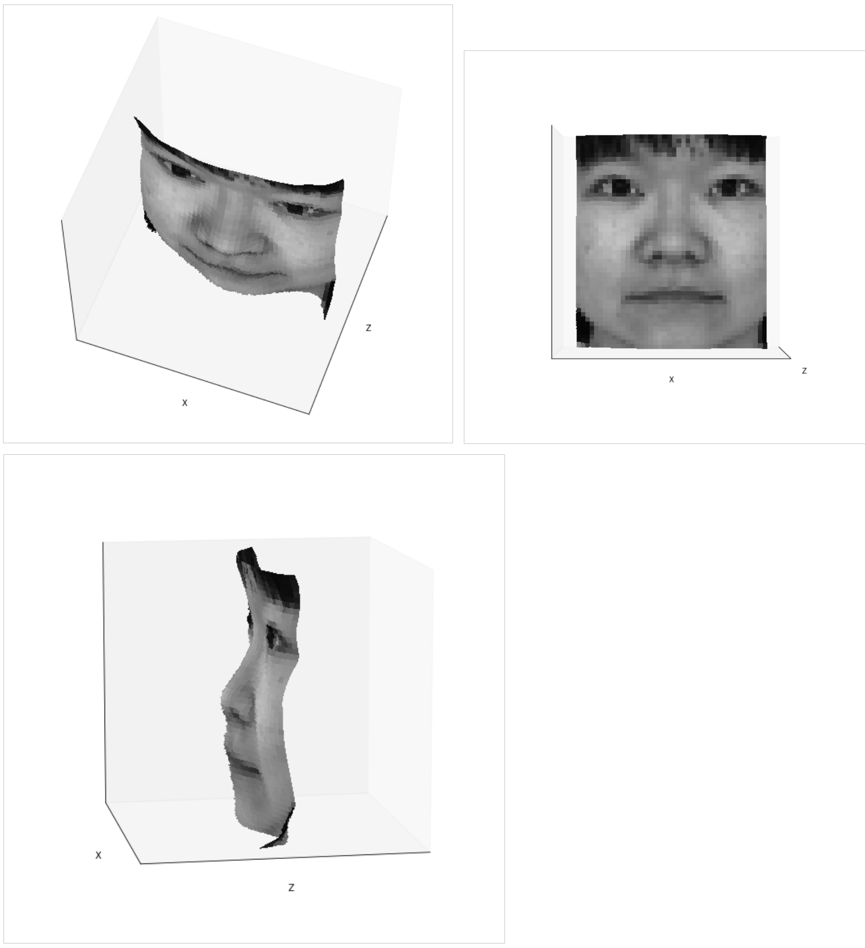


Figure 12 3D images for the Random method for subject B05

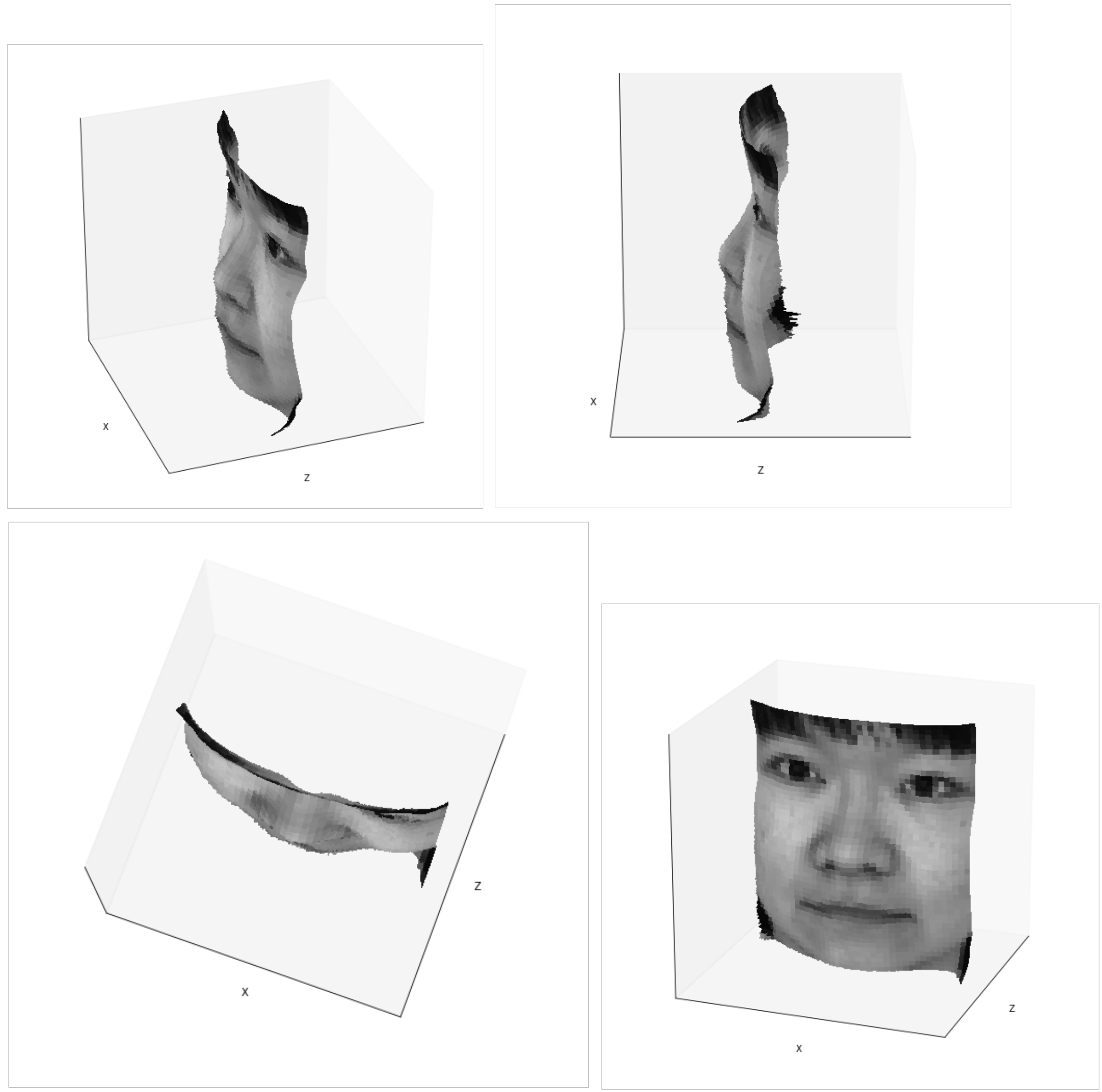


Figure 13 3D images for the Random method for subject B05

Computation Time for Subject B05-

```
Time Taken for the Various methods
Row       0.0018241405487060547
Column    0.0012159347534179688
Average   0.003742218017578125
Random    64.78003001213074
```

Subject: B07-

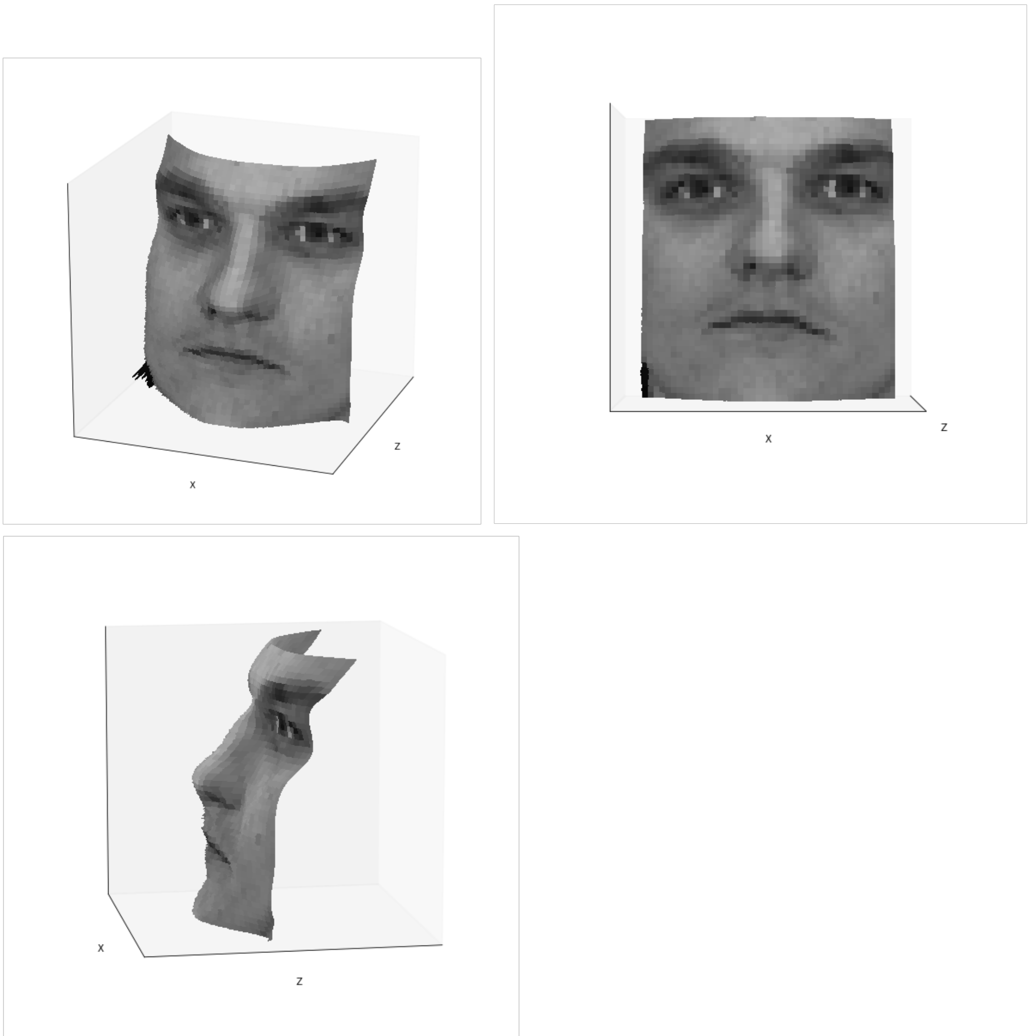


Figure 14 3D images for the Random method for subject B07

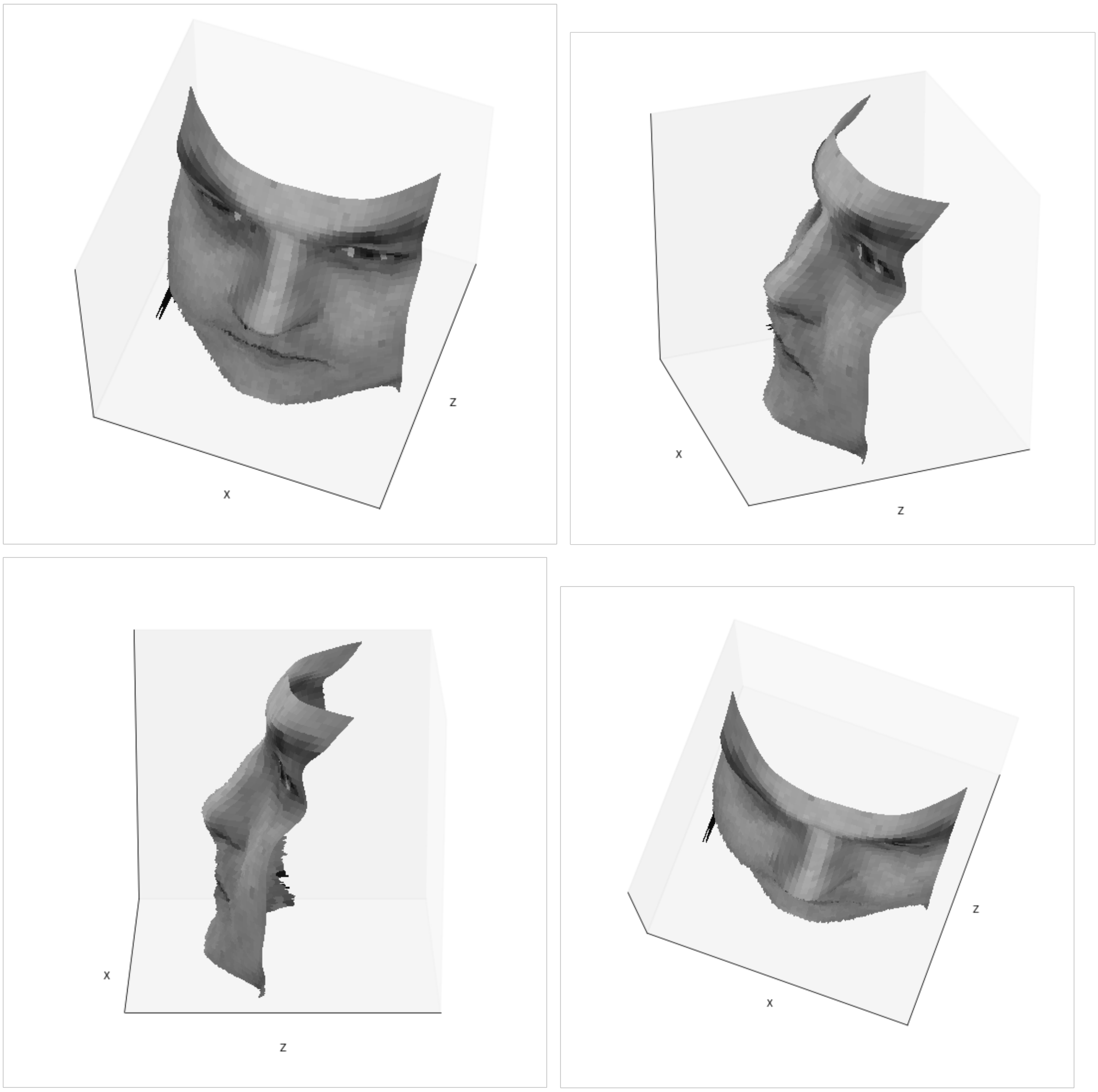


Figure 15 3D images for the Random method for subject B05

Computation Time for Subject B07-

```
Time Taken for the Various methods
Row      0.0008749961853027344
Column   0.001239776611328125
Average  0.006265163421630859
Random   65.66791725158691
```

Q3 (e)

The Reasons are-

1. We assume that Lambert's law is true for the reconstruction. So, the shiny parts like eyes and the oil on the face does not follow diffuse reflection which can be seen here-

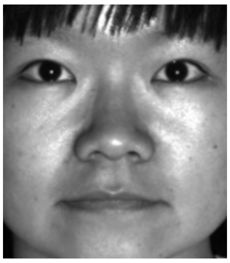


Figure 16 Image showing glares

This can be reduced if the face were washed before taking images so as to remove the oil from face.

2. The reconstruction of the face has losses at the shadow area like the area in the hair. Also, some images don't have the chin are cropped which doesn't help the reconstruction of the face.

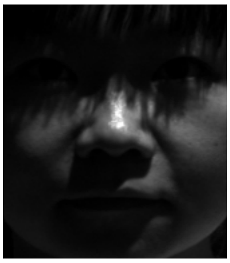


Figure 17 Image showing losses in reconstruction due to hair and chin

3. We use a local point shading model. This means that a point will receive light only from a source visible at that point and not from any other point's reflection.