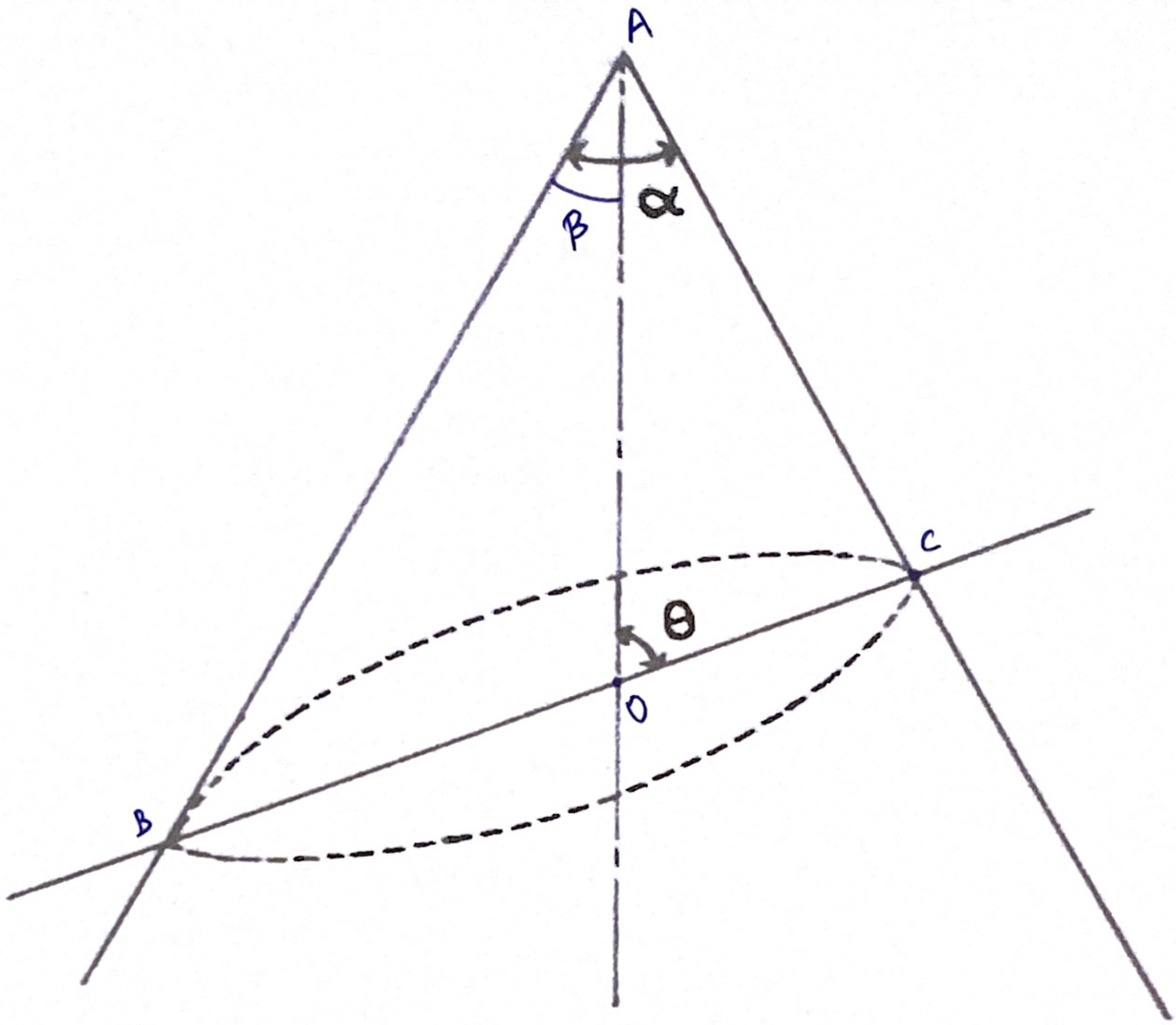

Q2 a)

Let the point where the plane of ellipse intersects the axis of the cone called O, ~~&~~ let the vertex of the cone be denoted by A. . Let OA be unit of our measurement so that OA = 1. Let other vertex of the cone (triangular section) be called B, C (with B on the left of O.

let

$AB = c$

$BC = a$

$AC = b$

$\alpha = 2\beta \Rightarrow \beta = \alpha/2$

Clearly in ΔAOC,

$$\frac{OA}{\sin\angle OCA} = \frac{AC}{\sin\angle AOC}$$

So that

$$b = AC = \frac{\sin\theta}{\sin(\theta+\beta)}$$

Similarly

$$c = AB = \frac{\sin\theta}{\sin(\theta-\beta)}$$

&

$$a = BC = \frac{b\sin 2\beta}{\sin(\theta-\beta)} = \frac{\sin\theta\sin 2\beta}{\sin(\theta+\beta)\sin(\theta-\beta)}$$

let the incircle of ΔABC touch BC on point D, so that D is the focus of ellipse. BC is the major axis of the ellipse & if we know the distance of CD we can find the eccentricity of the ellipse. If 'r' is the Inradius of ΔABC then we know

$$\tan\frac{C}{2} = \frac{r}{CD}$$

So that

$$CD = \frac{r}{\tan(C/2)} = \frac{r}{\tan\left(\frac{\pi-\theta-\beta}{2}\right)} = r\tan\left(\frac{\theta+\beta}{2}\right)$$

Note further that the inradius r is given by :→

$$r = \frac{\Delta}{s}$$

$$= \frac{bc \sin 2\beta}{a+b+c}$$

$$= \frac{\dfrac{\sin^2\theta \sin 2\beta}{\sin(\theta+\beta)\sin(\theta-\beta)}}{\dfrac{\sin\theta(\sin(\theta+\beta)+\sin(\theta-\beta)+\sin 2\beta)}{\sin(\theta+\beta)\sin(\theta-\beta)}}.$$

$$= \frac{\sin\theta \sin 2\beta}{2\sin\theta\cos\beta + 2\sin\beta\cos\beta}$$

$$= \frac{2\sin\theta\sin\beta\cos\beta}{2\cos\beta(\sin\theta+\sin\beta)}$$

$$= \frac{\sin\theta\sin\beta}{\sin\theta+\sin\beta}$$

if e is the eccentricity

$$e = \frac{\text{distance of focus from center}}{\text{length of semi major axis}} = \frac{(a/2) - CD}{a/2}$$

$$= \frac{a - 2CD}{a}.$$

$$= \frac{a - 2r\tan\left(\frac{\theta+\beta}{2}\right)}{a}.$$

$$= 1 - \frac{2\sin\theta\sin\beta}{\sin\theta+\sin\beta} \cdot \frac{\sin^2\theta - \sin^2\beta}{2\sin\theta\sin\beta\cos\beta} \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{\sin\theta - \sin\beta}{\cos\beta} \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{2\cos\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2}}{\cos\beta} \cdot \tan\left(\frac{\theta+\beta}{2}\right)$$

$$= 1 - \frac{2\sin\frac{\theta+\beta}{2}\sin\frac{\theta-\beta}{2}}{\cos\beta}$$

$$= 1 - \frac{\cos\beta - \cos\theta}{\cos\beta}$$

$$= \frac{\cos\theta}{\cos\beta} = \frac{\cos\theta}{\cos(\alpha/2)}$$

$$\therefore \quad e = \frac{\cos\theta}{\cos\beta}$$

for $\beta \Rightarrow \sin\beta = \dfrac{r}{\sqrt{x^2+z^2}}$

$$\cos\beta = \frac{\sqrt{x^2+z^2-r^2}}{\sqrt{x^2+z^2}}$$

The angle θ is the same as the angle made by the cone axis with XY plane. The axis of the cone is the line joining the optical centre with the center of sphere $(X, 0, Z)$.

$$\cos\theta = \frac{X}{\sqrt{X^2+Z^2}}$$

$\therefore$ eccentricity $(e) = \dfrac{\cos\theta}{\cos\beta}$

$$= \frac{X}{\sqrt{X^2+Z^2-r^2}}.$$

<u>Q2b,</u>

The eccentricity of the hyperbola is greater than 1, while on the other hand the eccentricity of a parabola is equal to 1.

For eccentricity equal to 1,

$$Z^2 - r^2 = 0 \Rightarrow r = Z.$$

For eccentricity greater than 1,

$$Z < r.$$