1.

a. The page-replacement algorithm can be outlined as follows:
    i. Initial value of counters is zero.
    ii. Counters increase whenever a page is accessed (read or written).
    iii. Counters decrease after a fixed interval or periodically; all counters are divided by 2 (integer division) to reduce their values.
    iv. When a page fault occurs and a new page needs to be brought into memory, the page frame with the smallest counter value is selected for replacement. In case of ties, any arbitrary tie-breaking rule can be used, such as selecting the oldest page or using a round-robin fashion.

b. Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2
Page Frames: 4
Initial State: Page Frames: [-, -, -, -] Counters: [0, 0, 0, 0] Page Faults: 0
1: Page fault (1) Page Frames: [1, -, -, -] Counters: [1, 0, 0, 0] Page Faults: 1
2: Page fault (2) Page Frames: [1, 2, -, -] Counters: [1, 1, 0, 0] Page Faults: 2
3: Page fault (3) Page Frames: [1, 2, 3, -] Counters: [1, 1, 1, 0] Page Faults: 3
4: Page fault (4) Page Frames: [1, 2, 3, 4] Counters: [1, 1, 1, 1] Page Faults: 4
5: Page fault (5) Page Frames: [5, 2, 3, 4] Counters: [1, 2, 1, 1] Page Faults: 5
3: Page fault (3) Page Frames: [5, 2, 3, 4] Counters: [1, 3, 1, 1] Page Faults: 6
4: Page fault (4) Page Frames: [5, 2, 4, 3] Counters: [1, 3, 2, 1] Page Faults: 7
1: Page fault (1) Page Frames: [5, 2, 4, 1] Counters: [2, 3, 2, 1] Page Faults: 8
6: Page fault (6) Page Frames: [5, 2, 6, 1] Counters: [2, 3, 2, 2] Page Faults: 9
7: Page fault (7) Page Frames: [5, 2, 7, 1] Counters: [2, 3, 3, 2] Page Faults: 10
8: Page fault (8) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 3, 3] Page Faults: 11
7: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 3] Page Faults: 11
8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 4] Page Faults: 11
9: Page fault (9) Page Frames: [5, 2, 9, 8] Counters: [2, 3, 4, 5] Page Faults: 12
7: Page fault (7) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 5] Page Faults: 13
8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 6] Page Faults: 13
9: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 7] Page Faults: 13
5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 4, 5, 7] Page Faults: 13
4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 5, 5, 7] Page Faults: 13
5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 6, 5, 7] Page Faults: 13
4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 7, 5, 7] Page Faults: 13

2: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 8, 5, 7] Page Faults: 13

Page faults for the algorithm with four page frames: 13

c. Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2
Page Frames: 4
1, 2, 3, 4 - No page fault (initial fill)
5 - Page fault (replace 1)
3 - Page fault (replace 2)
4 - Page fault (replace 3)
1 - Page fault (replace 4)
6 - Page fault (replace 5)
7 - Page fault (replace 1)
8 - Page fault (replace 6)
9 - Page fault (replace 7)
5 - Page fault (replace 8)
4 - Page fault (replace 9)
2 - Page fault (replace 5)

Total page faults for the optimal page replacement strategy with four page frames: 12

Therefore, the minimum number of page faults for an optimal page replacement strategy for the given reference string with four page frames is 12.