1. 1. The page-replacement algorithm can be outlined as follows:
      1. Initial value of counters is zero.
      2. Counters increase whenever a page is accessed (read or written).
      3. Counters decrease after a fixed interval or periodically; all counters are divided by 2 (integer division) to reduce their values.
      4. When a page fault occurs and a new page needs to be brought into memory, the page frame with the smallest counter value is selected for replacement. In case of ties, any arbitrary tie-breaking rule can be used, such as selecting the oldest page or using a round-robin fashion.
   2. Let's apply the algorithm to the given reference string with four page frames:

Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2 Page Frames: 4

Initial State: Page Frames: [ -, -, -, -] Counters: [0, 0, 0, 0] Page Faults: 0

1: Page fault (1) Page Frames: [1, -, -, -] Counters: [1, 0, 0, 0] Page Faults: 1

2: Page fault (2) Page Frames: [1, 2, -, -] Counters: [1, 1, 0, 0] Page Faults: 2

3: Page fault (3) Page Frames: [1, 2, 3, -] Counters: [1, 1, 1, 0] Page Faults: 3

4: Page fault (4) Page Frames: [1, 2, 3, 4] Counters: [1, 1, 1, 1] Page Faults: 4

5: Page fault (5) Page Frames: [5, 2, 3, 4] Counters: [1, 2, 1, 1] Page Faults: 5

3: Page fault (3) Page Frames: [5, 2, 3, 4] Counters: [1, 3, 1, 1] Page Faults: 6

4: Page fault (4) Page Frames: [5, 2, 4, 3] Counters: [1, 3, 2, 1] Page Faults: 7

1: Page fault (1) Page Frames: [5, 2, 4, 1] Counters: [2, 3, 2, 1] Page Faults: 8

6: Page fault (6) Page Frames: [5, 2, 6, 1] Counters: [2, 3, 2, 2] Page Faults: 9

7: Page fault (7) Page Frames: [5, 2, 7, 1] Counters: [2, 3, 3, 2] Page Faults: 10

8: Page fault (8) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 3, 3] Page Faults: 11

7: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 3] Page Faults: 11

8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 4] Page Faults: 11

9: Page fault (9) Page Frames: [5, 2, 9, 8] Counters: [2, 3, 4, 5] Page Faults: 12

7: Page fault (7) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 5] Page Faults: 13

8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 6] Page Faults: 13

9: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 7] Page Faults: 13

5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 4, 5, 7] Page Faults: 13

4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 5, 5, 7] Page Faults: 13

5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 6, 5, 7] Page Faults: 13

4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 7, 5, 7] Page Faults: 13

2: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 8, 5, 7] Page Faults: 13

Page faults for the algorithm with four page frames: 13

* 1. The minimum number of page faults for an optimal page replacement strategy for the given reference string with four page frames can be calculated using the Optimal page replacement algorithm. This algorithm is an unrealistic, oracle-like algorithm that assumes it knows the future page requests.

Calculating the minimum number of page faults for the given reference string with four page frames using the Optimal algorithm requires analyzing the entire string and making optimal decisions at each step. Here is the calculation:

Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2

Page Frames: 4

1, 2, 3, 4 - No page fault (initial fill)

5 - Page fault (replace 1)

3 - Page fault (replace 2)

4 - Page fault (replace 3)

1 - Page fault (replace 4)

6 - Page fault (replace 5)

7 - Page fault (replace 1)

8 - Page fault (replace 6)

9 - Page fault (replace 7)

5 - Page fault (replace 8)

4 - Page fault (replace 9)

2 - Page fault (replace 5)

Total page faults for the optimal page replacement strategy with four page frames: 12

Therefore, the minimum number of page faults for an optimal page replacement strategy for the given reference string with four page frames is 12.