1. 1. The page-replacement algorithm can be outlined as follows:
      1. Initial value of counters is zero.
      2. Counters increase whenever a page is accessed (read or written).
      3. Counters decrease after a fixed interval or periodically; all counters are divided by 2 (integer division) to reduce their values.
      4. When a page fault occurs and a new page needs to be brought into memory, the page frame with the smallest counter value is selected for replacement. In case of ties, any arbitrary tie-breaking rule can be used, such as selecting the oldest page or using a round-robin fashion.
   2. Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2

Page Frames: 4

Initial State: Page Frames: [ -, -, -, -] Counters: [0, 0, 0, 0] Page Faults: 0

1: Page fault (1) Page Frames: [1, -, -, -] Counters: [1, 0, 0, 0] Page Faults: 1

2: Page fault (2) Page Frames: [1, 2, -, -] Counters: [1, 1, 0, 0] Page Faults: 2

3: Page fault (3) Page Frames: [1, 2, 3, -] Counters: [1, 1, 1, 0] Page Faults: 3

4: Page fault (4) Page Frames: [1, 2, 3, 4] Counters: [1, 1, 1, 1] Page Faults: 4

5: Page fault (5) Page Frames: [5, 2, 3, 4] Counters: [1, 2, 1, 1] Page Faults: 5

3: Page fault (3) Page Frames: [5, 2, 3, 4] Counters: [1, 3, 1, 1] Page Faults: 6

4: Page fault (4) Page Frames: [5, 2, 4, 3] Counters: [1, 3, 2, 1] Page Faults: 7

1: Page fault (1) Page Frames: [5, 2, 4, 1] Counters: [2, 3, 2, 1] Page Faults: 8

6: Page fault (6) Page Frames: [5, 2, 6, 1] Counters: [2, 3, 2, 2] Page Faults: 9

7: Page fault (7) Page Frames: [5, 2, 7, 1] Counters: [2, 3, 3, 2] Page Faults: 10

8: Page fault (8) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 3, 3] Page Faults: 11

7: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 3] Page Faults: 11

8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 4, 4] Page Faults: 11

9: Page fault (9) Page Frames: [5, 2, 9, 8] Counters: [2, 3, 4, 5] Page Faults: 12

7: Page fault (7) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 5] Page Faults: 13

8: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 6] Page Faults: 13

9: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 3, 5, 7] Page Faults: 13

5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 4, 5, 7] Page Faults: 13

4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 5, 5, 7] Page Faults: 13

5: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 6, 5, 7] Page Faults: 13

4: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 7, 5, 7] Page Faults: 13

2: No page fault (already in memory) Page Frames: [5, 2, 7, 8] Counters: [2, 8, 5, 7] Page Faults: 13

Page faults for the algorithm with four page frames: 13

* 1. Reference String: 1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2

Page Frames: 4

1, 2, 3, 4 - No page fault (initial fill)

5 - Page fault (replace 1)

3 - Page fault (replace 2)

4 - Page fault (replace 3)

1 - Page fault (replace 4)

6 - Page fault (replace 5)

7 - Page fault (replace 1)

8 - Page fault (replace 6)

9 - Page fault (replace 7)

5 - Page fault (replace 8)

4 - Page fault (replace 9)

2 - Page fault (replace 5)

Total page faults for the optimal page replacement strategy with four page frames: 12

Therefore, the minimum number of page faults for an optimal page replacement strategy for the given reference string with four page frames is 12.