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Homework Assignment 2

"I pledge my honor that I have abided by the Stevens Honor System."

Page 67 #4

a.) Mystery(n) takes in all nonnegative integers 1 to n and computes the sum of all squares

b.) The basic operation is multiplication

c.) It is executed n times

d.) $\Theta(n)$

e.) Instead of iterating through each number to n you can use $\frac{(n)\cdot(n+1)\cdot(2n+1)}{6}$ which has a time complexity of $\Theta(1)$.

Page 76 #1

a.) $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$

Replace n with (n-1)

- $x(n-1) = x(n-2) + 5$
- $x(n) = (x(n-2) + 5) + 5$
- $x(n) = x(n-2) + 10$

Replace n with (n-2)

- $x(n-2) = x(n-3) + 5$
- $x(n) = (x(n-3) + 5) + 10$
- $x(n) = x(n-3) + 15$

Write the general form of the equation

- $x(n) = x(n-i) + 5i$

Make use of the initial condition

- $x(1) = 0 \rightarrow n - i = 1$ so $i = n - 1$

Make the substitution

- $x(n) = x(n-n+1) + 5(n-1)$
- $x(n) = x(1) + 5(n-1)$
- $x(n) = \boxed{5(n-1)}$ which has a time complexity of $\Theta(n)$

b.) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

Replace n with $(n-1)$

- $x(n-1) = 3x(n-2)$
- $x(n) = 3(3x(n-2))$
- $x(n) = 9x(n-2)$

Replace n with $(n-2)$

- $x(n-2) = 3x(n-3)$
- $x(n) = 9(3x(n-3))$
- $x(n) = 27x(n-3)$

Write the general form of the equation

- $x(n) = 3^i x(n-i)$

Make use of the initial condition

- $x(1) = 4 \rightarrow n - i = 1$ so $i = n - 1$

Make the substitution

- $x(n) = 3^i x(n-i)$
- $x(n) = 3^{(n-1)} x(n-n-1)$
- $x(n) = 3^{n-1} x(1)$ (plug in 4)
- $x(n) = 3^{n-1} \cdot 4$
- $x(n) = \boxed{4 \cdot 3^{n-1}}$ which has a time complexity of $\Theta(3^n)$

c.) $x(n) = x(n-1) + n$ for $n > 1$, $x(0) = 0$

Replace n with $(n-1)$

- $x(n-1) = x(n-2) + (n-1)$
- $x(n) = (x(n-2) + (n-1)) + n$
- $x(n) = x(n-2) + (n-1) + n$

Replace n with $(n-2)$

- $x(n-2) = x(n-3) + (n-2)$
- $x(n) = (x(n-3) + (n-2)) + (n-1) + n$

Write the general form of the equation

- $x(n) = x(n-i) + (n-(i-1)) + (n-(i-2)) + \ldots + n$

Make use of the initial condition

- $x(0) = 0 \rightarrow n - i = 0$ so... $i = n+0$

Make the substitution

- $x(n) = x(n-i) + (n-(i-1)) + (n-(i-2)) + \ldots + n$
- $x(n) = x(n-n) + (n-(n-1)) + (n-(n-2)) + \ldots + n$
- $x(n) = x(1-1) + (1-(1-1)) + (1-(1-2)) + \ldots + n$
- $x(n) = x(0) + 1 + 2 + \ldots + n$
- $x(n) = \boxed{\dfrac{n(n+1)}{2}}$ which has a time complexity of $\Theta(n^2)$

d.) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$

(Solve for $n = 2^k$)

Replace n with (2^{k-1})

- $x(2^{k-1}) = x(2^{k-2}) + 2^{k-2}$
- $x(2^k) = (x(2^{k-2}) + 2^{k-1}) + 2^k$

Replace n with (2^{k-2})

- $x(2^{k-2}) = x(2^{k-3}) + 2^{k-2}$
- $x(2^k) = (x(2^{k-3}) + 2^{k-2}) + 2^{k-1} + 2^k$

Write the general form of the equation

- $x(2^n) = x(2^{k-i}) + 2^{k-(i-1)} + 2^{k-(i-2)} + \ldots + 2^k$

Make use of the initial condition

- $x(1) = 1$, $2^{k-i} = 1 \rightarrow k-i=0$ so... $i=k$

Make the substitution

- $x(n) = x(n/2) + n$
- $x(2^k) = x(2^{k-k}) + 2^{k-k+1} + 2^{k-k+2} + \ldots + 2^k$
- $x(2^k) = x(2^{1-1}) + 2^{1-1+1} + 2^{1-1+2} + \ldots + 2^k$
- $x(2^k) = x(1) + 2^1 + 2^2 + \ldots + 2^k$
- $x(2^k) = 2^{k+1} - 1$
- $x(2^k) = 2 \cdot 2^k - 1$
- $x(2^k) = \boxed{2n-1}$ which has a time complexity of $\Theta(n)$

e.) $x(n) = x(\frac{n}{3}) + 1$ for $n>1$, $x(1)=1$

(solve for $n=3^k$) $x(3^k) = x(3^{k-1}) + 1$

Replace n with (3^{k-1})

- $x(3^{k-1}) = x(3^{k-2}) + 1$
- $x(3^k) = (x(3^{k-2}) + 1) + 1$
- $x(3^k) = x(3^{k-2}) + 2$

Replace n with (3^{k-2})

- $x(3^{k-2}) = x(3^{k-3}) + 1$
- $x(3^k) = (x(3^{k-3}) + 1) + 2$

- $x(3^k) = x(3^{k-3}) + 3$

Write the general form of the equation

- $x(3^k) = x(3^{k-i}) + i$

Make use of the initial condition

- $x(1) = 1 \quad 3^{k-i} = 1 \rightarrow k - i = 0$ so... $k = i$

Make the substitution

- $x(3^k) = x(3^{k-2}) + 1$
- $x(3^k) = x(3^{k-k}) + k$
- $x(3^k) = 1 + k \rightarrow \boxed{1 + \log_3 n}$ which has a time complexity of $\Theta(\log n)$

Page 76-77 #3

a.) $M(n) = M(n-1) + 2 \quad \forall\, n > 1, \quad M(1) = 0$

$M(n) = M(n-1) + 2$

Replace n with $(n-1)$

- $M(n-1) = M(n-2) + 2$
- $M(n) = (M(n-2) + 2) + 2$
- $M(n) = M(n-2) + 4$

Replace n with $(n-2)$

- $M(n-2) = M(n-3) + 2$
- $M(n) = (M(n-3) + 2) + 4$
- $M(n) = M(n-3) + 6$

Write the general form of the equation

- $M(n) = M(n-i) + 2i$

Make use of the initial condition

- $n - i = 1$ so.. $i = n - 1$

Make the substitution

- $M(n) = M(n - n - 1) + 2(n-1)$
- $M(n) = M(1) + 2(n-1)$
- $M(n) = 2(n-1)$ so... $2(n-1)$ multiplications (2 per loop run)

b.) This algorithm compares with the straightforward nonrecursive algorithm for computing this sum because they have different time complexities. The recursive algorithm's being $O(n)$ and the nonrecursive being $\Theta(n)$.