{Problem Set 9}

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"I pledge my honor that I have abided by the Stevens Honor System"

Problem 1

Show that the following language is decidable by giving a high-level description of a TM that decides the language.

$\{ \langle M \rangle : M$ is a PDA and $L(M)$ is an infinite language$\}$

On input $\langle M \rangle$:

1) Convert M to a CFG A and take note of A's pumping length (let this be p)
2) In addition to the CFG let B be a regular expression that is made up of all strings greater than or equal to length p.
3) Let CFG $C \rightarrow L(C) = L(A) \cap L(B)$
4) If $L(C) \neq 0$, then ACCEPT
5) If $L(C) = 0$, then REJECT

• Since PDAs can store an infinite amount the result of all strings of length p or greater intersected with the CFG of M itself, this proves the language is decidable.

2) Show $\{ \langle G \rangle : G$ is a CFG over $\{a,b\}$ and $a^* \cap L(G) \neq \emptyset\}$ decidable

On input $\langle G \rangle$:

1) Construct a CFG A such that $L(A) = a^* \cap L(G)$
2) Let R be a decider for a decidable language. Using R, test whether $L(H) = \emptyset$
3) If $L(H) = \emptyset$ then REJECT

If $L(H) \neq \emptyset$ then ACCEPT

3) Let A be a TM recognizable language of strings that encode TMs that are deciders. Prove that there is a decidable language which is not decided by an TM in A. (Hint: Start w/ an enumerator for A)

Let $\mathcal{E}$ be the enumerator A.
↳ Let $\langle M_k \rangle$ be the kth output of $\mathcal{E}$.

On input x:
1) If x is not within the language of the alphabet of A, REJECT
2) Use $\mathcal{E}$ to enumerate up until $\langle M_k \rangle$ (aka enumerate all values ~~[illegible]~~ ~~[illegible]~~ in A)
3) Run M_k on input x
4) If M_k accept, REJECT → If M_k rejects, ACCEPT
• In this case M_k is the decidable language "not decided by any TM in A", so this theorem holds.

4) Consider the problem of determining whether a TM on input w ever attempts to move its head left when its head is on the leftmost tape. a) Formulate this problem as a decision problem
$L = \{\langle M, w\rangle : M$ attempts to move its head left when its head is on the leftmost tape cell$\}$

b) On input $\langle M, w\rangle$:
1) Let A' be a turning machine constructed from A
2) Mark the ~~[illegible]~~ ~~[illegible]~~ left end of the tape as #
3) Run A' on $\langle M, w\rangle$, when # is seen move to the right ~~[illegible]~~ ~~[illegible]~~ and M attempts to reach the leftmost tape.
4) Move A' left when M accepts w
• If were to run ~~[illegible]~~ M on A, if M accepts we know that A will also accept. Therefore M ~~[illegible]~~ ~~does~~ decides L. However, when A continues to move left while on the furthest left of the tape, M will halt or accept w. Because of this we have a contradiction which proves that L is undecidable.