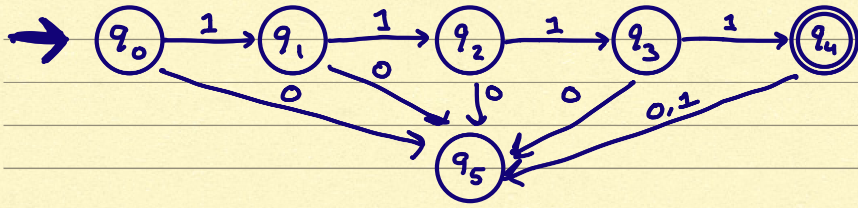
Jessica Noel & Jack Scheiderhan

Problem Set 2

"I pledge my honor that I have adibed by the Stevens Honor System"

Problem 1

a.) $L_1 = \{1111\}$

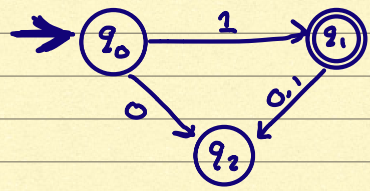


• You need 6 states because one extra state for the dead state.

b.) Induction Proof

Base Case: $K=3$

This means we need a 3-state FSA



Represents $\{1\}$
(a language of length 1)

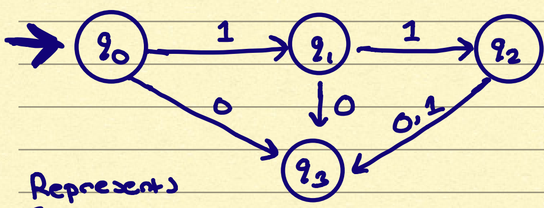
This cannot be represented with less States because then there would be either no accept state, Start state or dead State.

Inductive Hypothesis:

For $K \geq 3$ there is a language that can be represented by a K-state FSA that can't be represented by an FSA with fewer states.

Inductive Step:

Try using $K+1$, $K+1 = 3+1 = 4$ states



Represents $\{1, 1\}$ a language with a length of 2

This can't be represented with less states because there would be either no start, accept or dead state. It would also represent a different language.

• The theorem holds for both K and K+1, true. The number of states in an FSA must be greater than or equal to length + 2

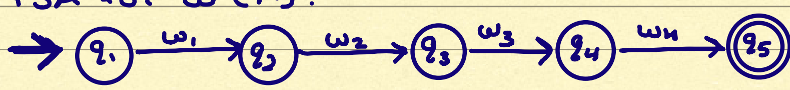
Problem 2

$A^R = \{w^R \mid w \in A\}$

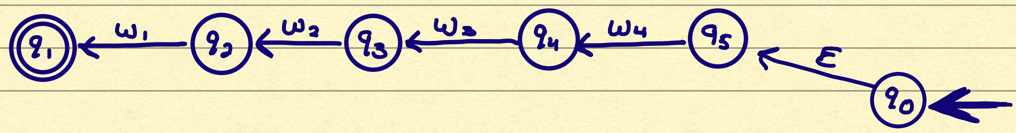
$w = w_1 w_2 w_3 w_4$ $\quad w^R = w_4\ w_3\ w_2\ w_1$

Example:

FSA for w (M):



FSA for w^R (M^R):

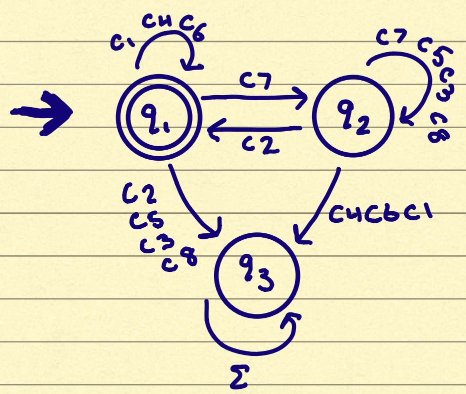


• Shown by the above graphs, if there's a path in M from q_1 to the accept state, there is the same path reversed in M^R. The epsilon in M^R shows that even with a new start state in M^R, this theorem holds, as the language is still w^R. $w \in A$ iff $w^R \in A^R$

Problem 3

$$\Sigma = \left\{ \underset{c_1}{\begin{bmatrix}0\\0\\0\end{bmatrix}}, \underset{c_2}{\begin{bmatrix}0\\0\\1\end{bmatrix}}, \underset{c_3}{\begin{bmatrix}0\\1\\0\end{bmatrix}}, \underset{c_4}{\begin{bmatrix}0\\1\\1\end{bmatrix}}, \underset{c_5}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \underset{c_6}{\begin{bmatrix}1\\0\\1\end{bmatrix}}, \underset{c_7}{\begin{bmatrix}1\\1\\0\end{bmatrix}}, \underset{c_8}{\begin{bmatrix}1\\1\\1\end{bmatrix}} \right\}$$

Represents B^R



Represents B