

CS 334 Fall 2020: Problem Set 8.

Problem 1. (15 points) Prove, using the pumping lemma for context-free languages, that the language of all palindromes over the alphabet $\{0,1\}$ in which the numbers of 0s and 1s are equal, is not context free.

Note: We will grade this problem very closely, so make sure that your argument is complete and that no details are left implicit. The problem is not hard, the reason for this exercise is for you write a complete and precise proof.

Problem 2. (10 points) Show that the class of TM-decidable languages is closed under the following operations: union, concatenation, star, intersection, and complement.

Problem 3. (10 points) Show that the class of TM-recognizable languages is closed under the following operations: union, concatenation, star, and intersection. Is it closed under complement?

Problem 4. (10 points) Show that every infinite TM-recognizable language has an infinite decidable subset.

Optional Problem 4. (10 points) A *queue automaton* is like a push-down automaton except that the stack is replaced by a queue. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a push) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a pull) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at any time. Show that a language can be recognized by a deterministic queue automaton iff the language is Turing-recognizable.