

CS 334 Fall 2020: Problem Set 5.

Problem 1. (10 points) Use the pumping lemma to show that the following languages are not regular.

- a) $\{0^i 1^j : i < j\}$
- b) $\{0^i 1^j : i > j\}$

Problem 2. (10 points) Prove that the language $B = \{0^i 1^j : i \neq j\}$ is not regular. Do not use the pumping lemma. Instead, express B as the result of regular operations between the non-regular language $\{0^i 1^i : i \geq 0\}$ and a regular language.

Problem 3. (10 points) The pumping lemma says that every regular language has a pumping length p , such that every string in the language can be pumped if it has length p or greater. If p is a pumping length for regular language A , then so is any length $p' \geq p$. The **minimum pumping length** for A is the smallest p that is a pumping length of A .

For example, the pumping length of 01^* cannot be 1 because the string $s = 0$ of length 1 cannot be pumped to give another string in the language. But any string of length 2 or more can be pumped by choosing $x = 0, y = 1$, and z to be the rest of the string.

What is the minimum pumping length for each of the following languages? Justify your answer in each case.

1. 0001^*
2. 0^*1^*
3. $0^*1^*0^*1^* \cup 10^*1$
4. $(01)^*$
5. $1^*01^*01^*$

Problem 4. (20 points)

- a) (7 points) Show that the language

$$L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = 1 \Rightarrow j = k\}$$

satisfies the three conditions of the pumping lemma. Hint: set the pumping threshold to 2 and argue that every string in L can be divided into three parts to satisfy the conditions of the pumping lemma.

- b) (8 points) Prove that L is not regular. Note that $L = b^*c^* \cup aaa^*b^*c^* \cup \{ab^i c^i : i \geq 0\}$, and use the fact that regular languages are closed under complement and difference.
- c) (5 points) Explain why parts (c) and (d) do not contradict the pumping lemma.