CS 334 Fall 2020: Problem Set 9.

Problem 1. (10 points) Show that the following language is decidable by giving a high-level description of a TM that decides the language.

$$\{< M >: M \; is \; a \; PDA \; and \; L(M) \; is \; an \; infinite \; language \}$$

Problem 2. (10 points) Let G be a context-free grammar that generates strings over the alphabet $\Sigma = \{a, b\}$. Show that the problem of determining if G generates a string in a^* is decidable. In other words, show that the following language is decidable:

$$\{< G >: G \; is \; a \; CFG \; over \; \{a, b\} \; and \; a^* \cap L(G) \neq \phi \}$$

Problem 3. (10 points) Let A be a TM-recognizable language of strings that encode TMs that are deciders. Prove that there is a decidable language which is not decided by any TM in A. (Hint: start with an enumerator for A.)

Problem 4. (10 points) Consider the problem of determining whether a TM M on input w ever attempts to move its head left when its head is on the leftmost tape cell.
a) Formulate this problem as a decision problem for a language, and
b) Show that the language is undecidable.