CS 334 Fall 2020: Problem Set 6.

Problem 1. (5 points) If G is a CFG in Chomsky Normal Form, show that a string of terminal symbols of length $n \geq 1$ is generated by the application of exactly $2n - 1$ rules of G.

Problem 2. (10 points) Let $G = (V, \Sigma, R, S)$ be a CFG where $V = \{S, T, U\}$, $\Sigma = \{0, \#\}$, and R is the set of rules:

$$S \to TT \mid U$$
$$T \to 0T \mid T0 \mid \#$$
$$U \to 0U00 \mid \#$$

Describe the language $L(G)$ in English and prove that it is not regular.

Problem 3. (10 points) Give a CFG for $\{a^i b^i c^k d^k : i, k \geq 0\} \cup \{a^i b^k c^k d^i : 1, k \geq 0\}$. Is your grammar ambiguous?

Problem 4. (15 points) Let $L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$ and $L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$. For each language, either give a CFG for it, or prove that it is not a CFL.

Optional Problem 5. (10 points) Let $\Sigma = \{a, b\}$. Give a CFG to generate all and only strings which contain twice as many $a's$ as $b's$. Give a proof that your grammar is correct.