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Problem Set 5

"I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

a.) $\{0^i 1^j : i < j\}$

Must satisfy
$|xy| \leq p$
$|y| > 0$
$\exists i, xy^iz \notin L$

Find S: $0^{p-1} 1^p$

Break into xyz: $x: \epsilon$ $y: 0^p$ $z: 1^{p+1}$

- If we pump y, we will always result with at least 0^{p+1}. $|0^{p+1}| = |1^{p+1}|$ which breaks the rule that $i < j$, so a contradiction is found. Every pump after this will also break the language as it will leave $0^{p+i} 1^{p+1}, j \geq 1$ so the language is irregular.

b.) $\{0^i 1^j : i > j\}$

Find S: $0^p 1^{p-1}$

Break into xyz: $x: \epsilon$ $y: 0^p$ $z: 1^{p-1}$

- If we make $i = 0$ then the y cannot be pumped because it will not exist. We do not have to worry about the epsilon, and in this case 1^{p-1} shows that $j > i$ which shows a contradiction. This proves that the language is irregular.

Problem 2

Prove that $B = \{0^i 1^j : i \neq j\}$ is not regular

- If B is regular, then so is $\bar{B}$, since regular languages are closed 0^*1^* under complement. Using the language which is regular, we can show that $\bar{B} \cap 0^*1^* = \{0^i1^j : 1 \geq 0\}$ which would show that B is regular. However, we know that from the given that this language is not regular, so there is a contradiction, so B is not regular.

Problem 3

Give the minimum pumping length for each of the following languages?

1) 0001^*

The minimum pumping length is 4. If $s=000$ of length 3, it can't be pumped to get another string in the language. So make $s=0001^*$, with subdivisions $x=000$ $y=1$ $z=$ The rest of the string, so the pumping length is 4.

2) 0^*1^*

The minimum pumping length is 1. If you make the division of s into $x=\epsilon$ $y=0$ or 1 and $z=$ the rest of the string. Therefore the pumping length is 1.

3) $0^*1^*0^*1^* \cup 10^*1$

The minimum pumping length is 1. For $0^*1^*0^*1^*$ you can split it up as $x=\epsilon$ $y=0$ or 1 and $z=$ is the rest of the string, so the pumping length is 1. For 10^*1

the pumping length is 3. We can split is as $x = 1$ $y = 0$ and $z = 1$. Because they are under union (OR) and we are trying to find the minimum pumping length we would choose 1 because $1 < 4$.

4) $(01)^*$

The minimum pumping length is 2. It can not be 0 since you cannot pump nothing. It can't be 1 because the language has to be a multiple of 2, and if $|xy| \leq P$, in this case $P = 1$, $|y|$ has to be equal to $1 \rightarrow x = \epsilon$ $y = 0$ which this pumped is easily not in the language of $(01)^*$. For $p = 2$ if $x = \epsilon$ $y = 01$ and $z =$ the rest of the string: this can be pumped, therefore the minimum pumping length is 2.

5) $1^*01^*01^*$

The minimum pumping length is 3. P cannot be 1 or 2, as the pumped strings could be 0 or 00, which if pumped will not be part of the given language.

If you make $p = 3$, it makes it so the pumped string y must be either 100, 010 or 101. All of these are valid to be pumped so the minimum pumping length is 3.

Problem 4

4a) $L = \{a^i b^j c^k : i, j, k \geq 0$ and $i = 1 \rightarrow j = k\}$

Satisfies the 3 conditions

The 3 conditions are:

1) For each $i \geq 0$, $xy^iz \in A$

2) $|y| > 0$

3) $|xy| \leq p$

- If the pumping lemma threshold (p) is 2, then the divisions of xyz are as follows:

  - If $i=1$, then $j=k$

    let the div → $x = \epsilon$ $y = a$ $z =$ rest of the string

    Pumping a any amount stays in language

  - If $i=2$

    Let the div → $x = \epsilon$ $y = aa$ $z =$ rest of string

    Pumping aa any amount stays in language (always stays even # of a)

  - If $i=3$

    Includes two other a's

    Let the div → $x = \epsilon$ $y = a$ $z =$ rest of string

    Pumping a in this case will add to the other as in z, so it stays in language.

  - The above case can apply to all $i \geq 3$

  - If $i=0$

    let the div → $x = \epsilon$ $y = b$ $z =$ rest of the string

    Since this pumping will always result in the form b^jc^k which stays in the language. All cases of i are covered, so the conditions of the pumping lemma are satisfied.

4b.) $L = b^*c^* \cup aab^*c^* \cup \{ab^ic^i : i \geq 0\}$

• b^*c^* and $aaa^*b^*c^*$ are regular

Set language b^*c^* as A

Set language $aaa^*b^*c^*$ as B

AUB is regular, because regular languages are closed over union. $L-(A \cup B) = \{ab^ic^i;\ i \geq 0\}$

If L & = AUB, are both regular, then so should the difference of the two (In this case $\{ab^ic^i;\ i \geq 0\}$

$S: ab^Pc^P$ $S:\ x = \epsilon \quad y = a \quad z = b^Pc^P$

• By pumping a any amount of times, you break out of the language, as the # of a's is greater than 1, so there is a contradiction, proving this language is irregular

Since this is irregular and (AUB) is regular, that proves that the language L is irregular.

4c) The pumping lemma says "If A is a regular language, then it will satisfy the conditions of the pumping lemma." Although most regular languages are put into the pumping lemma and pass, nonregular languages can also possibly satisfy the pumping lemma