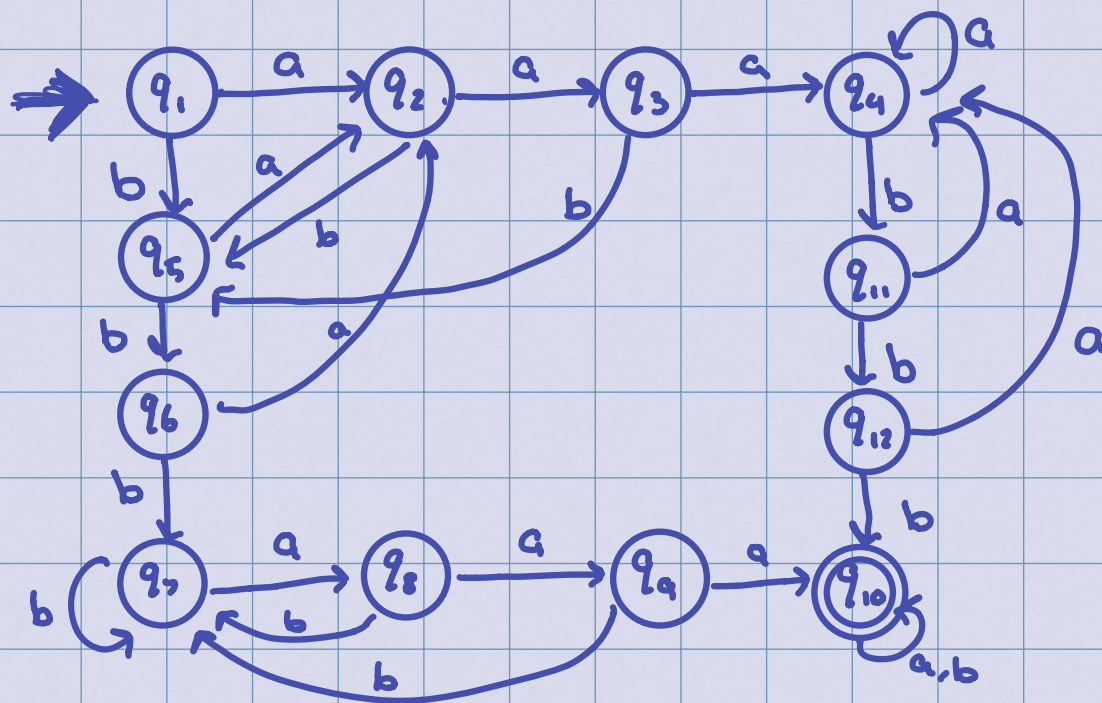


"I pledge my honor that I have abided by the Stevens honor system." *Paul*

### Problem 1

1)  $L_1 = \{w: w \text{ contains the string } aaa \text{ and the string } bbb\}$

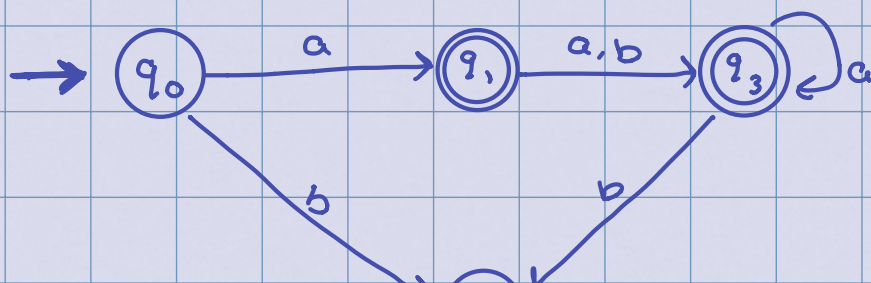


2)  $L_2 = \{w: w \text{ starts with an } a \text{ and has at most one } b\}$

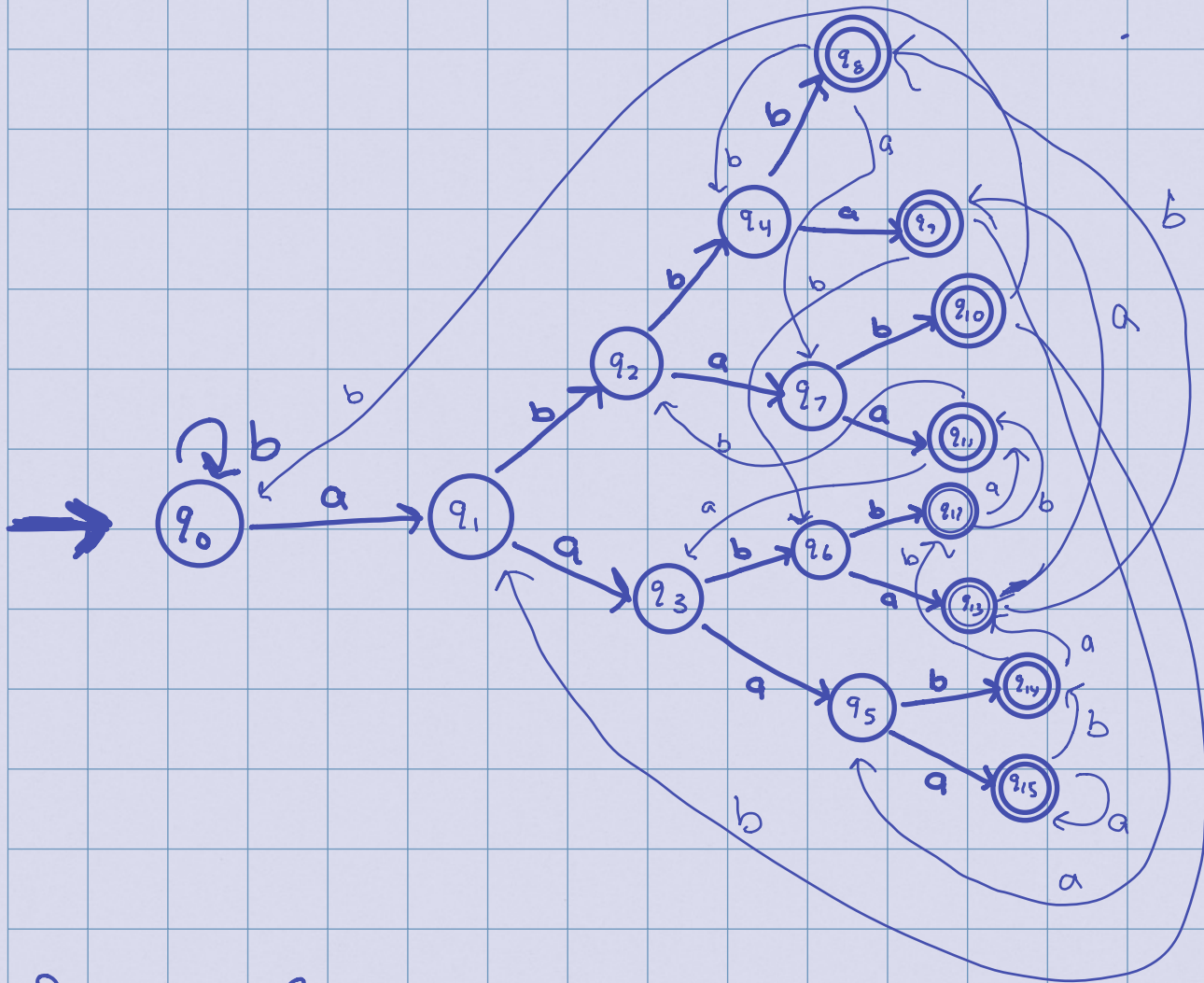
Break up to show intersection

$L_1 = \{w: w \text{ starts with an } a\}$

$L_2 = \{w: w \text{ has at most one } b\}$



3.)  $L_3 = \{w : \text{the 4th symbol from the end of } w \text{ is } a\}$



### Problem 2

$$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$$

$M$  be union of  $M_1$  &  $M_2$

$$M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$$

$$M = (Q, \Sigma, \cup \Sigma_2, \delta, q_0, F)$$

$$Q = Q_1 \times Q_2$$

$$\delta((r_1, r_2), \Sigma_1, \cap \Sigma_2) = (\delta(r_1, a), q_2)$$

↳ IF  $a$  is not in  $\Sigma_2$

$$\delta((r_1, r_2), \Sigma, \cap \Sigma_2) = (q_1, \delta(r_2, a))$$

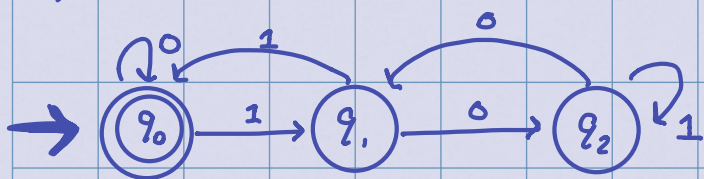
↳ IF  $a$  is not in  $\Sigma$

$$q_0 = (q_1, q_2)$$

$$F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$$

### Problem 3

1)



Example check:

$$9 = 01001 \quad \checkmark$$

$$60 = 01111100 \quad \checkmark$$

$$375 = 0101110111 \quad \checkmark$$

2.) Let  $D_1$  be the set of binary strings that represent numbers divisible by 1, and let  $D_{k+1}$  be the set of binary strings that represent numbers divisible by  $k+1$ .

$M_1 =$  finite automaton for  $D_1$

$$\hookrightarrow (Q_1, \Sigma_1, \delta_1, q_0, f_1)$$

$M_{k+1} =$  finite automaton for  $D_{k+1}$

$$\hookrightarrow (Q_{k+1}, \Sigma_{k+1}, \delta_{k+1}, q_{0,k+1}, f_{k+1})$$

In this case,  $\Sigma_1 = \Sigma_{k+1}$ , since the only strings in the alphabet is 1 & 0.

Define a new FSA:  $M = (Q, \Sigma, \delta, q_0, F)$

Theorem 1.25

$$Q = Q_1 \times Q_2$$

$$\delta = ((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$$

$$q_0 = (q_1, q_2)$$

$$F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$$

Since the union of 2 languages is closed under the union operation, and  $D_1$  &  $D_{k+1}$  have the same alphabet,

$D_{k+1}$  is regular