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"I pledge my honor that I have abided by the Stevens Honor System"

Problem 1

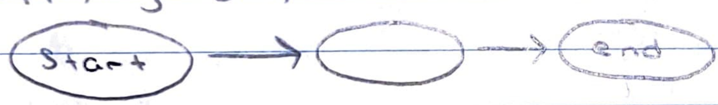
Show that a string of terminal symbols of length $n \geq 1$ is generated by the application of $2n-1$ rule of G

Two rules for CNF:

1) $A \to BC$ ← adds a variable
2) $A \to a$ ← adds a terminal, loses a variable

Let x = # of rule #1 applications
Let y = # of rule #2 applications

ex: String of length 3
applying only rule #1 $x = n-1$ times applied



then, applying just rule #2 to this, which essentially "converts" all nonterminals to terminals, $y = n$, since the rule applies to each non-terminal.
Adding $x + y$, we see that the total applications of the two rules is $2n-1$.

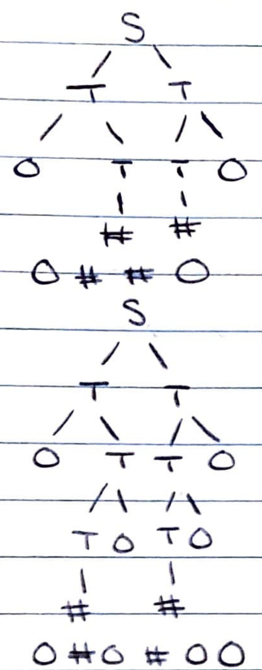
Problem 2

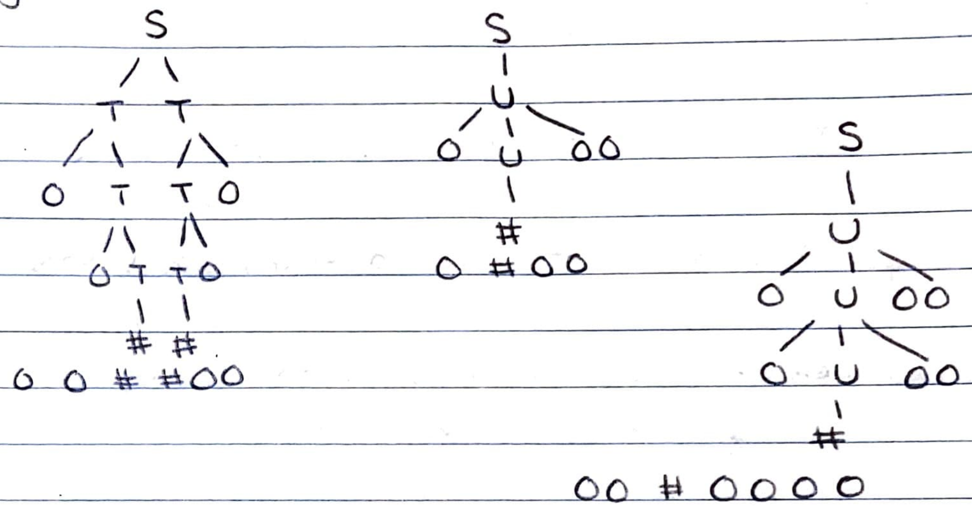
$S \to TT \mid U$ $T \to 0T \mid T0 \mid \#$ $U \to 0U00 \mid \#$

L(G) is a language that takes two "forms" depending on which path you go down from the start variable.
If you go down with TT at first, the language can be describe as "L_1 = (in english) the string contains an # of zeros and 2 hashtags. The order of these varies, but there are always 0s at the start and finish.
While if you choose U the language is $L_2 = \{0^i \# 0^{2i} \mid i \geq 0\}$

in english: the String contains n 0s, followed by a #, followed by 2n 0s. This option is much more constant and "organized"

Some example strings:





The first language L_1 cannot be described by a regular expression (in fact, can't be described by a language expression easily) so its not regular. L_2 is also irregular, as its language is $\{0^i \# 0^{2i}\}$, since $L = L_1 \cup L_2$ and non-regular languages are closed under union, then this language is irregular.

Problem 3

Give a CFG for $\{a^i b^i c^k d^k : i, k \geq 0\} \cup \{a^i b^k c^k d^i : 1, k \geq 0\}$

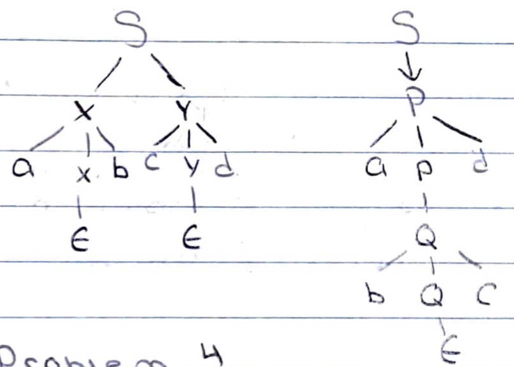
The nonterminal Symbols, N, are $\{a, b, c, d\}$

The set of terminals $\{S, X, Y, P, Q\}$

$S \rightarrow XY \mid P$ $\quad$ $Y \rightarrow cYd \mid \epsilon$ $\quad$ $Q \rightarrow bQc \mid \epsilon$

$X \rightarrow aXb \mid \epsilon$ $\quad$ $P \rightarrow aPd \mid Q \mid \epsilon$

The grammar is amigious because S does not have a unique leftmost derivation/parse tree. As shown above $S \rightarrow XY \mid P$, meaning S can go to XY or P.



Problem 4

$L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$ $\quad$ $L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$

The 3 conditions to satisfy a CFL are:

1) $uv^i xy^i z$ is in A for every $i \geq 0$
2) $|vy| > 0$
3) $|vxy| \leq P$

For condition 1: $(uv^i xy^i z \in L, \forall\ i \geq 0)$

If we make $i = 2$ $\rightarrow uv^2 xy^2 z \in L, i = 2$

If $U = a$ $\quad V = ab$ $\quad X = bb$ $\quad Y = bc$ $\quad Z = c$

When we pump v and y both twice, we reach aababbbbcbcc. Since an a appears after ab and ac before ab, the String leaves the language, proving that Ladd is not a CFL. For $L_{mult} = \{a^i b^{i \cdot j} c^j : i, j \geq 0\}$ Utilizing the same method as before: Set $i = 2$, so $uv^2 xy^2 z$. Let the division be: $u=a$ $v=ab$ $x=bb$ $y=bc$ $z=c$

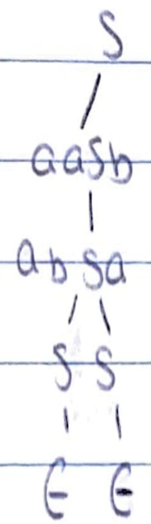
and pump v and y twice: aababbbbcbcc

(Same as Ladd, since add & mult act the same when $i=j=2$. For the same reason of an a appearing after a b and a c before a b, this string is **NOT** in the languages, so Lmult is not a CFL.

5) Let $\Sigma = \{a, b\}$. Give a CFG to gen. all & only strings that contain 2x as as bs.

$S \rightarrow SS \mid aaSb \mid abSa \mid baSa \mid \epsilon$

Proof that this works:



generates:

aaabbaaab

Looking at this parse tree, its clear that it generates a string that contains 2(S-1) as and (S-1) bs and with this general format it shows that the language holds for all S.