Supervised Learning

Neural Networks

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Introduction

Acknowledgements

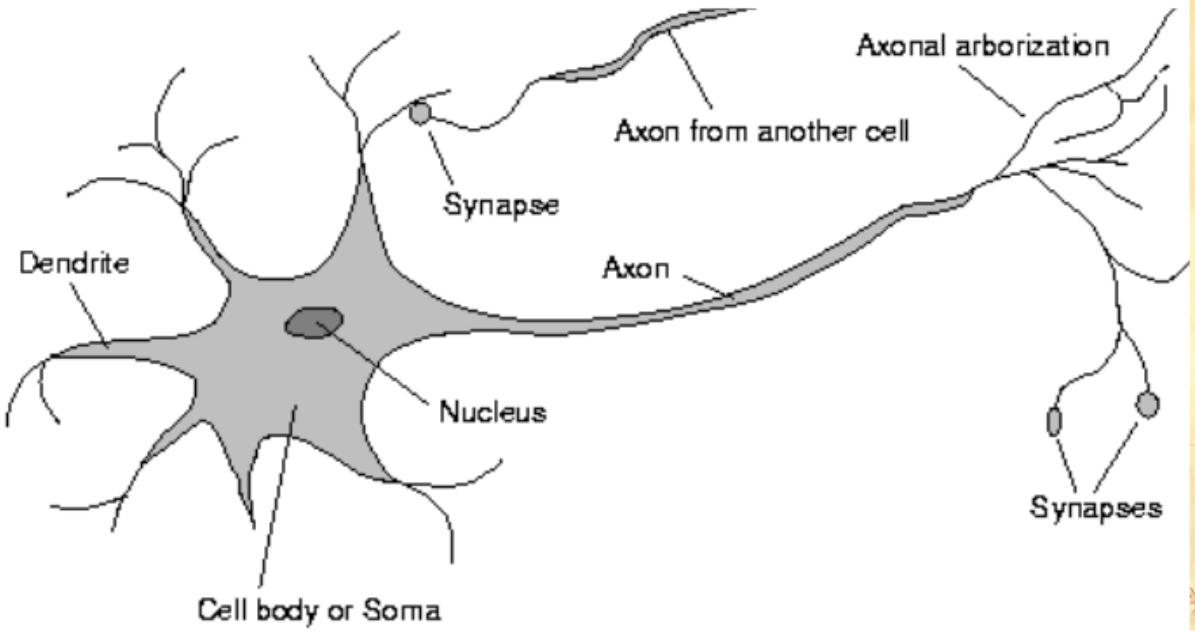
These slides have been adapted from the following Professors:

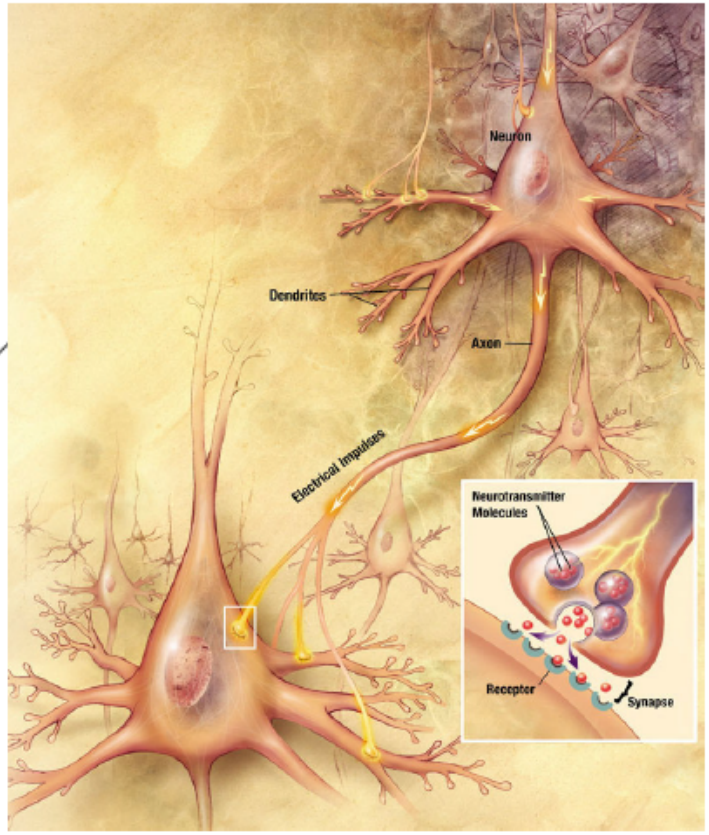
1) Andrew Ng - Stanford
2) Eric Eaton - UPenn
3) David Sontag - MIT
4) Alina Oprea - Northeastern

Neural Function

- Brain function (thought) occurs as the result of the firing of **neurons**
- Neurons connect to each other through **synapses**, which propagate **action potential** (electrical impulses) by releasing **neurotransmitters**
  - Synapses can be **excitatory** (potential-increasing) or **inhibitory** (potential-decreasing), and have varying **activation thresholds**
  - Learning occurs as a result of the synapses' **plasticicity**: They exhibit long-term changes in connection strength
- There are about 10^{11} neurons and about 10^{14} synapses in the human brain!

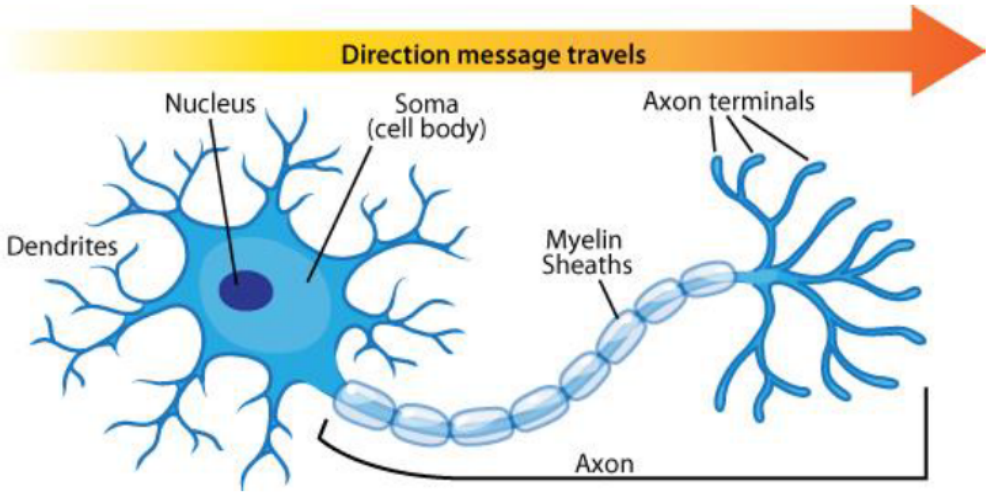
Biology of a Neuron



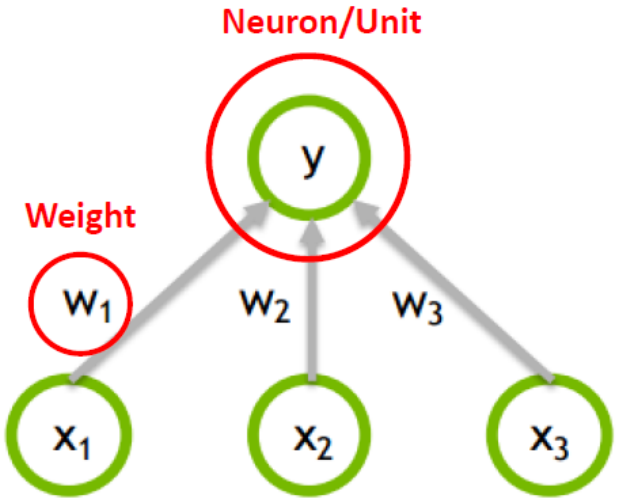


Analogy to Human Brain

Human Brain



Biological Neuron



$$y=F(w_1x_1+w_2x_2+w_3x_3)$$

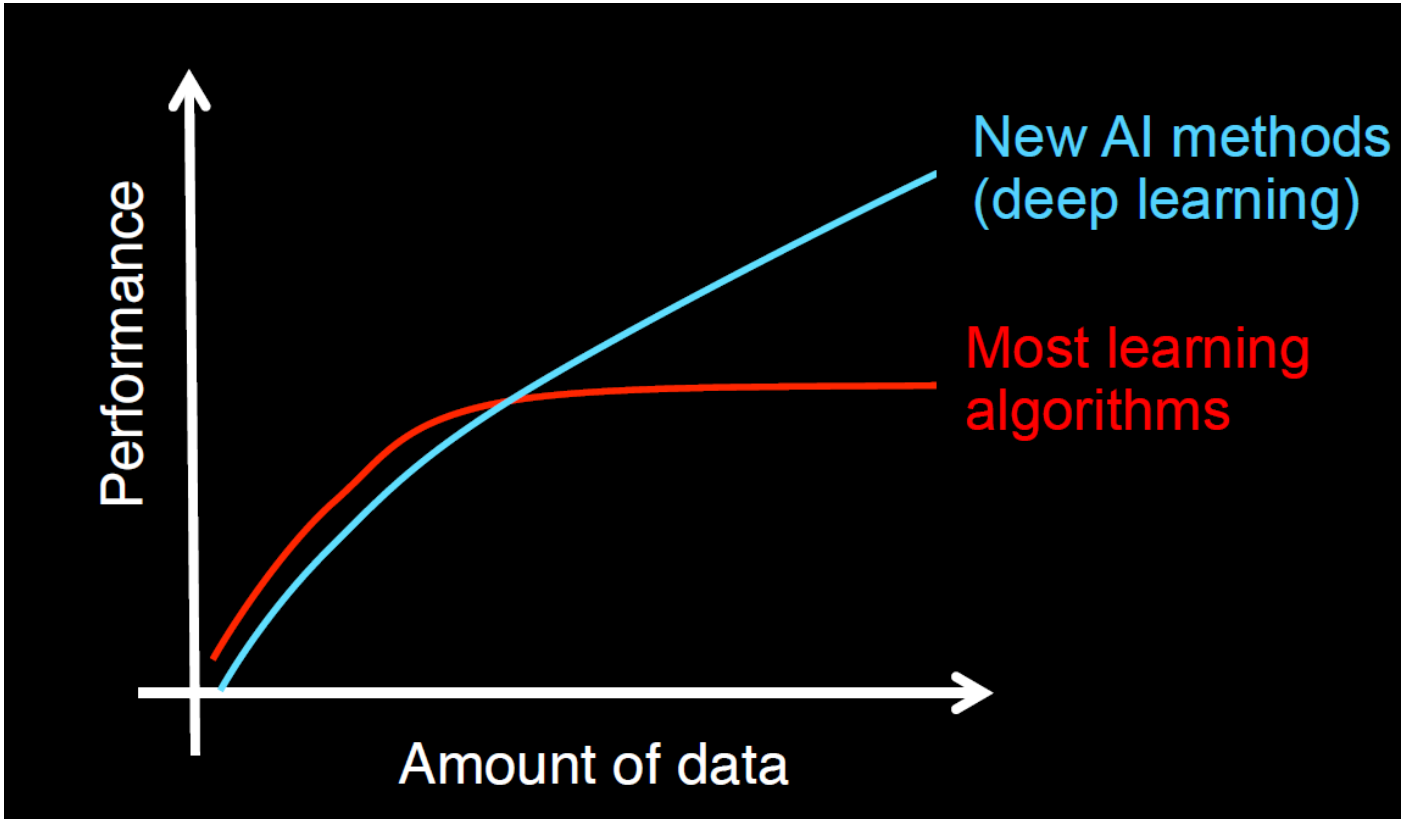
$$F(x)=\max(0,x)$$

Artificial Neuron

Neural Networks

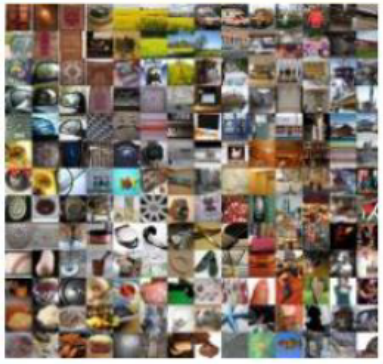
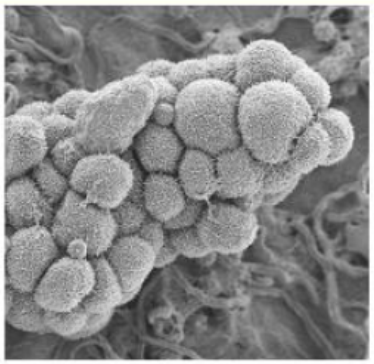
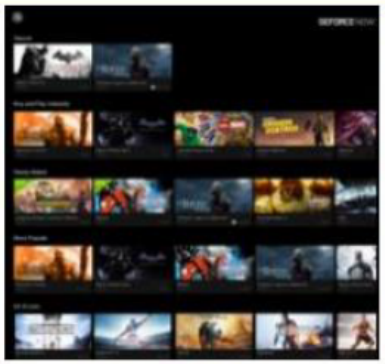
- Origins: Algorithms that try to mimic the brain.
- Very widely used in 80s and early 90s; popularity diminished in late 90s.
- Recent resurgence: State-of-the-art technique for many applications
- Artificial neural networks are not nearly as complex or intricate as the actual brain structure

Performance of Deep Learning



Deep Learning Applications

DEEP LEARNING EVERYWHERE

INTERNET & CLOUD

Image Classification
Speech Recognition
Language Translation
Language Processing
Sentiment Analysis
Recommendation

MEDICINE & BIOLOGY

Cancer Cell Detection
Diabetic Grading
Drug Discovery

MEDIA & ENTERTAINMENT

Video Captioning
Video Search
Real Time Translation

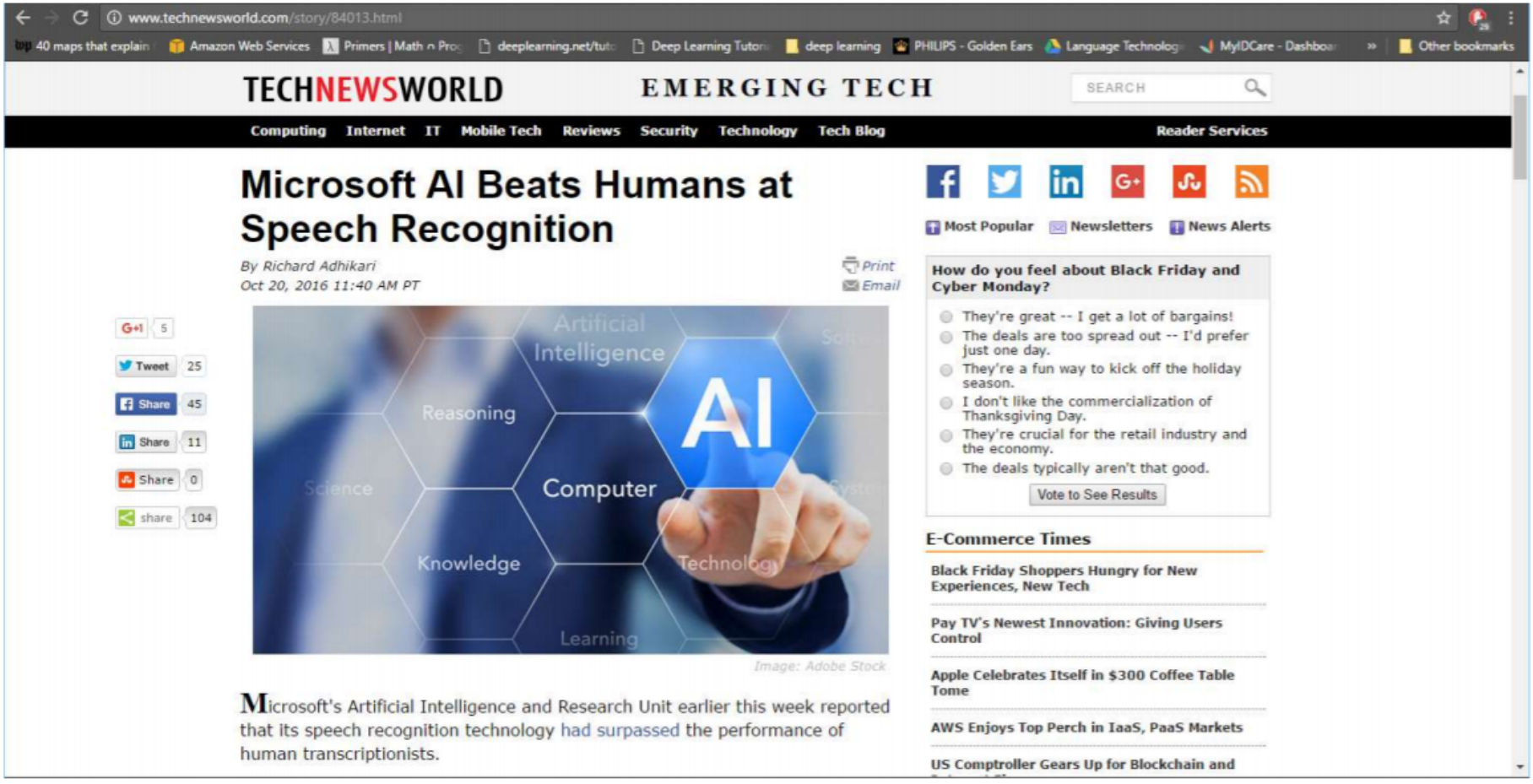
SECURITY & DEFENSE

Face Detection
Video Surveillance
Satellite Imagery

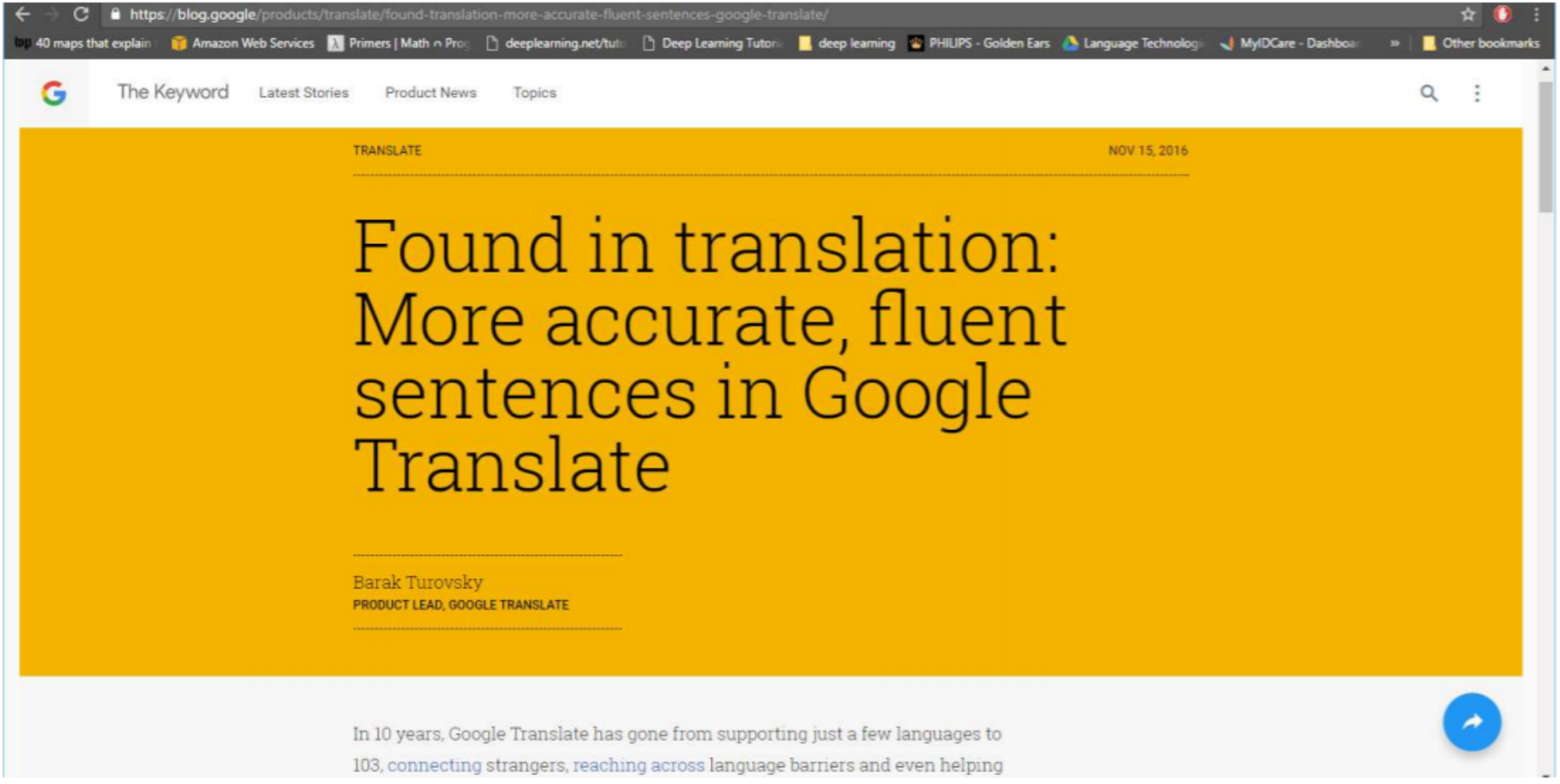
AUTONOMOUS MACHINES

Pedestrian Detection
Lane Tracking
Recognize Traffic Sign

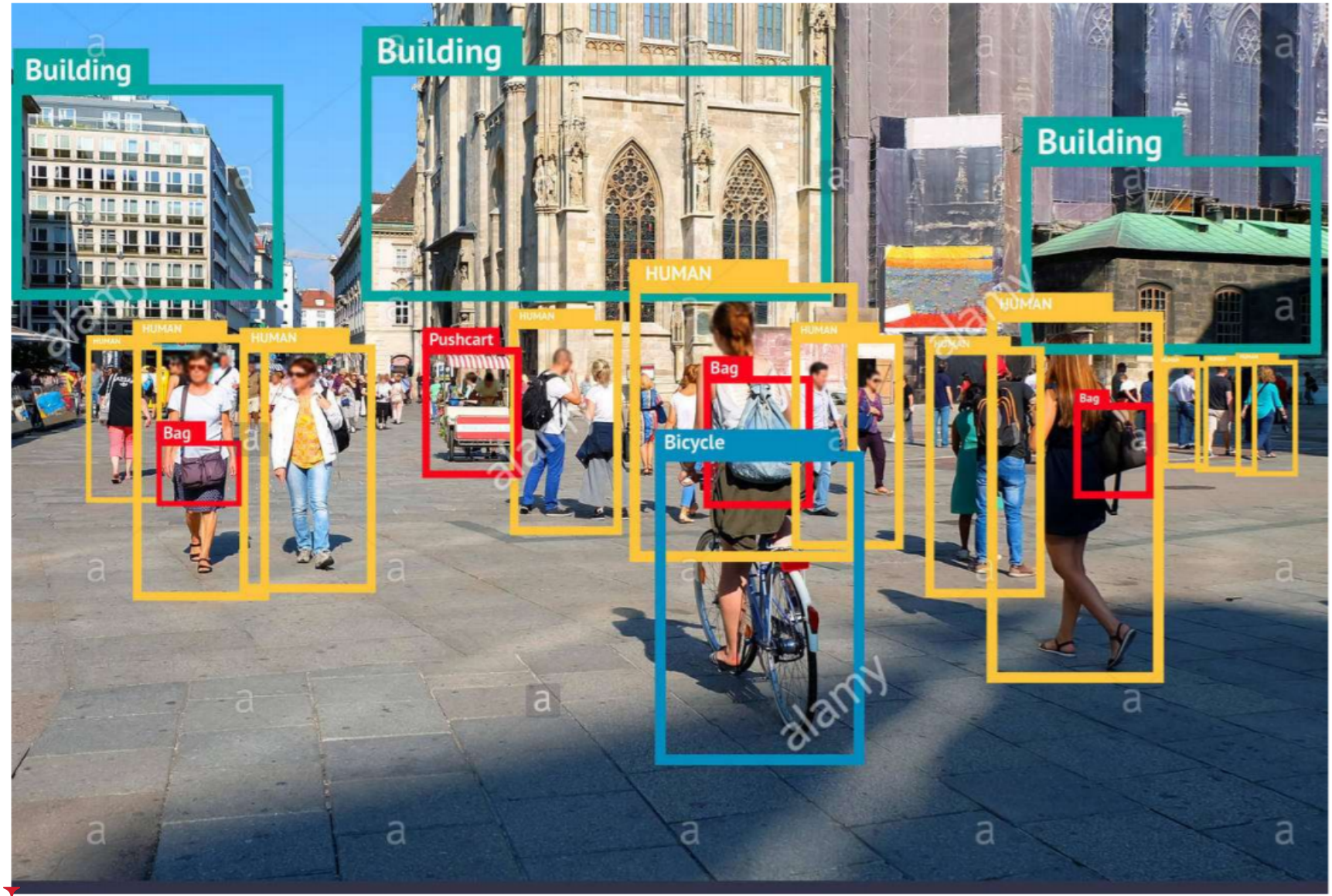
Success stories: Speech recognition



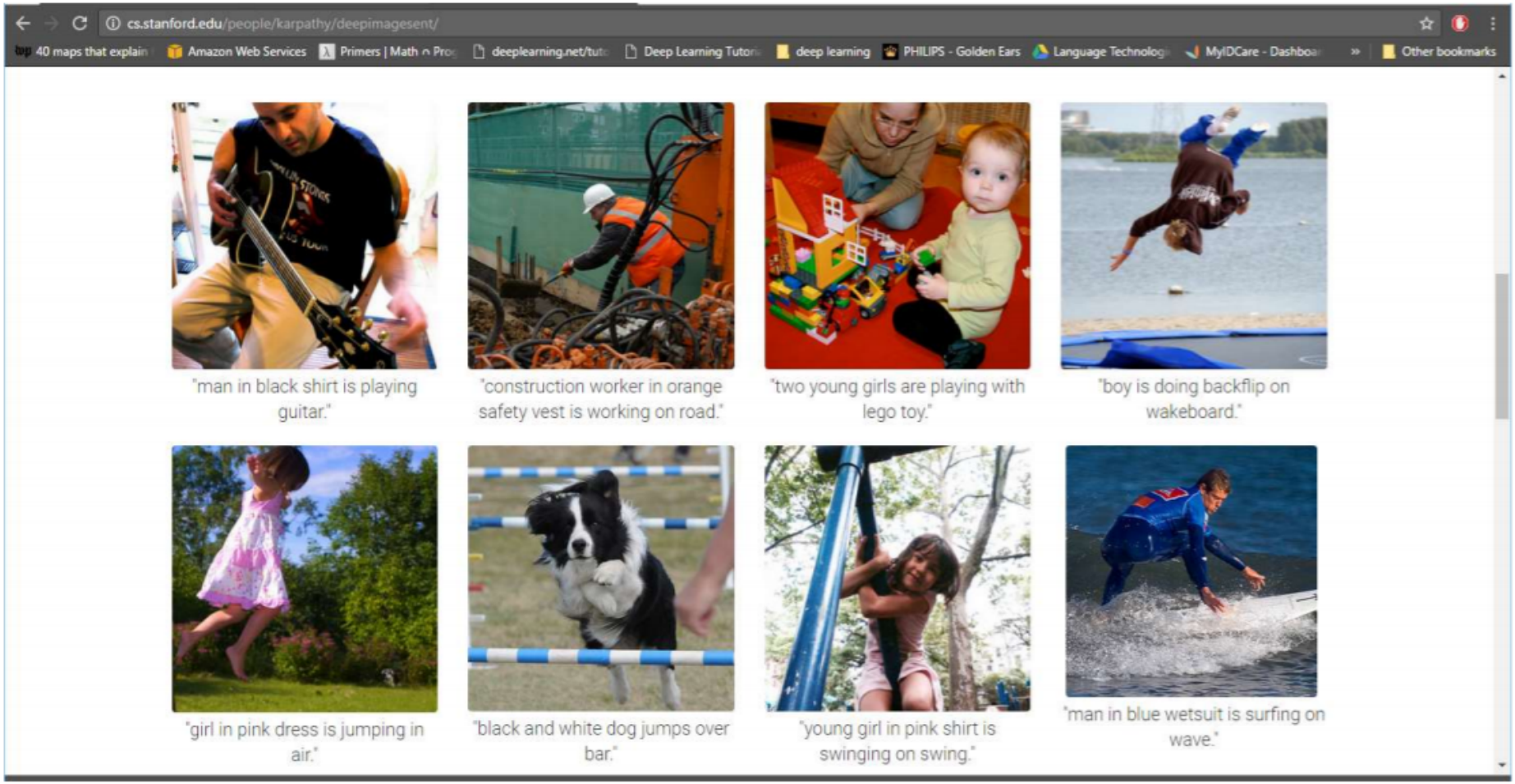
Success stories: Machine Translation



Success stories: Image segmentation



Success stories: Image captioning



References

- Deep Learning books
  - https://d2l.ai/ (D2L)
  - https://www.deeplearningbook.org/ (advanced)
- Stanford notes on deep learning
  - http://cs229.stanford.edu/notes/cs229-notes-deep_learning.pdf
- History of Deep Learning
  - https://beamandrew.github.io/deeplearning/2017/02/23/deep_learning_101_part1.html

Example

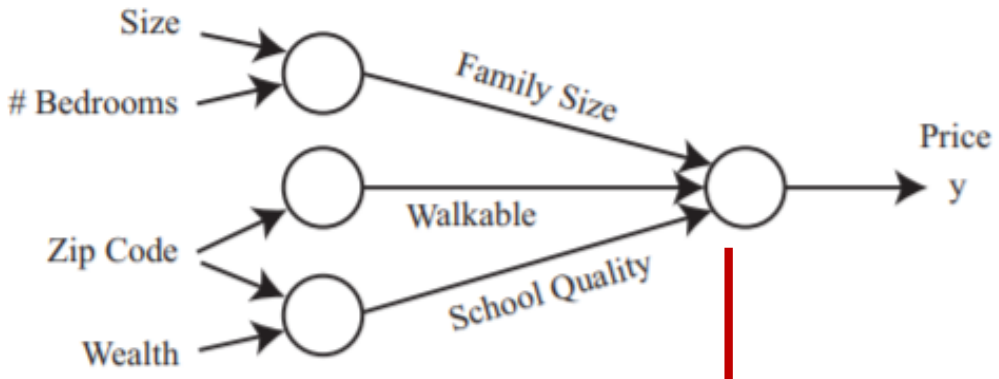
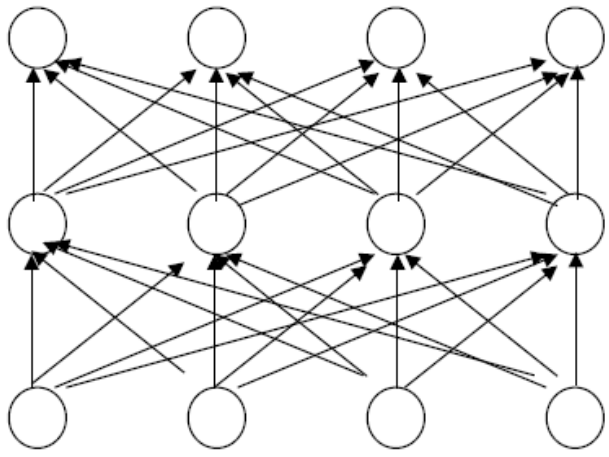


Figure 2: Diagram of a small neural network for predicting housing prices.

Intermediate Features

- Provide as input only training data: input and label
- Neural Networks automatically learn intermediate features!

Neural Networks



Layered feed-forward network

Training labels

Learned during training

Training data

- Neural networks are made up of **nodes** or **units**, connected by **links**
- Each link has an associated **weight** and **activation level**
- Each node has an **input function** (typically summing over weighted inputs), an **activation function**, and an **output**

Recall: The Perceptron

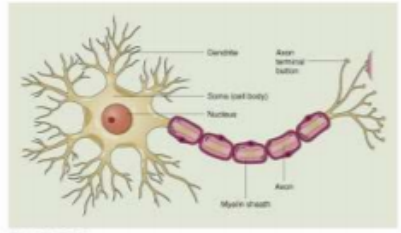
$$h(\boldsymbol{x}) = \text{sign}(\boldsymbol{\theta}^\mathsf{T}\boldsymbol{x}) \quad \text{where} \quad \text{sign}(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{if } z < 0 \end{cases}$$

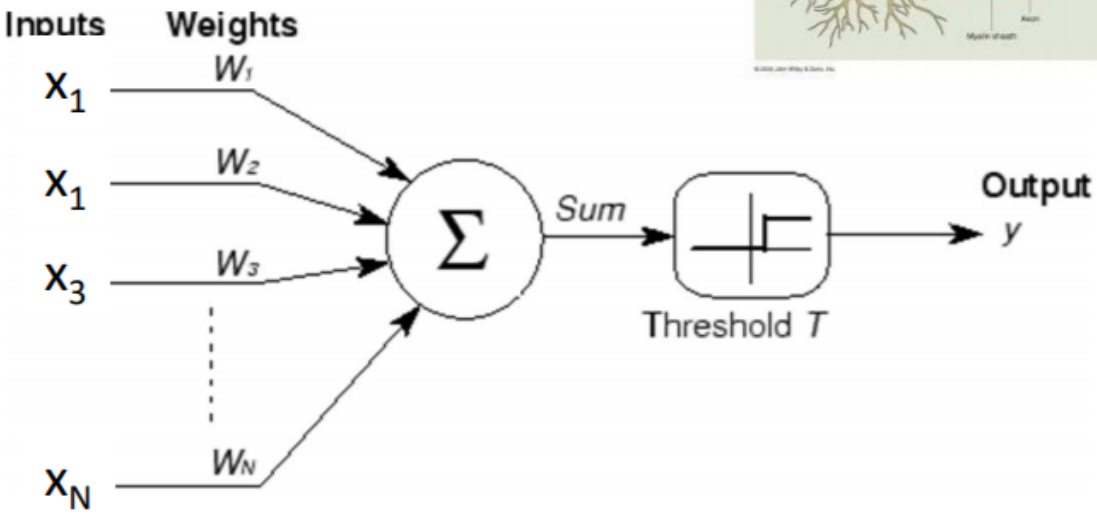
- The perceptron uses the following update rule each time it receives a new training instance $(\boldsymbol{x}^{(i)}, y^{(i)})$

$$\theta_j \leftarrow \theta_j - \frac{1}{2}\underbrace{\left(h_{\boldsymbol{\theta}}\left(\boldsymbol{x}^{(i)}\right) - y^{(i)}\right)}_{\text{either 2 or -2}} x_j^{(i)}$$

  - If the prediction matches the label, make no change
  - Otherwise, adjust $\boldsymbol{\theta}$

Perceptron

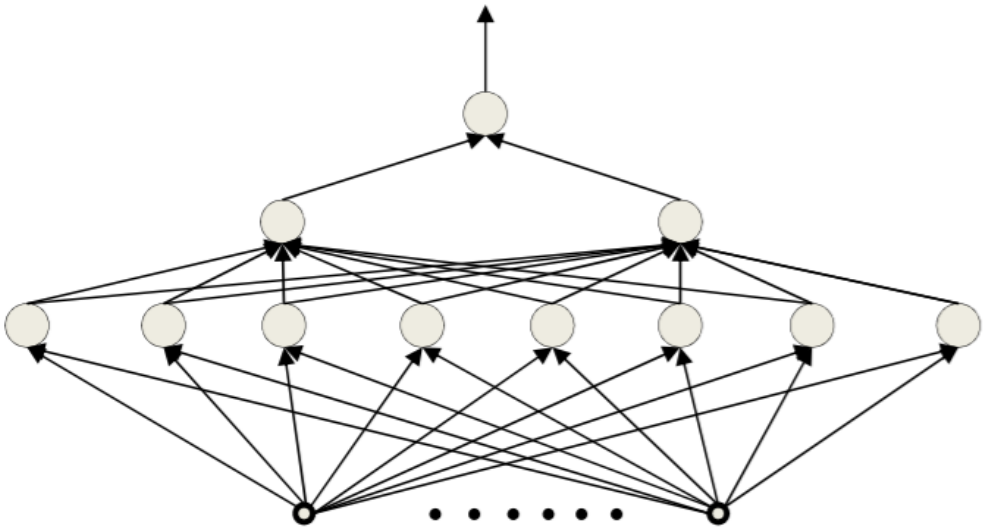


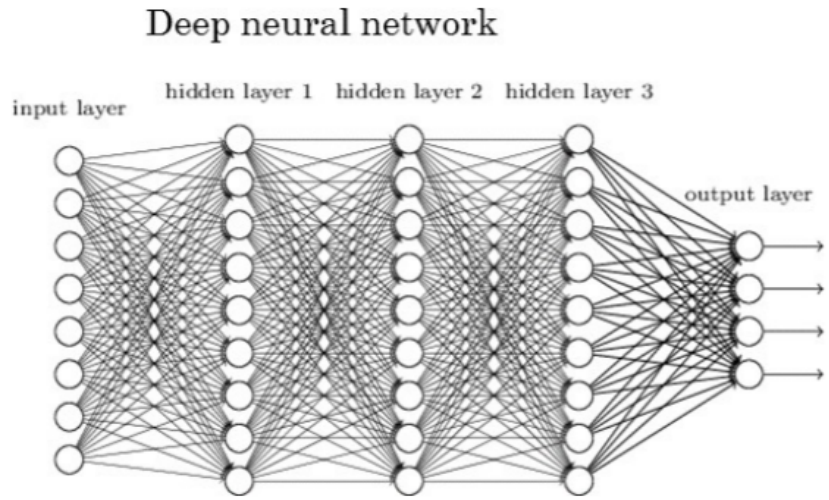


$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$

- A threshold unit
  - "Fires" if the weighted sum of inputs exceeds a threshold

Multi-Layer Perceptron



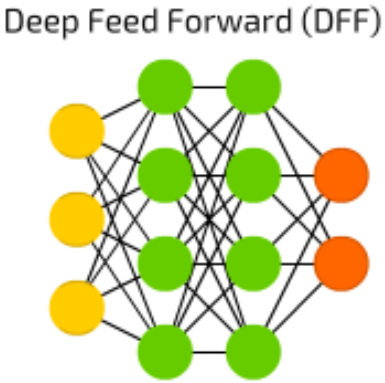


- A network of perceptrons
  - Generally “layered”

Neural Network Architectures

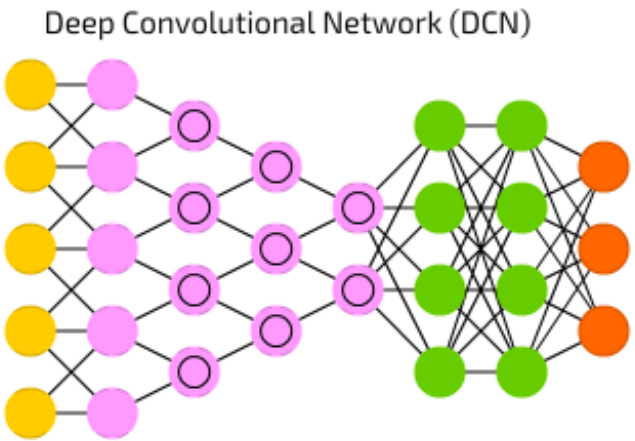
Feed-Forward Networks

- Neurons from each layer connect to neurons from next layer



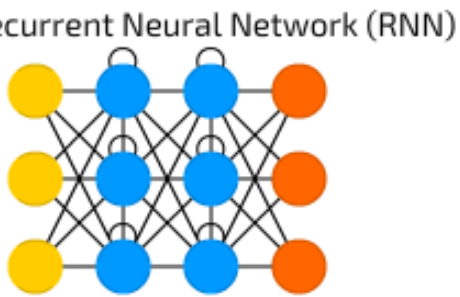
Convolutional Networks

- Includes convolution layer for feature reduction
- Learns hierarchical representations

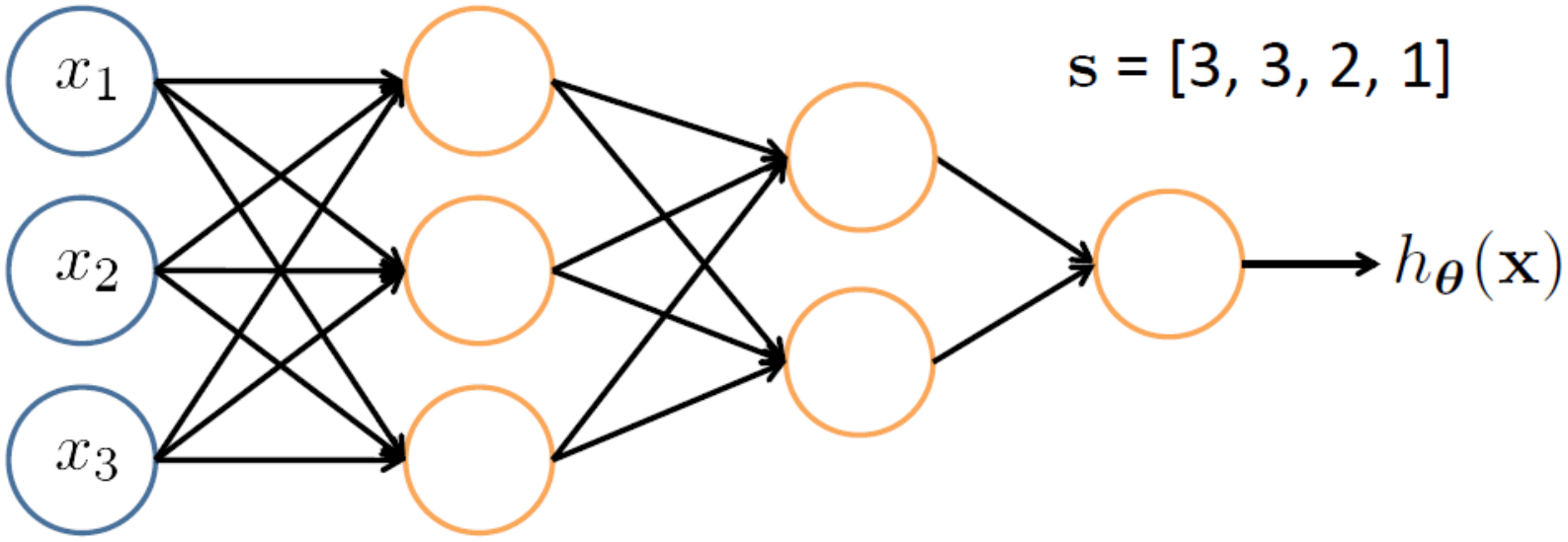


Recurrent Networks

- Keep hidden state
- Have cycles in computational graph



Feed-Forward Networks



L denotes the number of layers

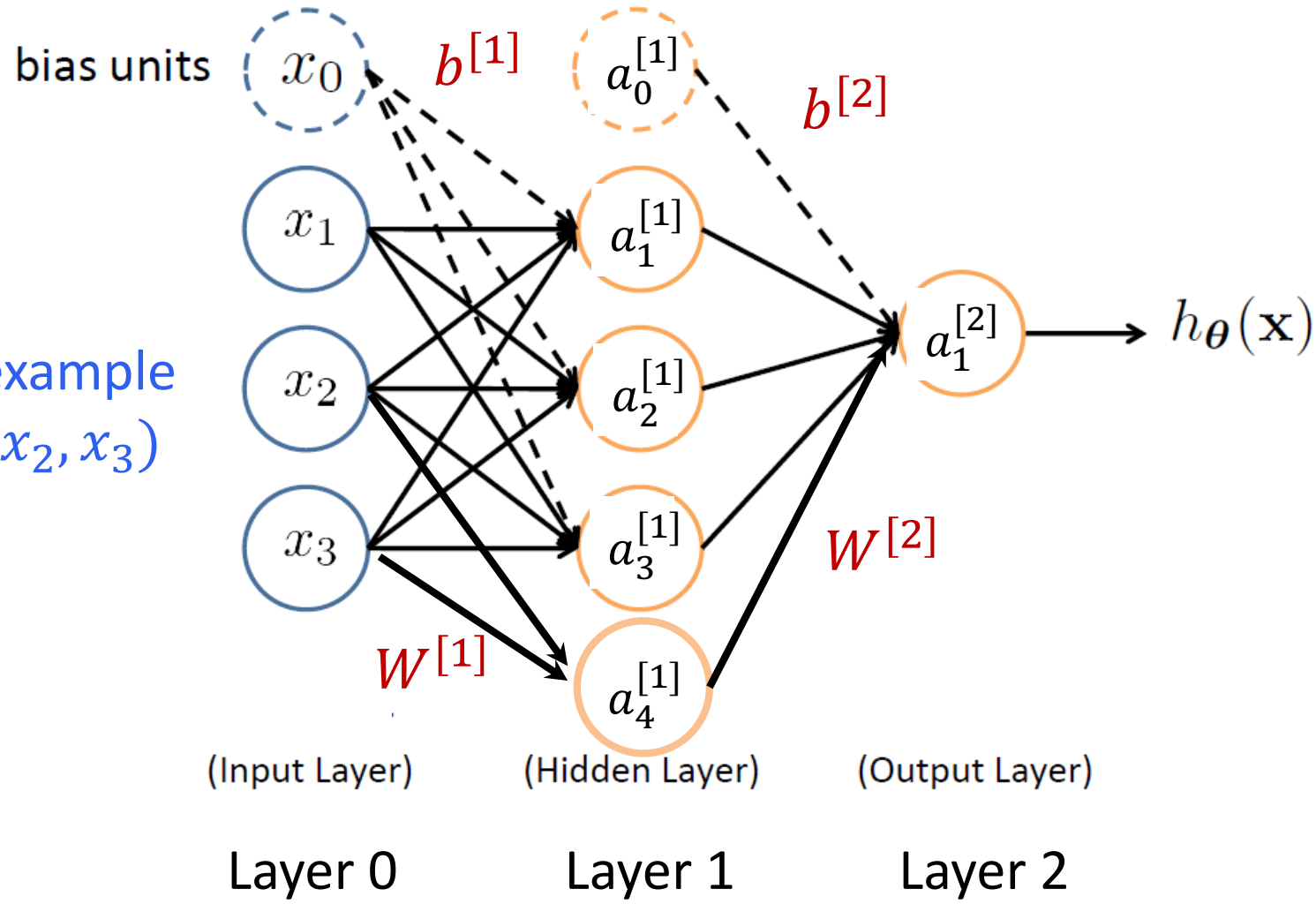
$\mathbf{s} \in \mathbb{N}^{+L}$ contains the numbers of nodes at each layer

- Not counting bias units
- Typically, $s_0 = d$ (# input features) and $s_{L-1} = K$ (# classes)

Feed-Forward NN

- Hyper-parameters
  - Number of layers
  - Architecture (how layers are connected)
  - Number of hidden units per layer
  - Number of units in output layer
  - Activation functions
- Other
  - Initialization
  - Regularization

Feed-Forward Neural Network



Training example
$x = (x_1, x_2, x_3)$

No cycles

$\theta = (b^{[1]}, W^{[1]}, b^{[2]}, W^{[2]})$

Vectorization

$$z_1^{[1]} = {W_1^{[1]}}^T x + b_1^{[1]} \quad \text{and} \quad a_1^{[1]} = g(z_1^{[1]})$$
$$\vdots \qquad\qquad\qquad \vdots \qquad\qquad \vdots$$
$$z_4^{[1]} = {W_4^{[1]}}^T x + b_4^{[1]} \quad \text{and} \quad a_4^{[1]} = g(z_4^{[1]})$$

$$\underbrace{\begin{bmatrix} z_1^{[1]} \\ \vdots \\ z_4^{[1]} \end{bmatrix}}_{z^{[1]} \in \mathbb{R}^{4\times 1}} = \underbrace{\begin{bmatrix} — \; {W_1^{[1]}}^T \; — \\ — \; {W_2^{[1]}}^T \; — \\ \vdots \\ — \; {W_4^{[1]}}^T \; — \end{bmatrix}}_{W^{[1]} \in \mathbb{R}^{4\times 3}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x \in \mathbb{R}^{3\times 1}} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ \vdots \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]} \in \mathbb{R}^{4\times 1}}$$

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

Linear

$$a^{[1]} = g(z^{[1]})$$

Non-Linear

Vectorization

Output layer

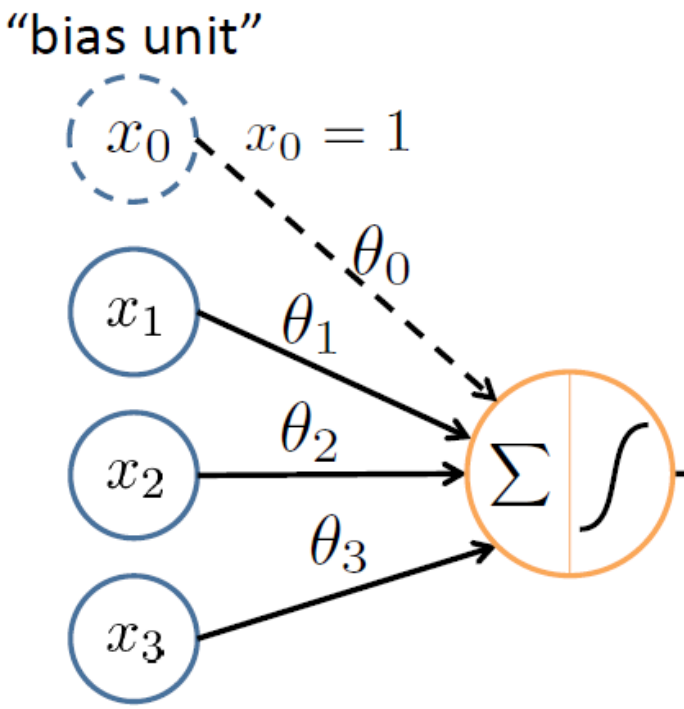
$$z_1^{[2]} = W_1^{[2]^T} a^{[1]} + b_1^{[2]} \quad \text{and} \quad a_1^{[2]} = g(z_1^{[2]})$$

$$\underbrace{z^{[2]}}_{1\times1} = \underbrace{W^{[2]}}_{1\times4}\underbrace{a^{[1]}}_{4\times1} + \underbrace{b^{[2]}}_{1\times1} \quad \text{and} \quad \underbrace{a^{[2]}}_{1\times1} = g(\underbrace{z^{[2]}}_{1\times1})$$

Hidden Units

- Layer 1
  - First hidden unit:
    - Linear: $z_1^{[1]} = \mathrm{W}_1^{[1]\,\mathrm{T}} x + \mathrm{b}_1^{[1]}$
    - Non-linear: $a_1^{[1]} = \mathrm{g}(\mathrm{z}_1^{[1]})$
  - …
  - Fourth hidden unit:
    - Linear: $z_4^{[1]} = \mathrm{W}_4^{[1]\,\mathrm{T}} x + \mathrm{b}_4^{[1]}$
    - Non-linear: $a_4^{[1]} = \mathrm{g}(\mathrm{z}_4^{[1]})$
- Terminology
  - $a_i^{[j]}$ - Activation of unit i in layer j
  - g - Activation function
  - $W^{[j]}$ - Weight vector controlling mapping from layer j-1 to j
  - $b^{[j]}$ - Bias vector from layer j-1 to j

Logistic Unit: A simple NN



$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

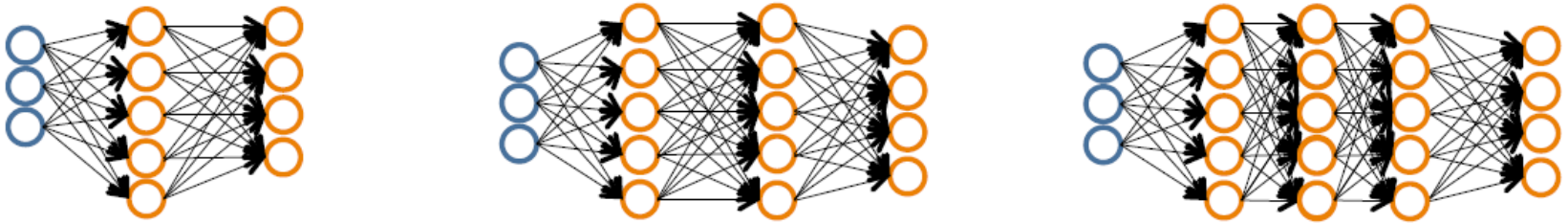
$$h_{\boldsymbol{\theta}}(\mathbf{x}) = g\left(\boldsymbol{\theta}^{\mathsf{T}}\mathbf{x}\right) = g(w^T x + b)$$
$$= \frac{1}{1 + e^{-\boldsymbol{\theta}^{\mathsf{T}}\mathbf{x}}}$$

Sigmoid (logistic) activation function: $g(z) = \dfrac{1}{1 + e^{-z}}$

No hidden layers

How to pick architecture?

Pick a network architecture (connectivity pattern between nodes)



- # input units = # of features in dataset
- # output units = # classes

Reasonable default: 1 hidden layer

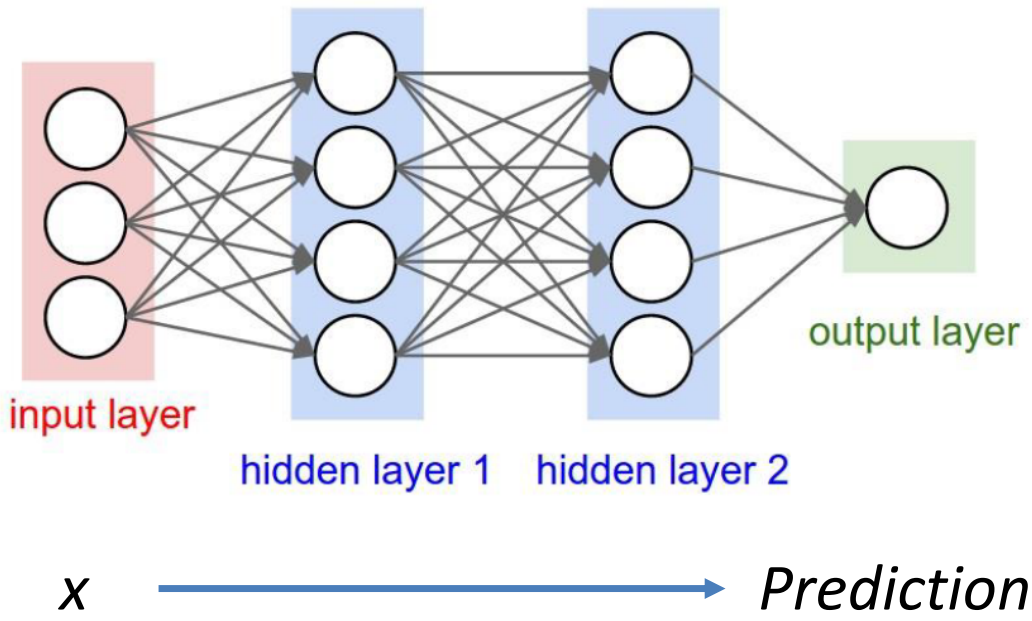
- or if >1 hidden layer, have same # hidden units in every layer (usually the more the better)

Training Neural Networks

- Input training dataset D
  - Number of features: d
  - Labels from K classes
- First layer has d+1 units (one per feature and bias)
- Output layer has K units
- Training procedure determines parameters that optimize loss function
  - Backpropagation
  - Learn optimal $W^{[i]}, b^{[i]}$ at layer i
- Testing done by forward propagation

Forward Propagation

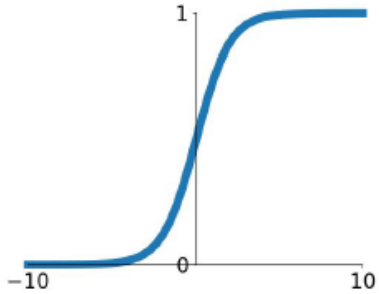
- The input neurons first receive the data features of the object. After processing the data, they send their output to the first hidden layer.
- The hidden layer processes this output and sends the results to the next hidden layer.
- This continues until the data reaches the final output layer, where the output value determines the object's classification.
- This entire process is known as Forward Propagation, or Forward prop.



Activation Functions

Sigmoid

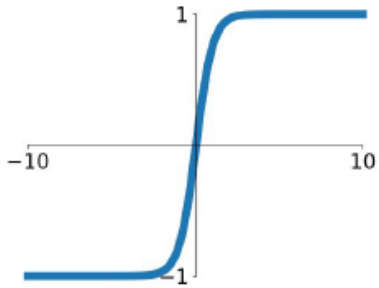
$$\sigma(x) = \frac{1}{1+e^{-x}}$$



Binary Classification

tanh

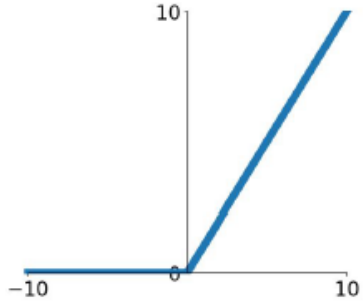
$$\tanh(x)$$



Regression

ReLU

$$\max(0, x)$$



Intermediary layers

Why Non-Linear Activations?

- Assume g is linear: $\mathrm{g}(z) = Uz$
  - At layer 1: $z^{[1]} = \mathrm{W}^{[1]} x + \mathrm{b}^{[1]}$
  - $a^{[1]} = \mathrm{g}\left(\mathrm{z}^{[1]}\right) = \mathrm{Uz}^{[1]} = \mathrm{UW}^{[1]}\mathrm{x} + \mathrm{Ub}^{[1]}$
- Layer 2:
  - $a^{[2]} = \mathrm{g}\left(\mathrm{z}^{[2]}\right) = \mathrm{Uz}^{[2]} = \mathrm{UW}^{[2]}\mathrm{a}^{[1]} + \mathrm{Ub}^{[2]} =$
  - $= \mathrm{UW}^{[2]}\,\mathrm{UW}^{[1]} x + \mathrm{UW}^{[2]}\mathrm{Ub}^{[1]} + \mathrm{Ub}^{[2]}$
- Last layer
  - Output is linear in input!
  - Then NN will only learn linear functions

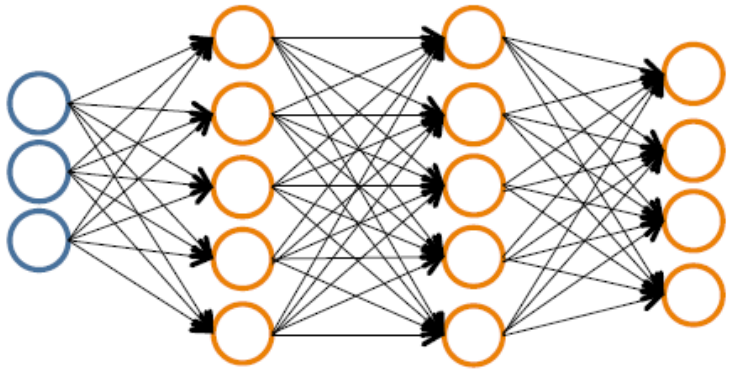
Multiple Output Units: One-vs-Rest

Pedestrian

Car

Motorcycle

Truck



$$h_\Theta(\mathbf{x}) \in \mathbb{R}^K$$

We want:

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

when pedestrian

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

when car

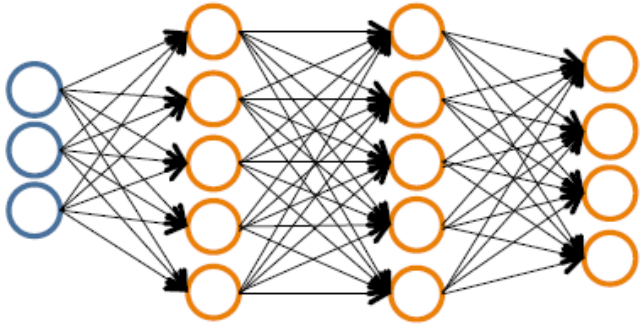
$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

when motorcycle

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

when truck

Multiple Output Units: One-vs-Rest



$$h_\Theta(\mathbf{x}) \in \mathbb{R}^K$$

We want:

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ when pedestrian

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$ when car

$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$ when motorcycle

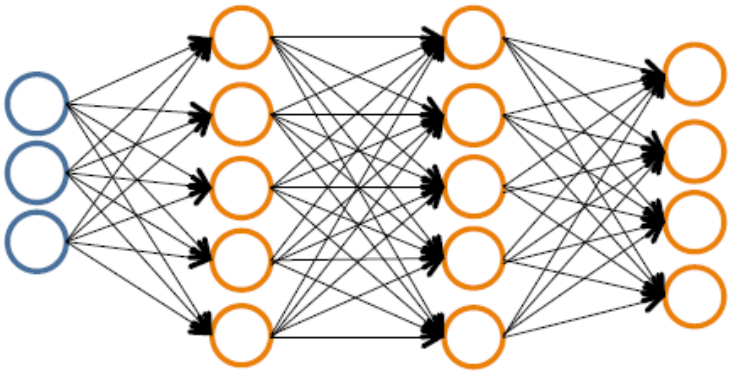
$$h_\Theta(\mathbf{x}) \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ when truck

---

- Given $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)\}$
- Must convert labels to 1-of-K representation
  - e.g., $\mathbf{y}_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ when motorcycle, $\mathbf{y}_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ when car, etc.

Based on slide by Andrew Ng

Neural Network Classification



Given:

$\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)\}$

$\mathbf{s} \in \mathbb{N}^{+L}$ contains # nodes at each layer

- $s_0 = d$ (# features)

Binary classification

y = 0 or 1

1 output unit $(s_{L-1} = 1)$

Sigmoid

Multi-class classification (K classes)

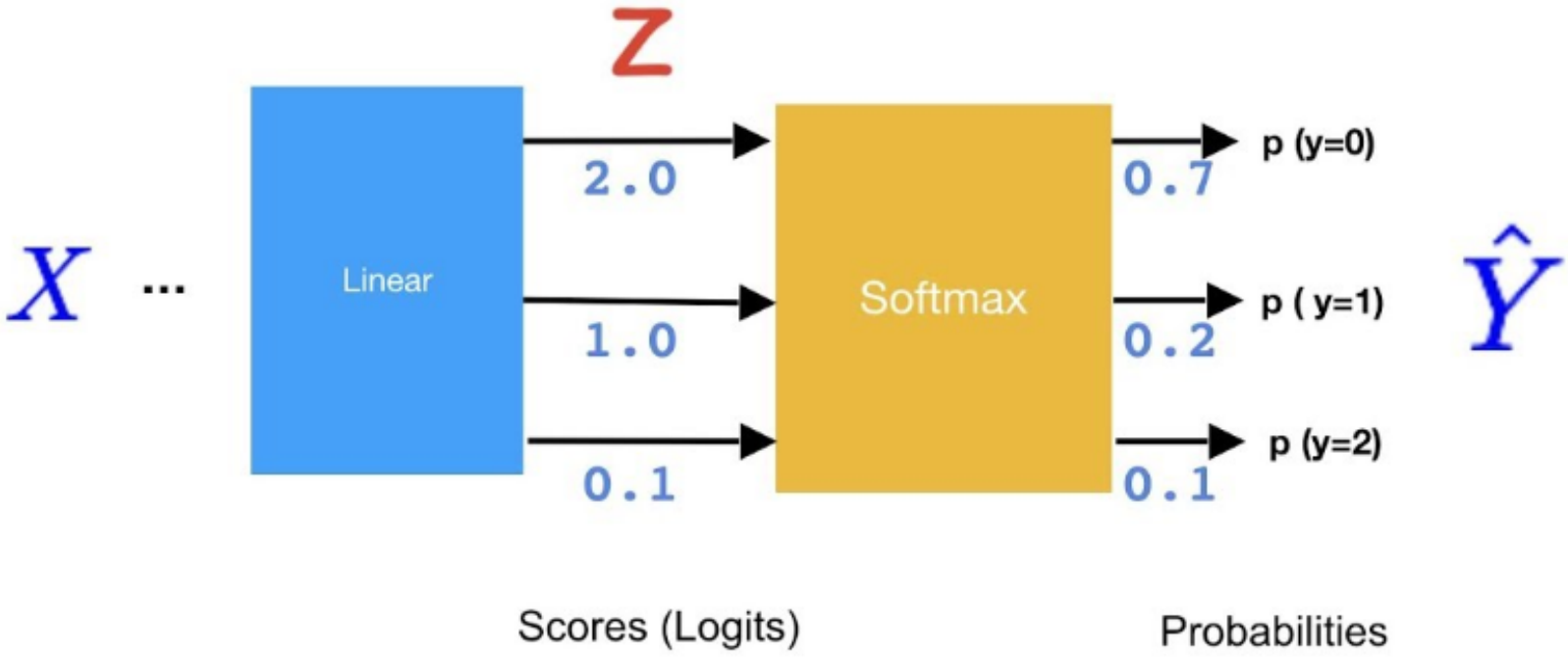
$\mathbf{y} \in \mathbb{R}^K$ e.g. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

pedestrian car motorcycle truck

K output units $(s_{L-1} = K)$

Softmax

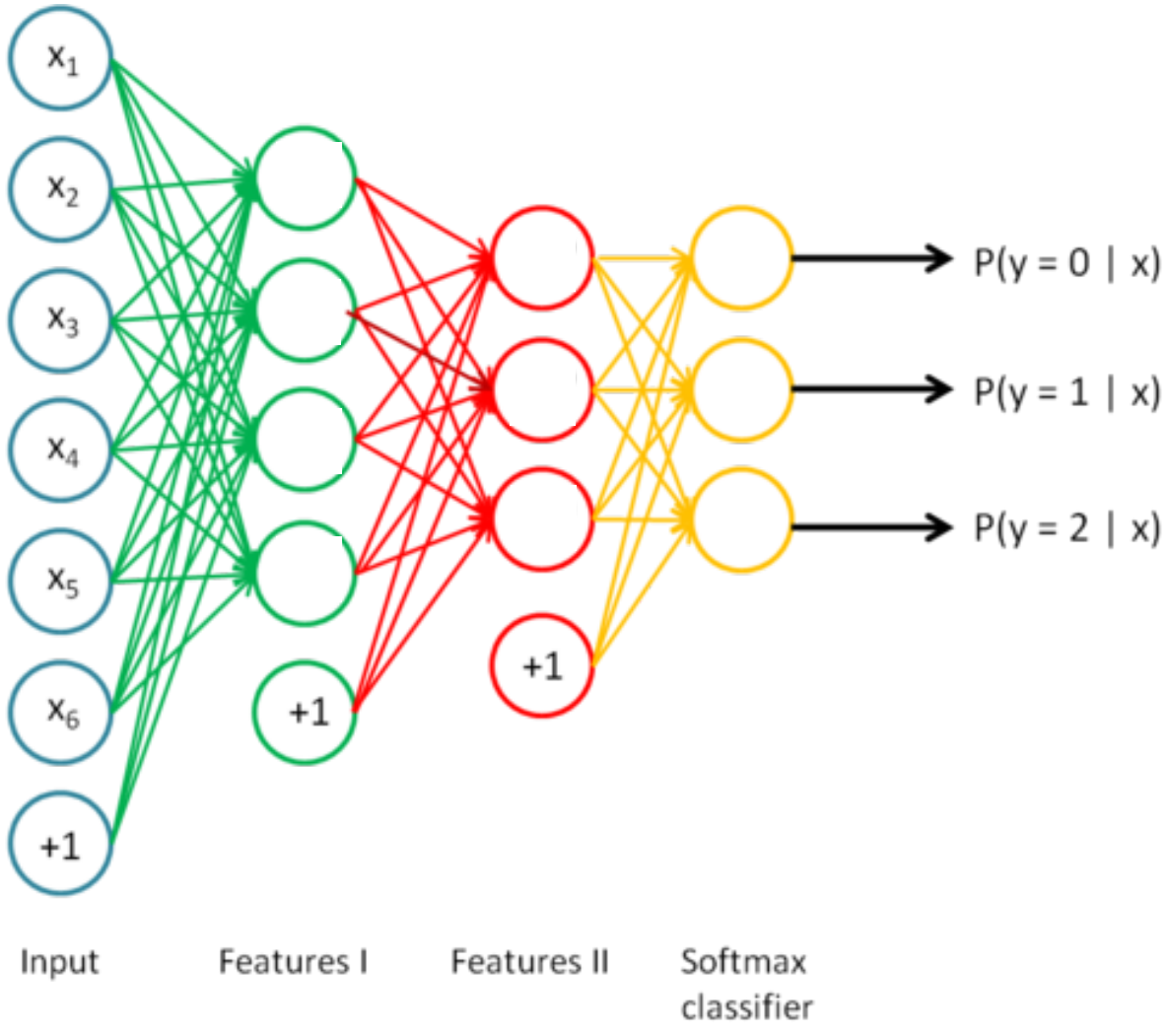
Softmax classifier



$$\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}} \quad \text{for } j = 1, \ldots, K.$$

- Predict the class with highest probability
- Generalization of sigmoid/logistic regression to multi-class

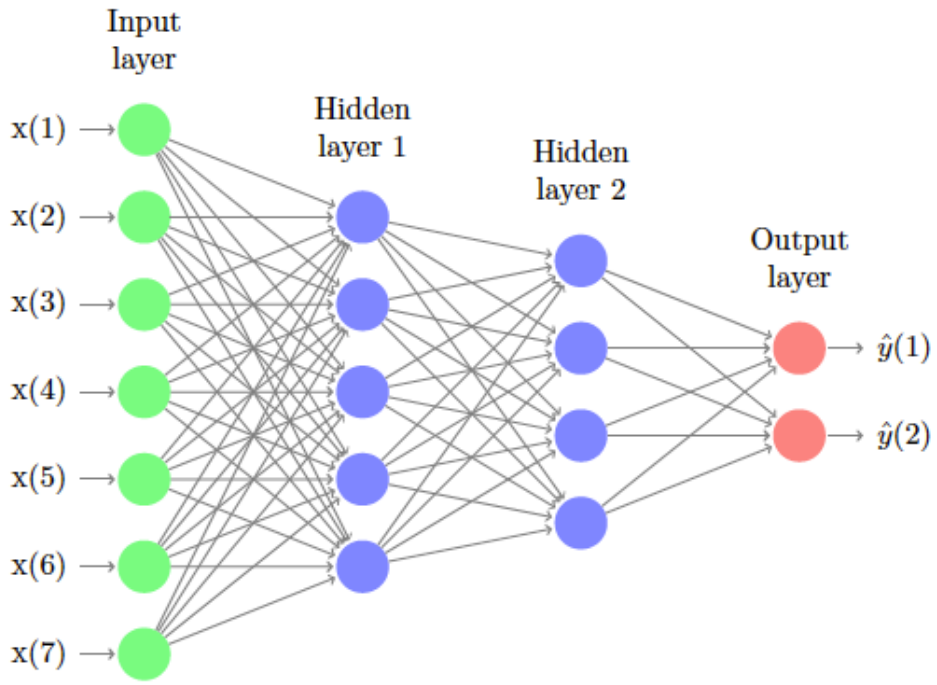
Multi-class classification



Review Feed-Forward Neural Networks

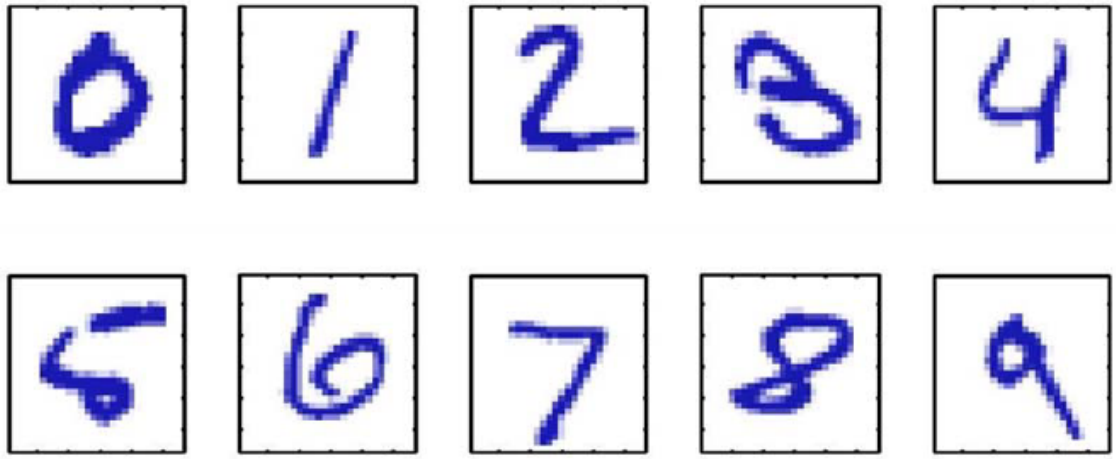
- Simplest architecture of NN
- Neurons from one layer are connected to neurons from next layer
  - Input layer has feature space dimension
  - Output layer has number of classes
  - Hidden layers use linear operations, followed by non-linear activation function
  - Multi-Layer Perceptron (MLP): fully connected layers
- Activation functions
  - Sigmoid for binary classification in last layer
  - Softmax for multi-class classification in last layer
  - ReLU for hidden layers
- Forward propagation is the computation of the network output given an input
- Back propagation is the training of a network
  - Determine weights and biases at every layer

FFNN Architectures



- Input and Output Layers are completely specified by the problem domain
- In the Hidden Layers, number of neurons in Layer i+1 is always smaller than number of neurons in Layer i

MNIST: Handwritten digit recognition



Images are 28 x 28 pixels

Represent input image as a vector $\mathbf{x} \in \mathbb{R}^{784}$

Learn a classifier $f(\mathbf{x})$ such that,

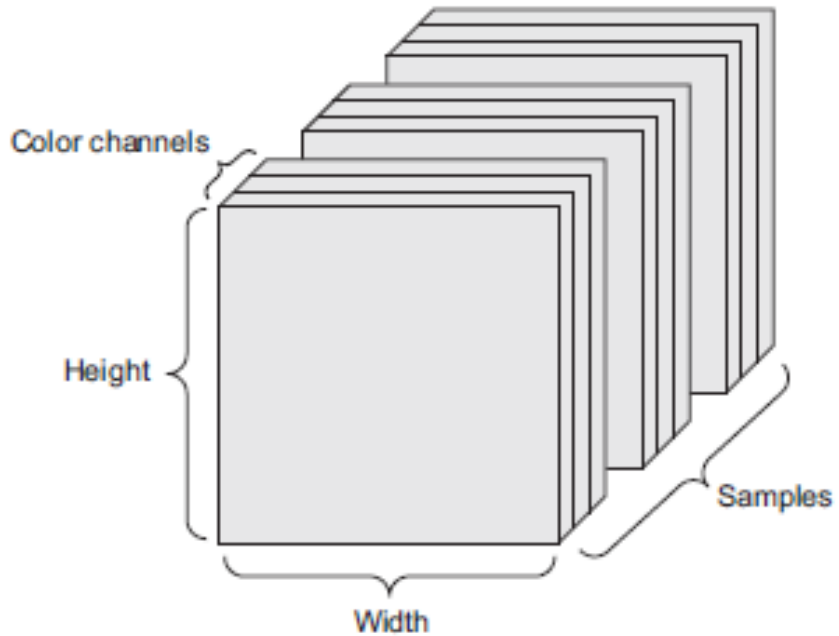
$$f : \mathbf{x} \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Predict the digit

Multi-class classifier

Image Representation

- Image is 3D “tensor”: height, width, color channel (RGB)
- Black-and-white images are 2D matrices: height, width
  - Each value is pixel intensity



Lab – Feed Forward NN

```
import time
import numpy as np
from keras.utils import np_utils
import keras.callbacks as cb
from keras.models import Sequential
from keras.layers.core import Dense, Dropout, Activation
from keras.optimizers import RMSprop
from keras.datasets import mnist

import matplotlib
matplotlib.use('agg')
import matplotlib.pyplot as plt
```

Import modules

```
def load_data():
    print("Loading data")
    (X_train, y_train), (X_test, y_test) = mnist.load_data()

    X_train = X_train.astype('float32')
    X_test = X_test.astype('float32')

    # Normalize
    X_train /= 255
    X_test /= 255

    y_train = np_utils.to_categorical(y_train, 10)
    y_test = np_utils.to_categorical(y_test, 10)

    X_train = np.reshape(X_train, (60000, 784))
    X_test = np.reshape(X_test, (10000, 784))

    print("Data loaded")
    return [X_train, X_test, y_train, y_test]
```

Load MNIST data
Processing

Vector
representation

Neural Network Architecture

```
def init_model():
    start_time = time.time()

    print("Compiling Model")
    model = Sequential()
    model.add(Dense(10, input_dim=784))
    model.add(Activation('relu'))

    model.add(Dense(10))
    model.add(Activation('softmax'))

    rms = RMSprop()
    model.compile(loss='categorical_crossentropy', optimizer=rms, metrics=['accuracy'])

    print("Model finished"+format(time.time() - start_time))
    return model
```

10 hidden units
ReLU activation

Output Layer
Softmax activation

Loss function

Optimizer

Feed-Forward Neural Network Architecture

- 1 Hidden Layer (“Dense” or Fully Connected)
- 10 neurons
- Output layer uses softmax activation

Train and evaluate

```
def run_network(data=None, model=None, epochs=10, batch=256):
    try:
        start_time = time.time()
        if data is None:
            X_train, X_test, y_train, y_test = load_data()
        else:
            X_train, X_test, y_train, y_test = data

        if model is None:
            model = init_model()

        print("Training model")
        history = model.fit(X_train, y_train, nb_epoch=epochs, batch_size=batch,
                  validation_data=(X_test, y_test), verbose=2)

        print("Training duration:"+format(time.time() - start_time))
        score = model.evaluate(X_test, y_test, batch_size=16)

        print("\nNetwork's test loss and accuracy:"+format(score))
        return model, history
    except KeyboardInterrupt:
        print("KeyboardInterrupt")
        return model, history
```

Training/testing results

```
2s - loss: 0.9114 - acc: 0.7649 - val_loss: 0.4499 - val_acc: 0.8819
Epoch 2/10
0s - loss: 0.3935 - acc: 0.8907 - val_loss: 0.3378 - val_acc: 0.9049
Epoch 3/10
0s - loss: 0.3296 - acc: 0.9063 - val_loss: 0.3042 - val_acc: 0.9128
Epoch 4/10
0s - loss: 0.3036 - acc: 0.9132 - val_loss: 0.2889 - val_acc: 0.9181
Epoch 5/10
0s - loss: 0.2888 - acc: 0.9189 - val_loss: 0.2874 - val_acc: 0.9185
Epoch 6/10
0s - loss: 0.2785 - acc: 0.9210 - val_loss: 0.2703 - val_acc: 0.9257
Epoch 7/10
0s - loss: 0.2705 - acc: 0.9241 - val_loss: 0.2718 - val_acc: 0.9239
Epoch 8/10
0s - loss: 0.2649 - acc: 0.9257 - val_loss: 0.2694 - val_acc: 0.9240
Epoch 9/10
0s - loss: 0.2601 - acc: 0.9264 - val_loss: 0.2616 - val_acc: 0.9261
Epoch 10/10
0s - loss: 0.2561 - acc: 0.9277 - val_loss: 0.2607 - val_acc: 0.9274
Training duration:10.31288456916809
 9840/10000 [============================>.] - ETA: 0s
Network's test loss and accuracy:[0.26067940444946291, 0.9274]
```

Epoch Output

Metrics

- Loss
- Accuracy

Reported on both training and validation

Changing Number of Neurons

```
def init_model():
    start_time = time.time()

    print("Compiling Model")
    model = Sequential()
    model.add(Dense(500, input_dim=784))
    model.add(Activation('relu'))

    model.add(Dense(10))
    model.add(Activation('softmax'))

    rms = RMSprop()
    model.compile(loss='categorical_crossentropy', optimizer=rms, metrics=['accuracy'])

    print("Model finished"+format(time.time() - start_time))
    return model
```

500 hidden units

```
2s - loss: 0.3169 - acc: 0.9088 - val_loss: 0.1652 - val_acc: 0.9502
Epoch 2/10
0s - loss: 0.1277 - acc: 0.9626 - val_loss: 0.1071 - val_acc: 0.9679
Epoch 3/10
0s - loss: 0.0847 - acc: 0.9749 - val_loss: 0.0861 - val_acc: 0.9731
Epoch 4/10
0s - loss: 0.0607 - acc: 0.9822 - val_loss: 0.0746 - val_acc: 0.9767
Epoch 5/10
0s - loss: 0.0471 - acc: 0.9863 - val_loss: 0.0655 - val_acc: 0.9796
Epoch 6/10
0s - loss: 0.0359 - acc: 0.9895 - val_loss: 0.0636 - val_acc: 0.9813
Epoch 7/10
0s - loss: 0.0280 - acc: 0.9920 - val_loss: 0.0599 - val_acc: 0.9810
Epoch 8/10
0s - loss: 0.0223 - acc: 0.9937 - val_loss: 0.0678 - val_acc: 0.9795
Epoch 9/10
0s - loss: 0.0174 - acc: 0.9952 - val_loss: 0.0607 - val_acc: 0.9815
Epoch 10/10
0s - loss: 0.0134 - acc: 0.9964 - val_loss: 0.0672 - val_acc: 0.9806
Training duration:10.458189249038696
 9456/10000 [============================>..] - ETA: 0s
Network's test loss and accuracy:[0.067179036217656543, 0.98060000000000003]
```

Two Layers

```
def init_model():
    start_time = time.time()

    print("Compiling Model")
    model = Sequential()

    # Hidden Layer 1
    model.add(Dense(500, input_dim=784))
    model.add(Activation('relu'))

    # Hidden Layer 2
    model.add(Dense(300))
    model.add(Activation('relu'))

    model.add(Dense(10))
    model.add(Activation('softmax'))

    rms = RMSprop()
    model.compile(loss='categorical_crossentropy', optimizer=rms, metrics=['accuracy'])

    print("Model finished"+format(time.time() - start_time))
    return model
```

Layer 1

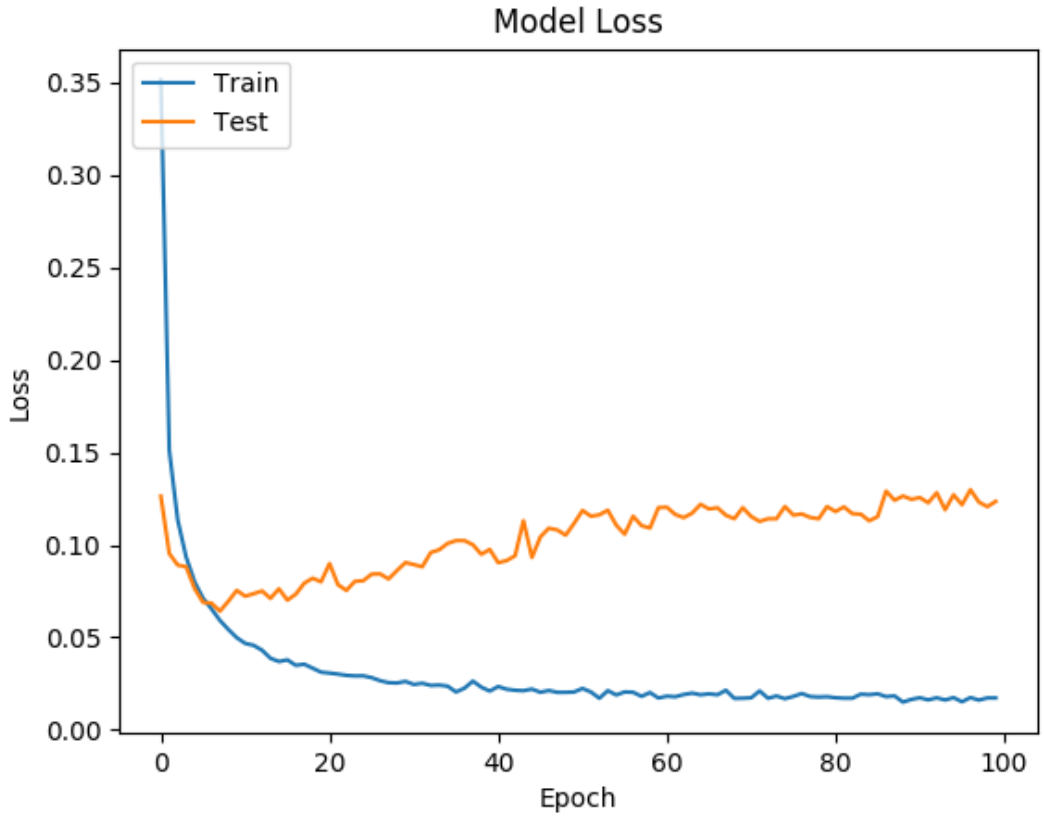
Layer 2

Output Softmax Layer

```
2s - loss: 0.2800 - acc: 0.9132 - val_loss: 0.1821 - val_acc: 0.9409
Epoch 2/10
1s - loss: 0.0974 - acc: 0.9699 - val_loss: 0.0951 - val_acc: 0.9703
Epoch 3/10
0s - loss: 0.0616 - acc: 0.9803 - val_loss: 0.0843 - val_acc: 0.9754
Epoch 4/10
0s - loss: 0.0429 - acc: 0.9862 - val_loss: 0.0670 - val_acc: 0.9809
Epoch 5/10
0s - loss: 0.0303 - acc: 0.9904 - val_loss: 0.0820 - val_acc: 0.9777
Epoch 6/10
0s - loss: 0.0233 - acc: 0.9922 - val_loss: 0.0794 - val_acc: 0.9783
Epoch 7/10
0s - loss: 0.0180 - acc: 0.9941 - val_loss: 0.0866 - val_acc: 0.9792
Epoch 8/10
0s - loss: 0.0137 - acc: 0.9956 - val_loss: 0.0788 - val_acc: 0.9814
Epoch 9/10
0s - loss: 0.0116 - acc: 0.9963 - val_loss: 0.0901 - val_acc: 0.9795
Epoch 10/10
1s - loss: 0.0100 - acc: 0.9966 - val_loss: 0.0812 - val_acc: 0.9827
Training duration:11.816290140151978
 9744/10000 [============================>.] - ETA: 0s
```

Monitor Loss

```
def plot_losses(history):
    plt.plot(history.history['loss'])
    plt.plot(history.history['val_loss'])
    plt.title('Model Loss')
    plt.ylabel('Loss')
    plt.xlabel('Epoch')
    plt.legend(['Train', 'Test'], loc='upper left')
    plt.show()
    plt.savefig('output.png')
```



Model Parameters

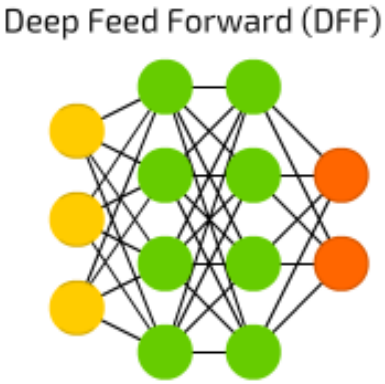
```
model.summary()
```

```
Using TensorFlow backend.
Loading data
Data loaded
Compiling Model
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_1 (Dense)              (None, 500)               392500
_________________________________________________________________
activation_1 (Activation)    (None, 500)               0
_________________________________________________________________
dense_2 (Dense)              (None, 300)               150300
_________________________________________________________________
activation_2 (Activation)    (None, 300)               0
_________________________________________________________________
dense_3 (Dense)              (None, 10)                3010
_________________________________________________________________
activation_3 (Activation)    (None, 10)                0
=================================================================
Total params: 545,810
Trainable params: 545,810
Non-trainable params: 0
```

Neural Network Architectures

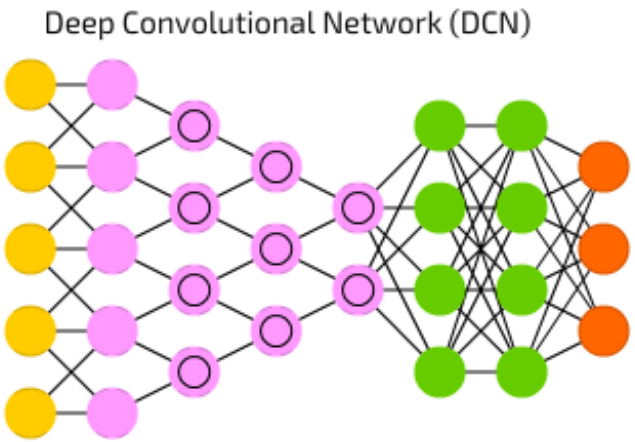
Feed-Forward Networks

- Neurons from each layer connect to neurons from next layer



Convolutional Networks

- Includes convolution layer for feature reduction
- Learns hierarchical representations



Recurrent Networks

- Keep hidden state
- Have cycles in computational graph