

Dynamic modeling and motion control of quadrotor vehicles



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Lisboa, March, 2021



Why are unmanned aerial vehicles (drones)
on the spotlight?



Inspection, Precision Agriculture, Recreation

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TÉCNICO
LISBOA





Drones on the news



BBC News, Oct. 2020
Covid-19: NHS trials
tests and equipment
A hybrid drone is being trialed
typical drone and the wings
good at flying long distances

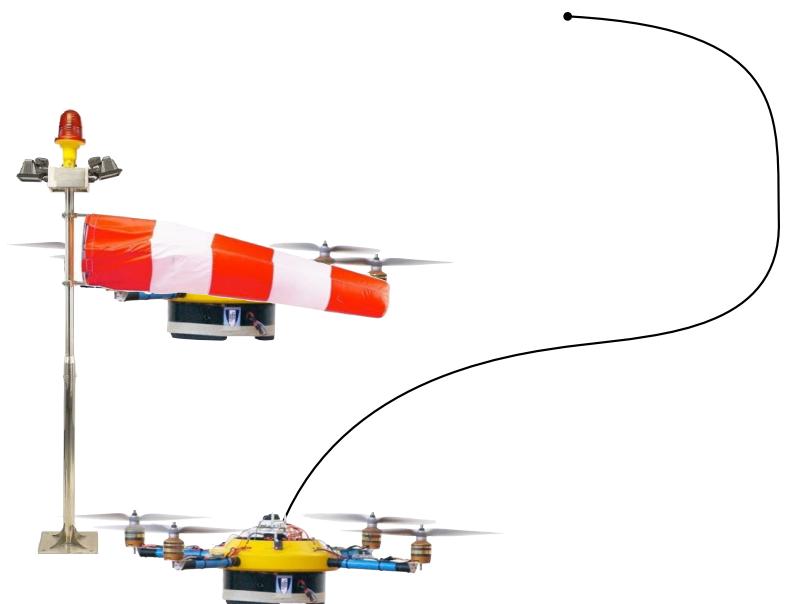




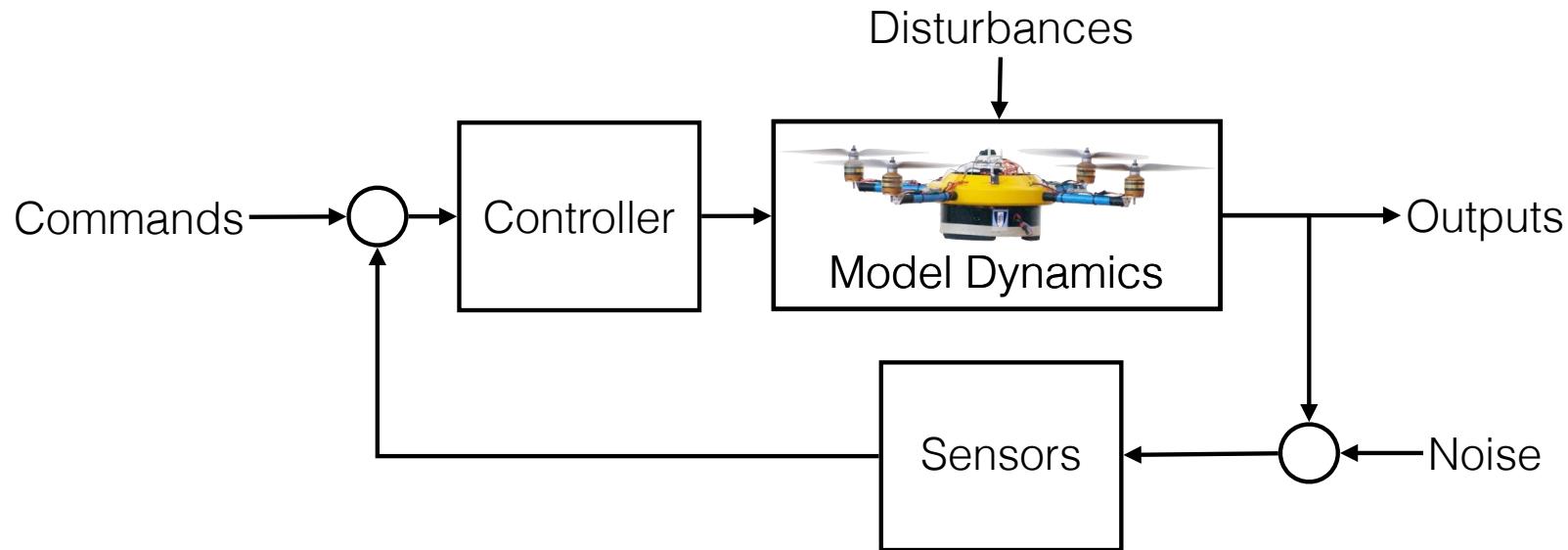
Motion Control of Aerial Vehicles

Trajectory Tracking - Control Objectives

- Track a trajectory
- Realistic model
- Robustness to disturbances
- Actuation bounds
- Large basin of attraction



The quadrotor as a control system



Let's start with the model

The quadrotor as a rigid body

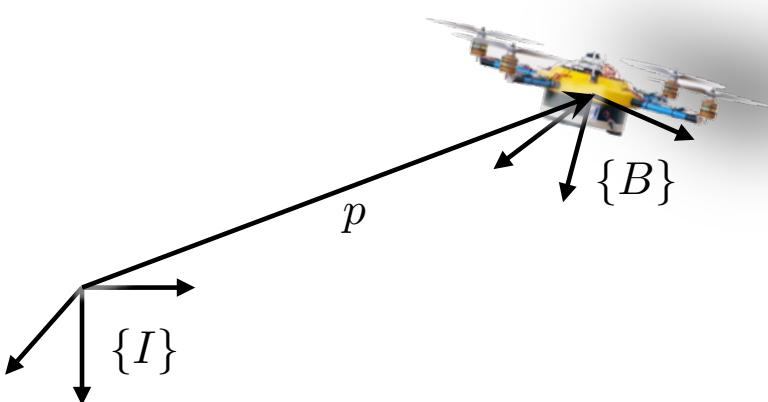
Kinematics

$$\begin{cases} \dot{p} = v \\ \dot{R} = RS(\omega) \end{cases}$$

Dynamics

$$\begin{cases} m\dot{v} = f \\ J\dot{\omega} = -S(\omega)J\omega + n \end{cases}$$

$$S(\omega)a = \omega \times a, \text{ for } \omega, a \in \mathbb{R}^3$$



$m \in \mathbb{R}$ – mass

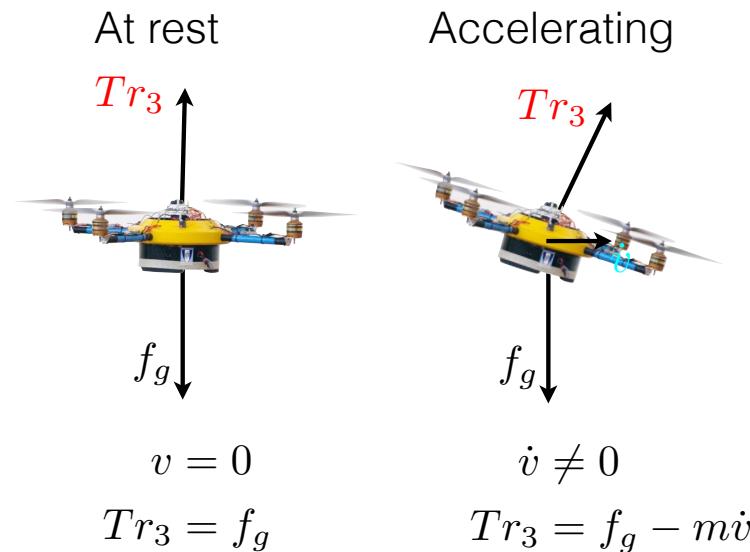
$J \in \mathbb{R}^{3 \times 3}$ – Tensor of inertia expressed in $\{B\}$

$f \in \mathbb{R}^3$ – External forces expressed in $\{I\}$

$n \in \mathbb{R}^3$ – External moments expressed in $\{B\}$

Quadrotor forces

Underactuated system: 12 states, 4 inputs



Control objective:

Steer the position to
a desired trajectory

$$p(t) \rightarrow p_d(t)$$

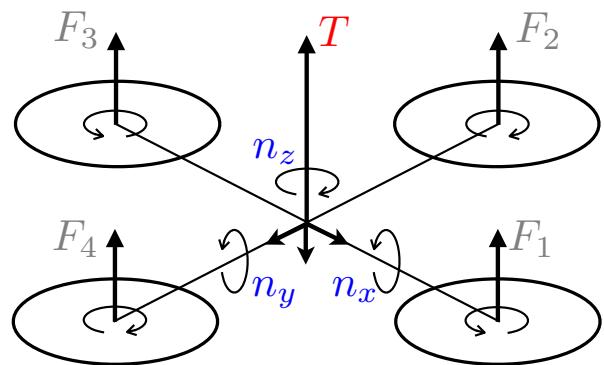
using (T , n) as inputs.

Can only prescribe 4 states:
e.g. 3D position and rotation about z-axis

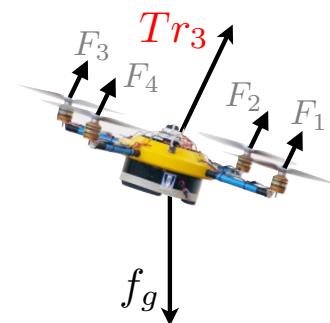
Desired output (p_d, ψ_d)

Quadrotor forces and moments

Two pairs of counter-rotating rotors



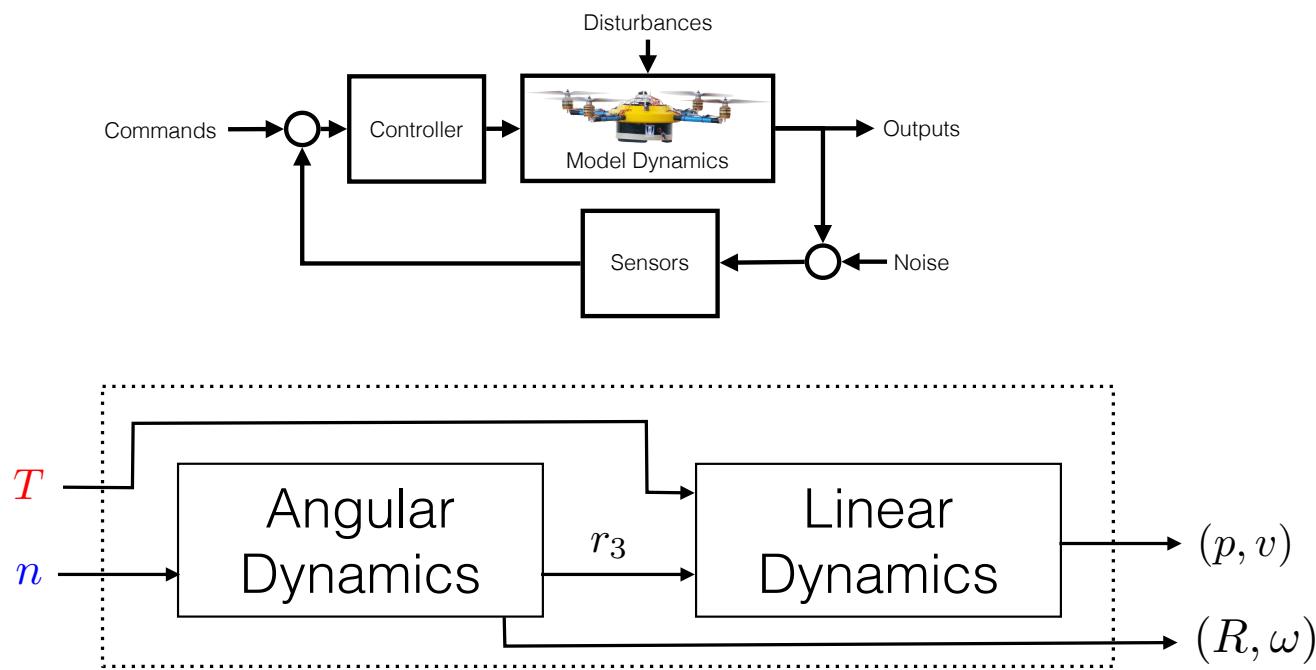
$$\begin{bmatrix} \textcolor{red}{T} \\ n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & b & 0 & -b \\ -b & 0 & b & 0 \\ c & -c & c & -c \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$



$$\textcolor{red}{r}_3 = Re_3 = R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

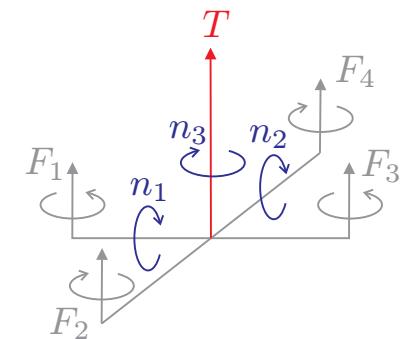
$$f_g = mge_3$$

Quadrotor dynamic model



Underactuated system: 12 states, 4 inputs

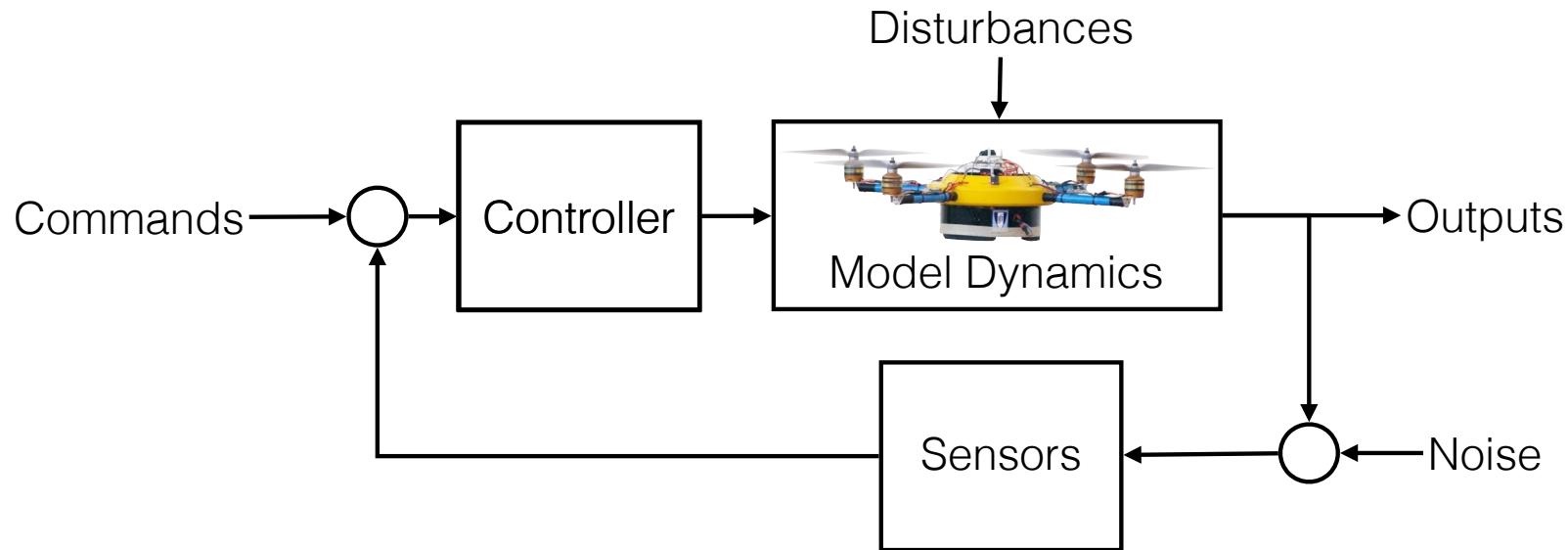
Can prescribe $(p, \psi) \in \mathbb{R}^4$



State (p, v, R, ω)

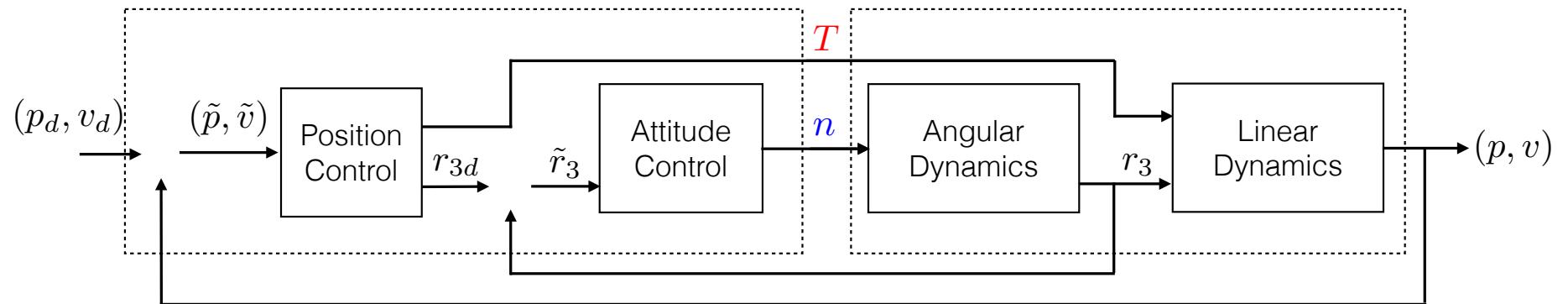
Input (T, n)

The quadrotor as a control system



Now the controller

Trajectory Tracking Control

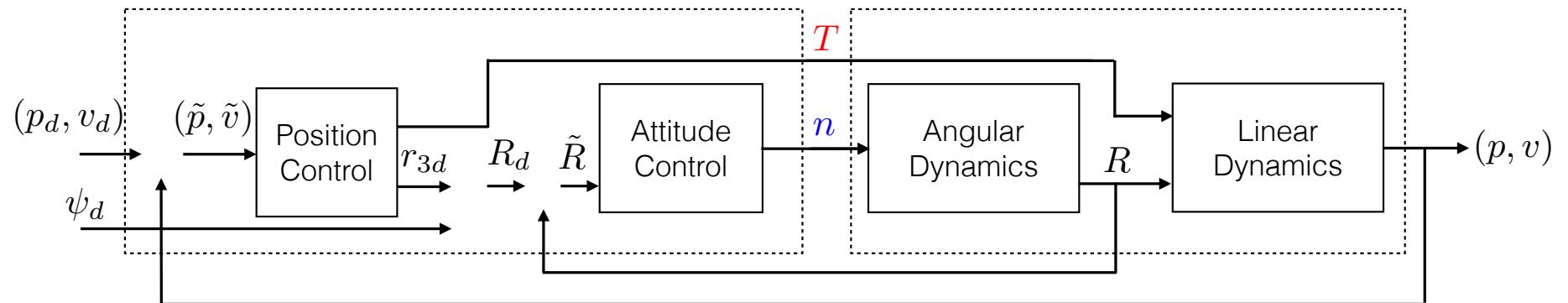


*Simplest Position Controller
yields a mass-spring-damper
system*

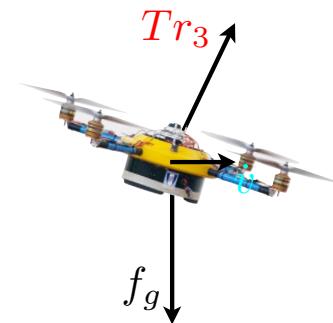
$$m\ddot{v} = -k_1\tilde{p} - k_2\tilde{v}$$



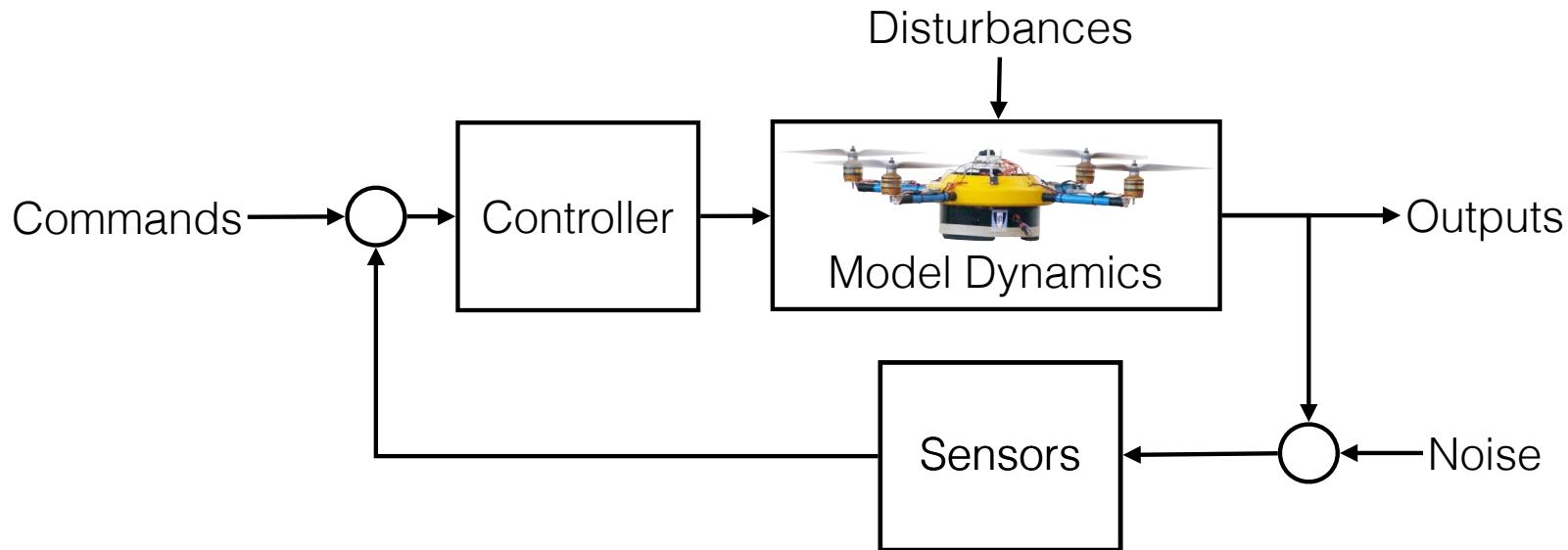
Trajectory Tracking Control



Desired position and yaw angle
 (p_d, ψ_d)



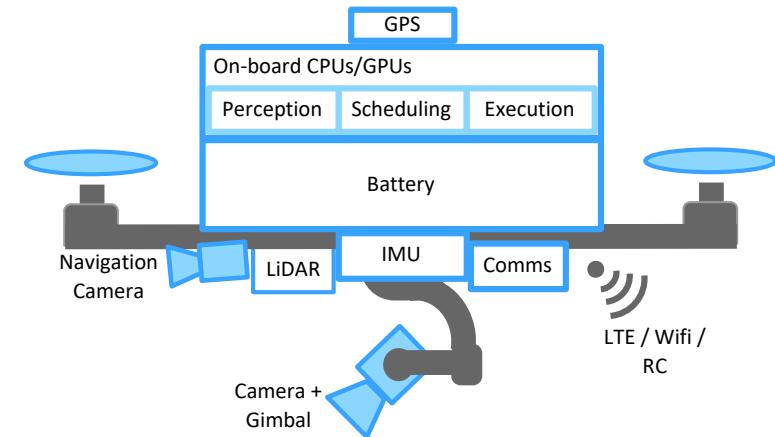
The quadrotor as a control system



Now the sensors

Sensors and Onboard Architecture

- Sensors
 - GPS - position estimates
 - IMU (accelerometers, gyroscopes, magnetometers) - attitude estimates
 - Cameras and LiDARs - mainly used for obstacle avoidance and task execution (surveying, inspection, ...)
- Onboard Computers
- Communication Systems
- Batteries



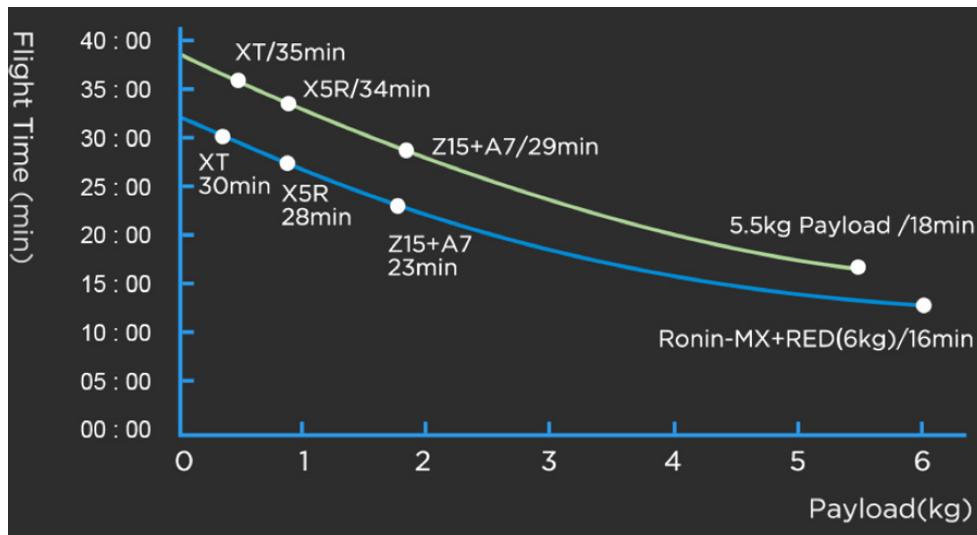
Biggest limitation to enhanced autonomy

- Flight time vs. available payload

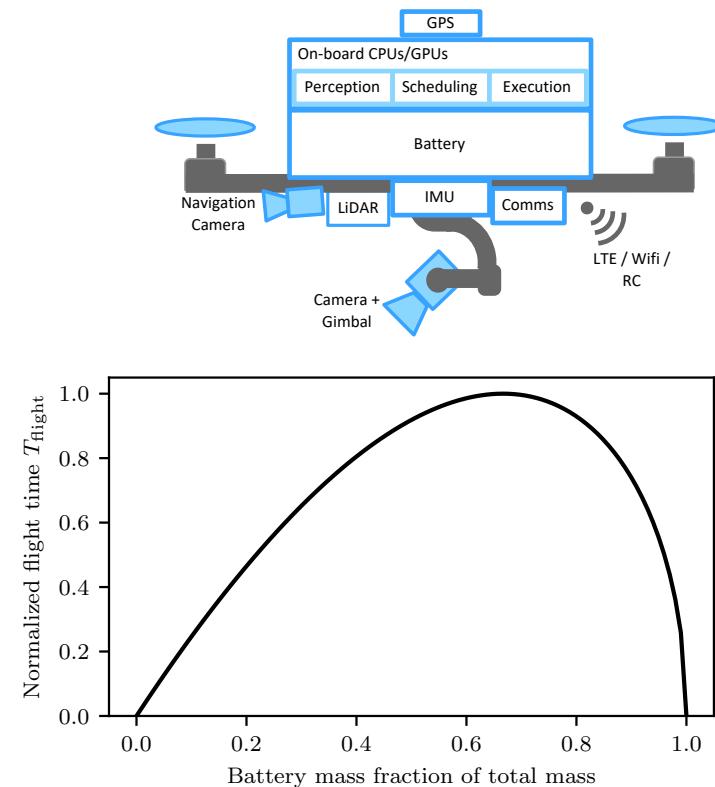
Onboard Architecture

Biggest limitation to enhanced autonomy

- Flight duration vs. available payload



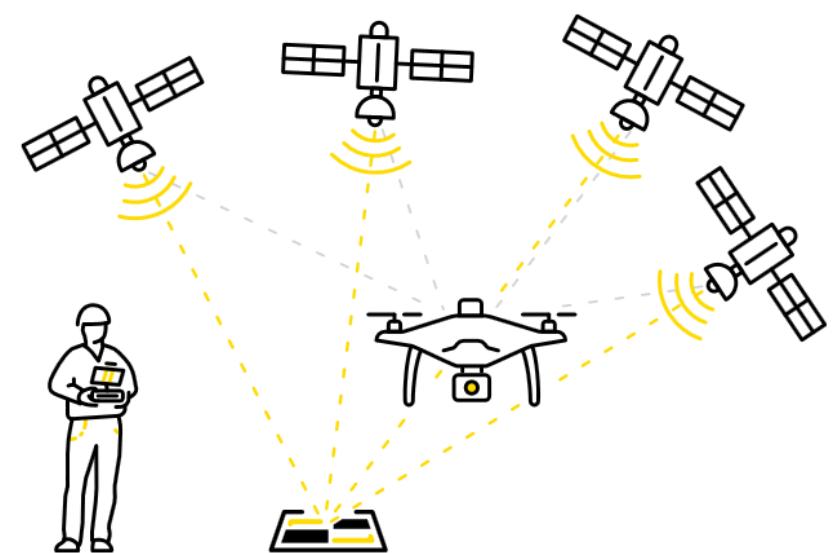
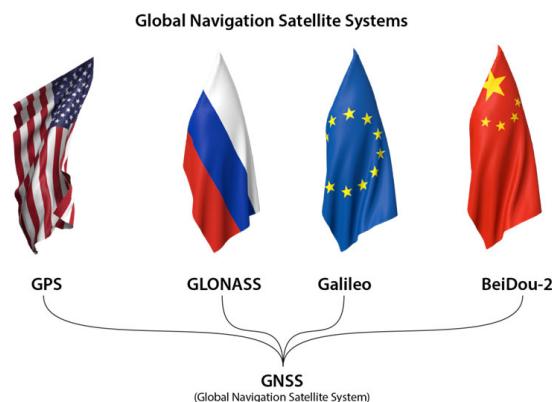
Source: <https://www.dji.com/pt/matrice600-pro>



Source: <https://spectrum.ieee.org/automaton/robotics/drones/swappable-flying-batteries-keep-drones-aloft-almost-forever>

GNSS - Global Navigation Satellite System

- Used to estimate **position** based on time and known position of dedicated satellites.

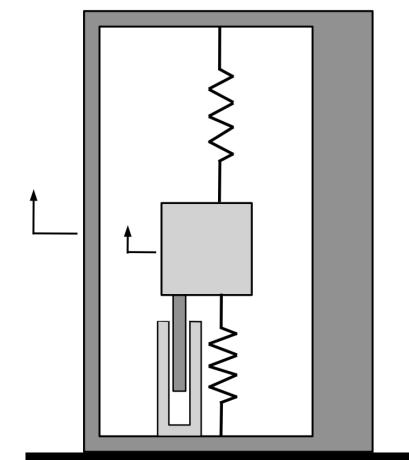
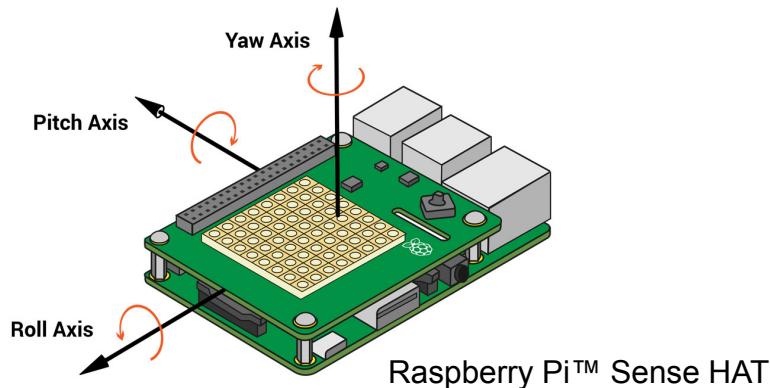


IMU - Inertial Measurement Unit

- Three-axis accelerometers, gyroscopes, and magnetometers
- Used to estimate the orientation of the vehicle

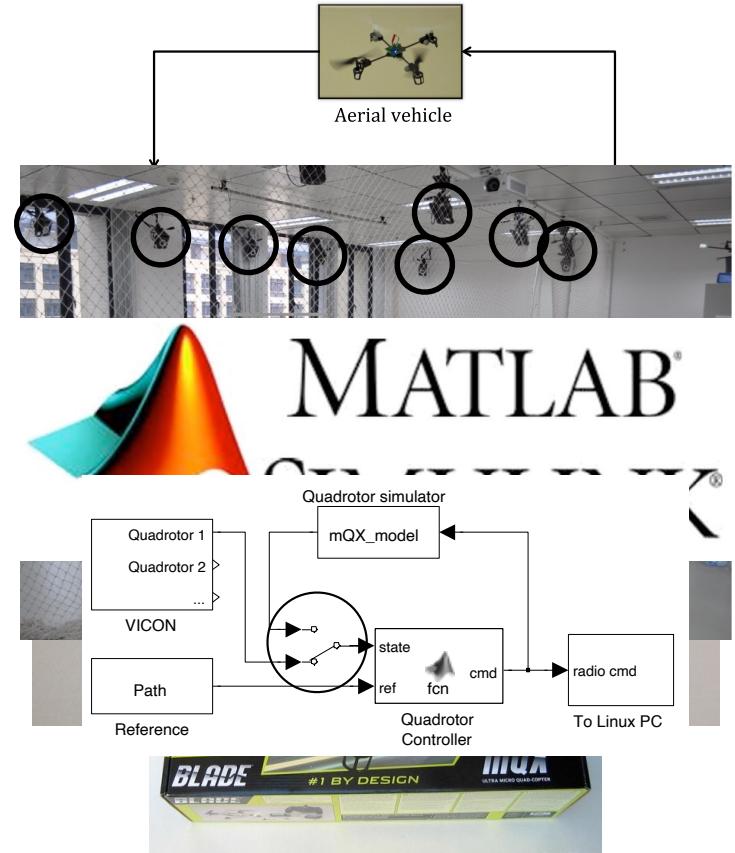
How a Smartphone Knows Up from Down (accelerometer)

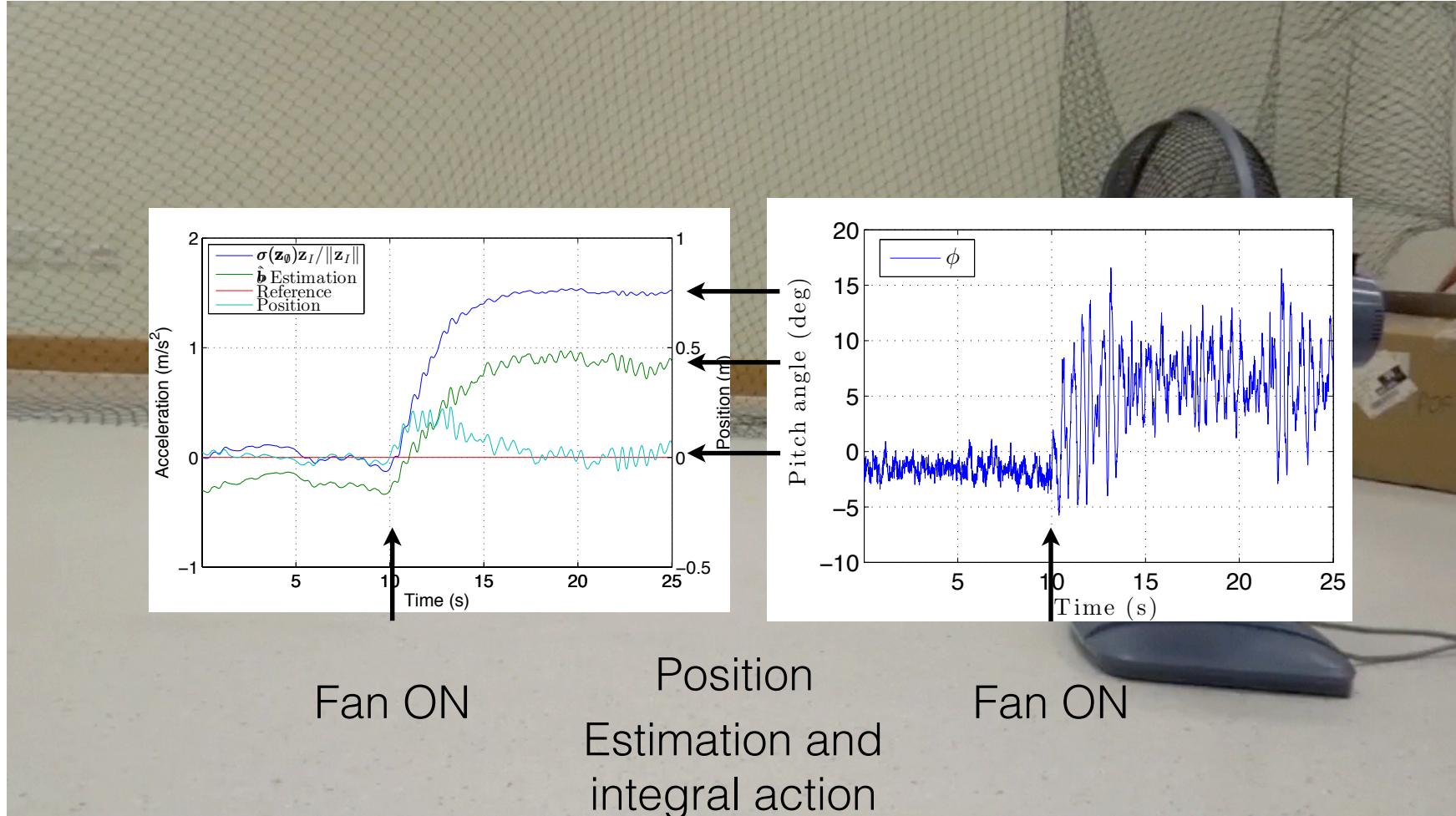
<https://www.youtube.com/watch?v=KZVgKu6v808>



Indoors - Experimental Setup

- Readily available quadrotor solution
- Computer controlled radio transmitters
- Motion Capture system for state measurements
- Integrated system
- Rapid prototyping and experimental testing of controllers







<http://users.isr.ist.utl.pt/~rmac/videos/aggressive.mp4>



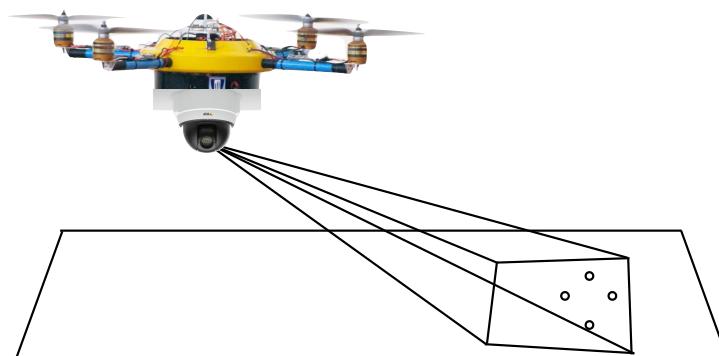
Landing on a moving platform
using vision-based control

Landing on a moving platform

- Quadrotor
- IMU
- Wide-angle camera
- *Goal:* land on a target

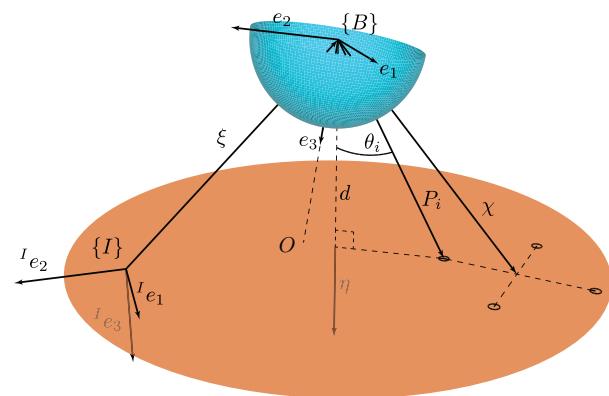
Assumptions

- Planar target
- Moving platform
 - Translation
- Non-aggressive maneuvers



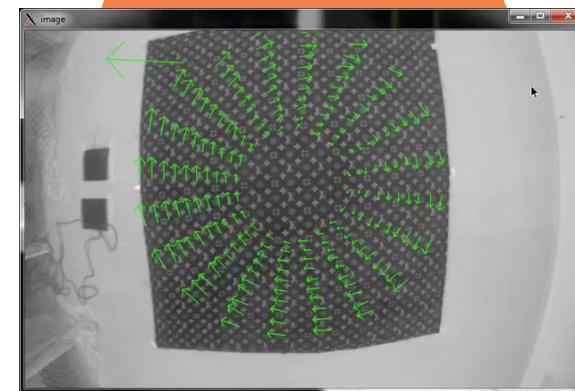
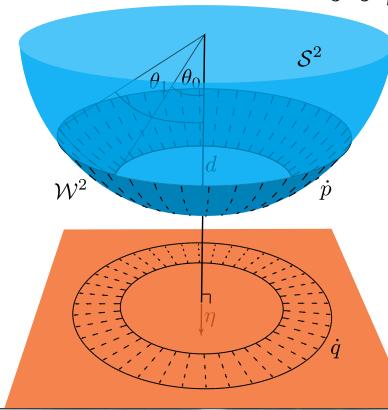
Vision-based controller

Control objective: $\chi \rightarrow 0$



- Optical flow field $\phi = \iint_{\mathcal{W}^2} \dot{p} \, dp$

$$\frac{\dot{\chi}}{d}$$



Available measurements:

- Images of target points

$$p_i = \frac{P_i}{\|P_i\|}$$

$$\frac{P_i}{d}$$

$$\frac{\chi}{d}$$



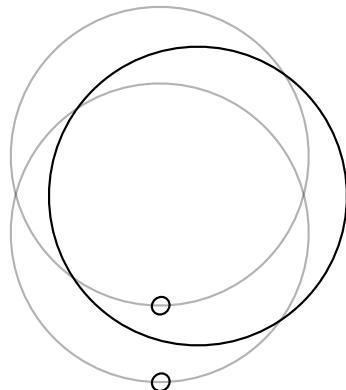
http://users.isr.ist.utl.pt/~rmac/videos/interaction_2.mp4



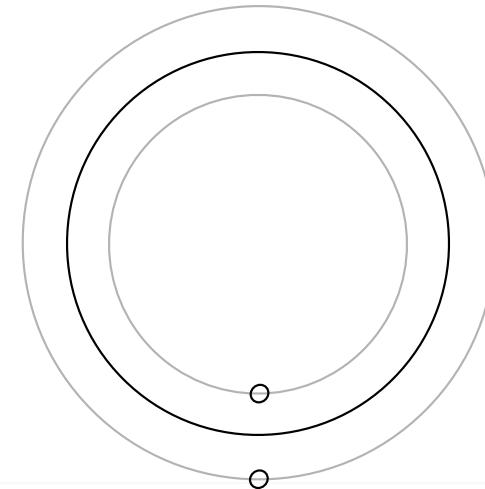
Leader-following for formation control

Leader-following for formation control

- Main idea:
Trailer-like behavior for the followers.



In inertial frame:
Translated identical paths



In trailer frame:
Different paths, no superposition



http://users.isr.ist.utl.pt/~rmac/videos/multi_vehicles.mp4

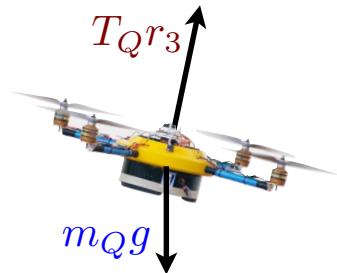




Slung-load transportation using quadrotors

Modeling

Quadrotor



State

$$(p_Q, v_Q, R_Q, \omega_Q)$$

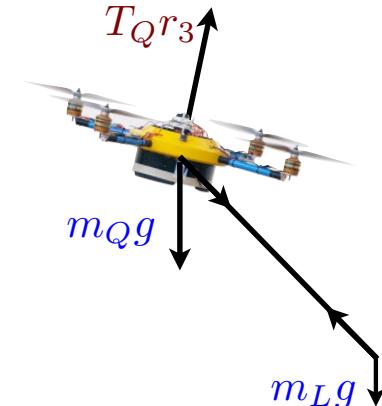
Input

$$(T_Q, n_Q) \in \mathbb{R}^4$$

Flat output

$$(p_Q, \psi_Q) \in \mathbb{R}^4$$

Quadrotor + Load



State

$$(p_Q, v_Q, R_Q, \omega_Q, p_L, v_L)$$

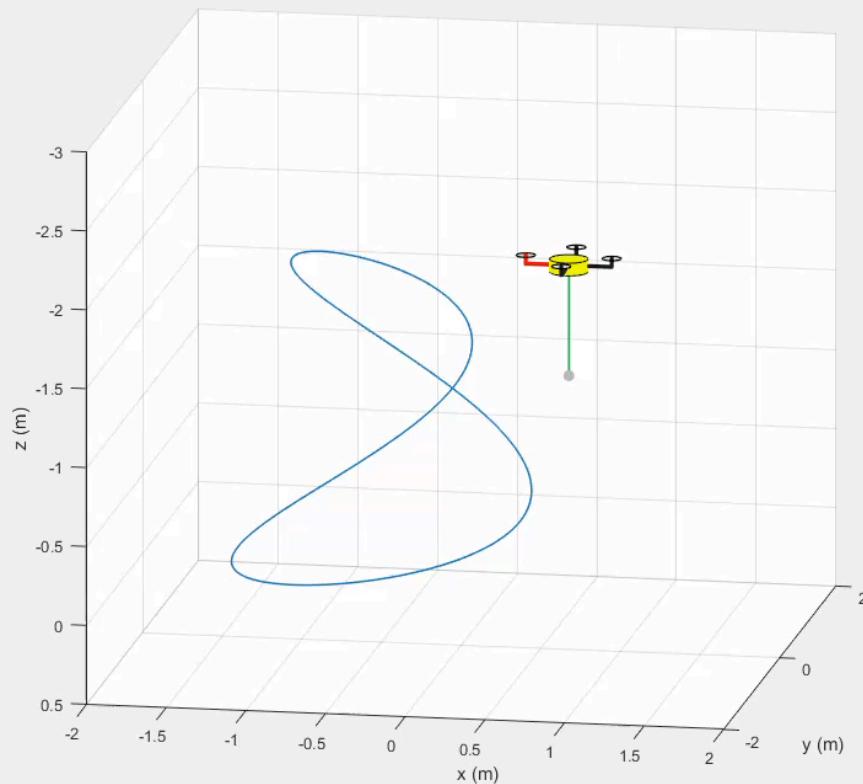
Input

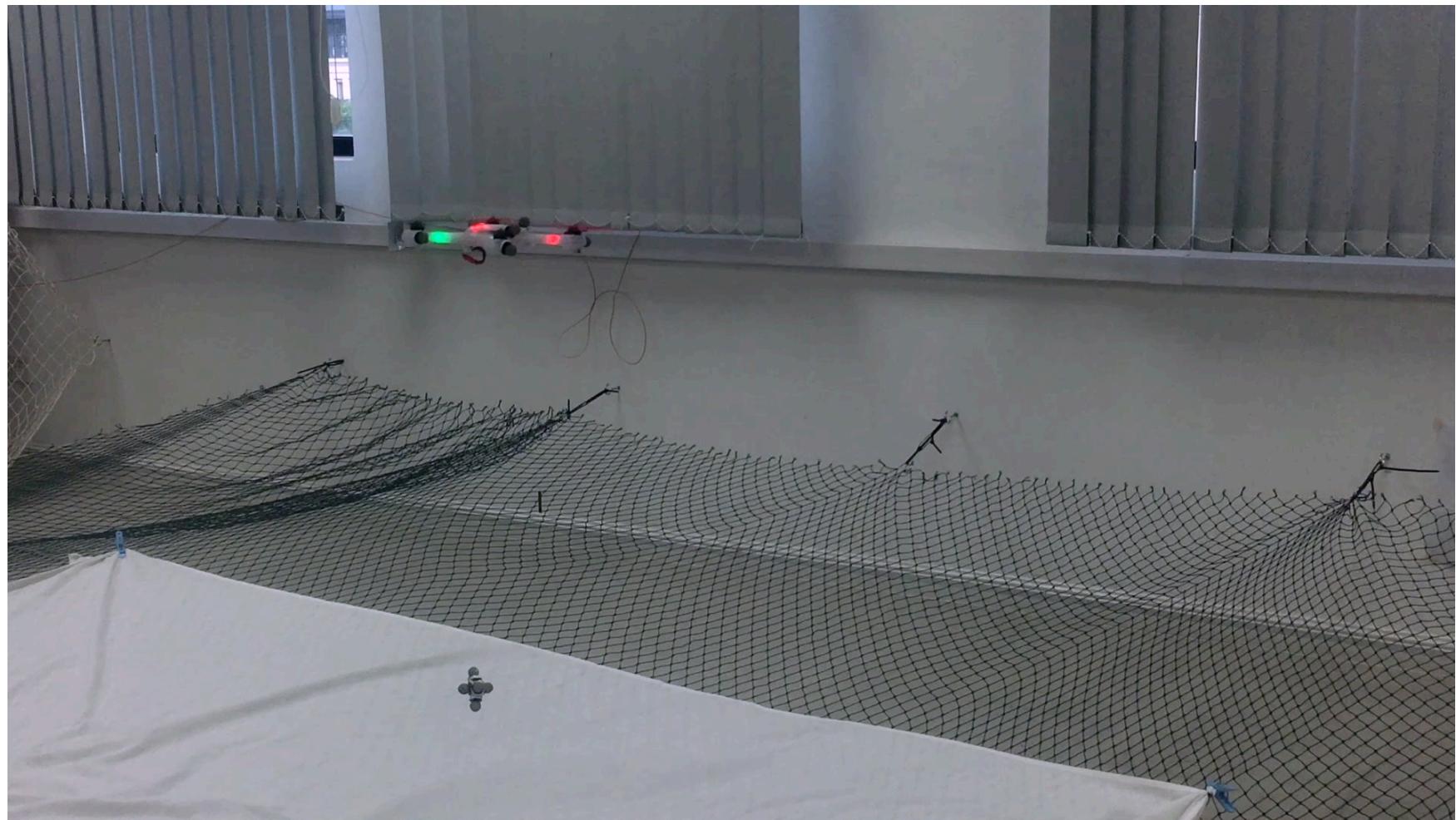
$$(T_Q, n_Q) \in \mathbb{R}^4$$

Flat output

$$(p_L, \psi_Q) \in \mathbb{R}^4$$

Simulation results





http://users.isr.ist.utl.pt/~rmac/videos/load_transportation.mp4





Acknowledgments

The team (current and past members)

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