TÉCNICO LISBOA

Integrated Masters in Aerospace Engineering
Unmanned Aerial Vehicles
Exam Questions

2019/2020 - 2nd Semester

1 Dynamic Modeling

Consider the system shown in Figure 1, where two identical planar quadrotors are cooperatively carrying an object. It is assumed that each quadrotor is rigidly attached to the object. A reference $\{B\}$ attached to the ensemble is placed at the center of mass of the object and the center of mass of each quadrotor is at a distance l from the origin of $\{B\}$ (see Figure 1).

The position and orientation of the body frame $\{B\}$ with respect to the inertial frame $\{I\}$ are described by the coordinates $(x, z) \in \mathbb{R}^2$ and $\theta \in \mathbb{R}$, respectively. Let m denote the mass of each vehicle, m_o the mass of the carried object, g the gravitational acceleration, J_{yy} the moment of inertia the entire system about the y axis, and f_{i1} and f_{i2} the force inputs generated by each the rotors of vehicle $i \in \{1, 2\}$, which are placed at a distance b from the vehicle's center of mass.

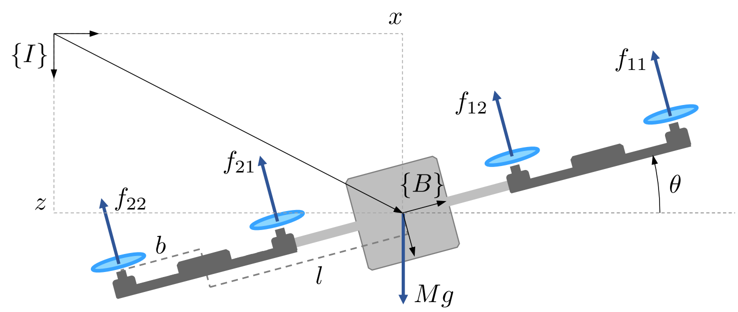

Figura 1: *Cooperative load transportation.*

1.1 Considering the forces and moments generated by each of the planar quadrotors $i \in \{1, 2\}$, show that the equations of motion for the complete system can be written as

$$M\ddot{x} = -T \sin\theta \tag{1}$$

$$M\ddot{z} = -T \cos\theta + Mg \tag{2}$$

$$J_{yy}\ddot{\theta} = \tau_y \tag{3}$$

where $M = 2m + m_o$ and the total thrust T along the negative z axis of $\{B\}$ and the torque τ_y about the y axis are given by the underdetermined system of equations

$$\begin{bmatrix} T \\ \tau_y \end{bmatrix} = W \begin{bmatrix} f_{11} \\ f_{12} \\ f_{21} \\ f_{22} \end{bmatrix} \tag{4}$$

Write explicit expressions for the elements of the constant matrix $W \in \mathbb{R}^{2\times4}$.

1.2 Show that by imposing the constraints $f_{i1} = f_{i2}$ for both quadrotors $i \in \{1, 2\}$, the standard equations of motions for a planar quadrotor with two inputs are recovered.

1.3 Show that to pass the complete structure through a horizontal tube-like passage that requires keeping the absolute value of the pitch angle below 30º, i.e. $|\theta(t)| < 30^{o}$, the forward acceleration must satisfy $|\ddot{x}(t)| < \frac{\sqrt{3}}{3} g$ ms^{-2}.

2 Controller and Observer Design

Consider the model described by (1)-(3) and let $(x_r(t), z_r(t))$ denote the x and z coordinates of a reference trajectory with continuous time-derivatives up to the fourth-order. To design a trajectory tracking controller with a hierarchical structure (see Figure 2), the position dynamics are approximated by $\ddot{x} = u_x^*$ and $\ddot{z} = u_z^*$, where u_x^* and u_z^* are virtual control inputs.

2.1 As an initial step to design a trajectory tracking controller for the outer loop system, show that the error system for the x coordinate can be written in state-space form as $\dot{\mathbf{x}}_e = A\mathbf{x}_e + Bu_e$, where $\mathbf{x}_e = \begin{bmatrix}(x - x_r) & (\dot{x} - \dot{x}_r)\end{bmatrix}^T$, $A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, and $B = \begin{bmatrix}0\\ 1\end{bmatrix}$. Write the expression for u_e. *Remark:* using the inner-outer loop system approximation, the dynamics for the x and z coordinates are decoupled and can be considered independently.

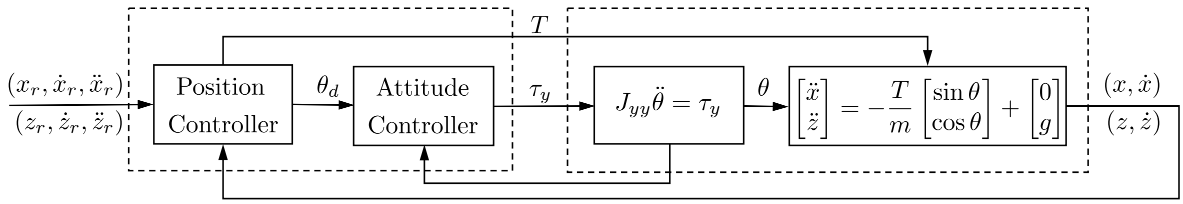

Figura 2: Hierarchical Control System.

2.2 Given the error system defined in **2.1**, consider the problem of minimizing the LQR criterion

$$J = \int_0^{\infty} \mathbf{x}_e(t)^T G^T G \mathbf{x}_e(t) + \rho^2 u_e(t)^2 dt \tag{5}$$

Show that, with $G = \begin{bmatrix}1 & 0\\ 0 & \gamma\end{bmatrix}$ and $\rho > 0$, the gain matrix $K = \begin{bmatrix}k_1 & k_2\end{bmatrix}$ for the LQR control law $u_e = -K\mathbf{x}_e$ is given by $K = \frac{1}{\rho}\begin{bmatrix}1 & \sqrt{\gamma^2 + 2\rho}\end{bmatrix}$.

2.3 Recalling that the closed-loop system described by $\dot{\mathbf{x}}_e = (A - BK)\mathbf{x}_e$ can also be written as $\ddot{x}_e = -k_1 x_e - k_2 \dot{x}_e$, where $x_e = x - x_r$, discuss the impact of changing γ and ρ on the performance of the system.

2.4 Assume that a GNSS system is installed on the carried object, providing latitude, longitude, and altitude measurements that after an adequate transformation provide position measurements $(x_m(t), z_m(t))$, expressed in the (local) inertial frame $\{I\}$. Considering once again the motion along the x coordinate only, propose a structure for an observer that provides estimates for both the position $x(t)$ and the velocity $\dot{x}(t)$. Discuss what additional information, sensor or assumptions are needed to define such an observer, bearing in mind that no sensor directly provides velocity measurements.

2.5 Redraw the block diagram from Fig. 2 and make the necessary changes to include in the block diagram an extended version of the observer that provides estimates for both the x and z coordinates. Clearly indicate the inputs and outputs of each block.