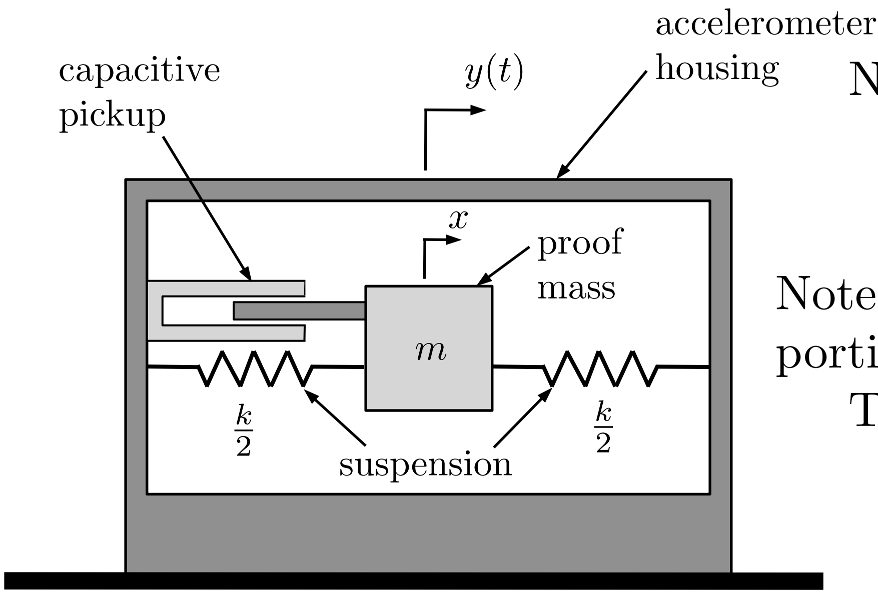
Chapter 7

Sensors

Sensors for MAVs

- The following types of sensors are commonly used for guidance and control of MAVs
  - accelerometers
  - rate gyros
  - pressure sensors
  - magnetometers (digital compasses)
  - GPS

MEMS Accelerometer



Newtons 2^{nd} law gives

$$m\ddot{x} + kx = k(y \cancel{- x})$$

Note that the acceleration of the proof mass proportional to deflection of the suspension

Taking the Laplace transform gives

$$\frac{X(s)}{Y(s)} = \frac{1}{\frac{m}{k}s^2 + 1}$$

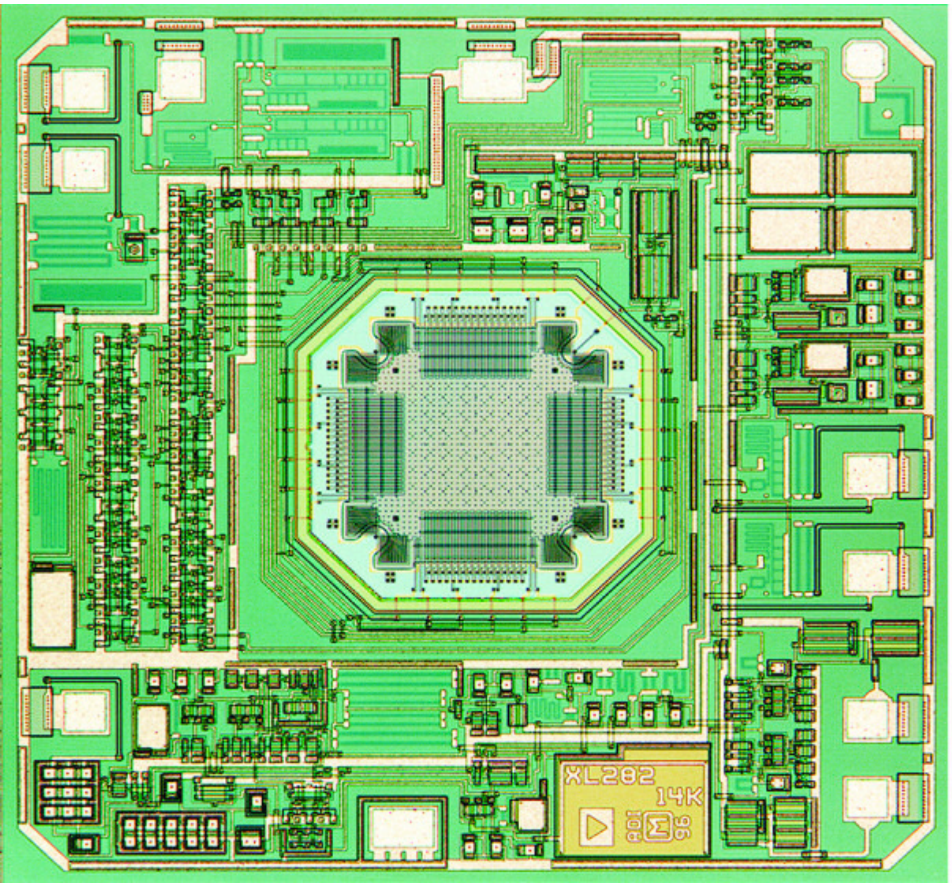
Also, since $X(s)/Y(s) = s^2X(s)/s^2Y(s)$, we can also write

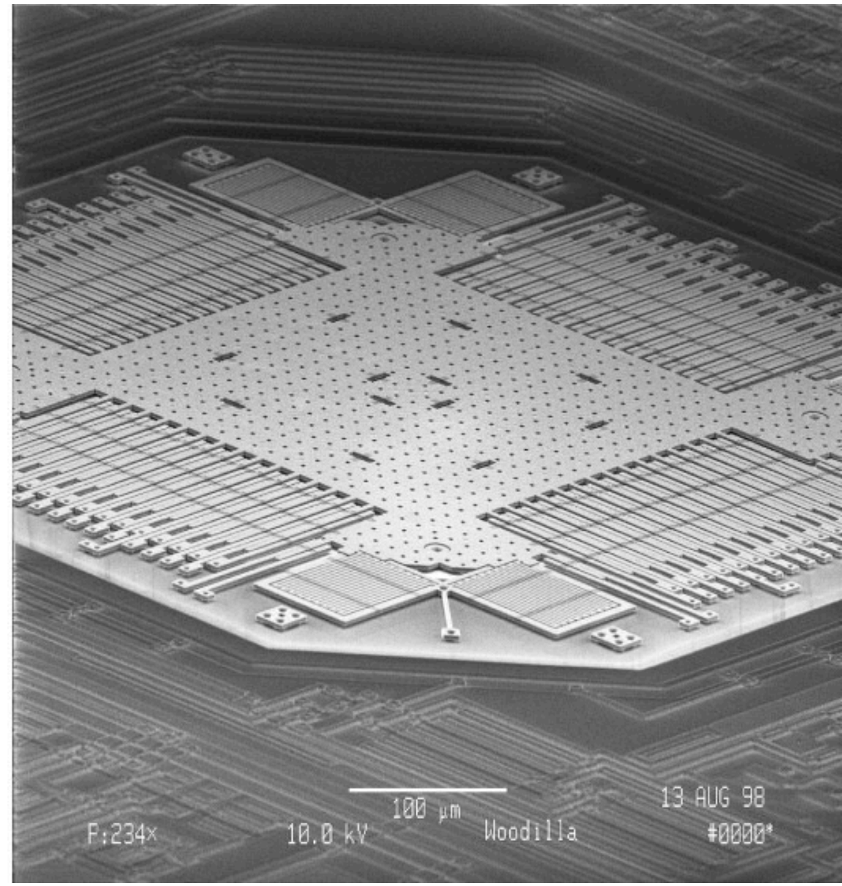
$$\frac{A_X(s)}{A_Y(s)} = \frac{1}{\frac{m}{k}s^2 + 1}$$

Accelerometers also have bias and zero mean Gaussian noise. Therefore, the sensor model is

$$\boxed{\Upsilon_{\text{accel}} = k_{\text{accel}}a + \beta_{\text{accel}} + \eta'_{\text{accel}}.}$$

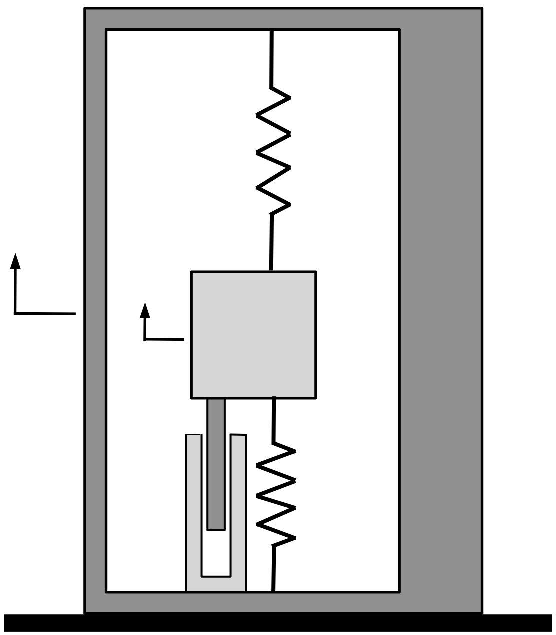
MEMS Accelerometer





Acceleration Measurement

Tricky concept: Measured acceleration is the total acceleration of the accelerometer casing minus the acceleration of gravity



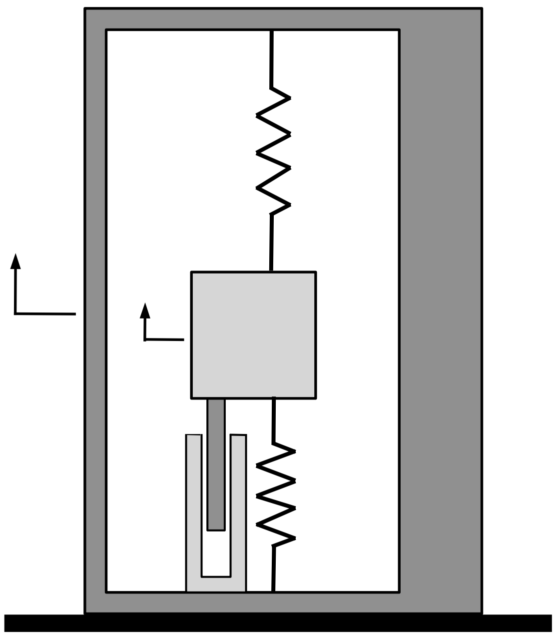
$$\mathbf{a} = \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\mathrm{total}} - \mathbf{F}_{\mathrm{gravity}}\right)$$

Example: Set the accelerometer on a table top. What does it measure?

Accels measure components of linear, coriolis, and externally applied acceleration. They do not measure gravity, since both the proof mass and the casing are acted on by gravity in exactly the same way

Acceleration Measurement

Said another way, accelerometers measure *specific force*, which is defined as the sum of the non-gravitational forces divided by the mass

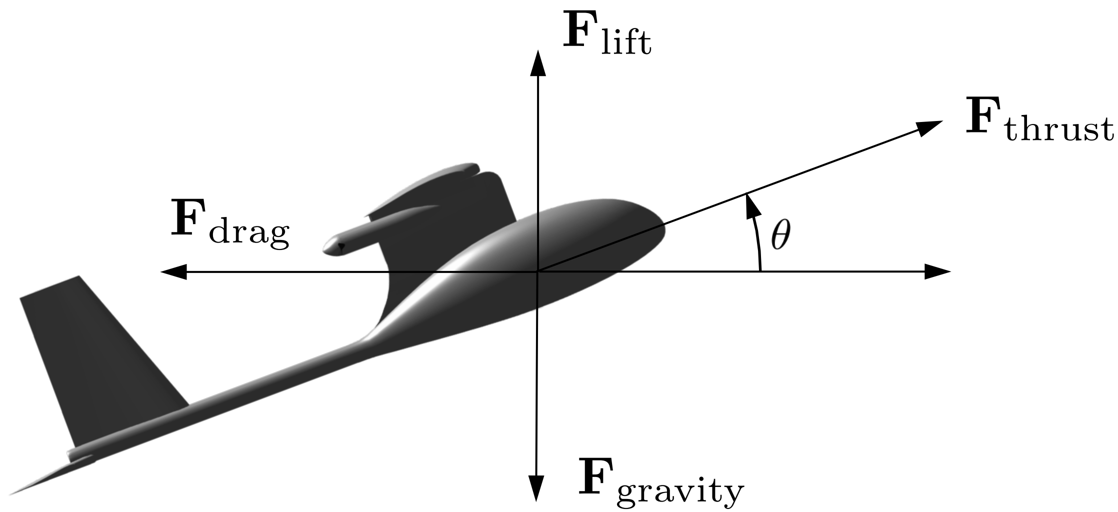


$$\mathbf{a}_{\text{measured}} = \frac{1}{\mathsf{m}}\left(\sum \mathbf{F}_{\text{non-gravitational}}\right)$$
$$= \frac{1}{\mathsf{m}}\left(\sum \mathbf{F} - \mathbf{F}_{\text{gravitational}}\right)$$

Example: Set the accelerometer on a table top.
What does it measure?

Hint: (Draw FBD of accel housing)

Acceleration on Fixed-Wing Aircraft



$$
\begin{aligned}
\mathbf{a}_{\text{measured}} &= \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{total}} - \mathbf{F}_{\text{gravity}}\right) \\
&= \frac{1}{\mathsf{m}}\left(\left(\mathbf{F}_{\text{lift}} + \mathbf{F}_{\text{drag}} + \mathbf{F}_{\text{thrust}} + \mathbf{F}_{\text{gravity}}\right) - \mathbf{F}_{\text{gravity}}\right) \\
&= \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{lift}} + \mathbf{F}_{\text{drag}} + \mathbf{F}_{\text{thrust}}\right)
\end{aligned}
$$

Acceleration on Fixed-Wing Aircraft

Recall from Chapter 3, that

$$\mathsf{m}\left(\frac{d\mathbf{v}}{dt_b} + \boldsymbol{\omega}_{b/i} \times \mathbf{v}\right) = \mathbf{F}_{\text{total}}.$$

Using the expression

$$\mathbf{a}_{\text{measured}} = \frac{1}{\mathsf{m}}\left(\mathbf{F}_{\text{total}} - \mathbf{F}_{\text{gravity}}\right),$$

the output of the accelerometer can be expressed as

$$\mathbf{a}_{\text{measured}} = \frac{d\mathbf{v}}{dt_b} + \boldsymbol{\omega}_{b/i} \times \mathbf{v} - \frac{1}{\mathsf{m}}\mathbf{F}_{\text{gravity}}.$$

Expressing this relationship in the body frame gives

$$\begin{aligned} a_x &= \dot{u} + qw - rv + g\sin\theta \\ a_y &= \dot{v} + ru - pw - g\cos\theta\sin\phi \\ a_z &= \dot{w} + pv - qu - g\cos\theta\cos\phi \end{aligned}$$

Accelerometer Models

$$\boxed{\begin{aligned} y_{\text{accel},x} &= \dot{u} + qw - rv + g\sin\theta + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \dot{v} + ru - pw - g\cos\theta\sin\phi + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \dot{w} + pv - qu - g\cos\theta\cos\phi + \eta_{\text{accel},z} \end{aligned}}$$

or

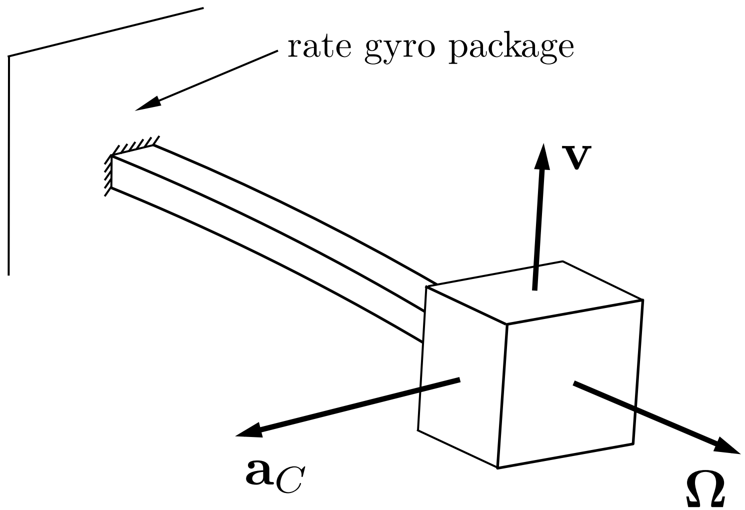
$$\boxed{\begin{aligned} y_{\text{accel},x} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_X(\alpha) + C_{X_q}(\alpha)\frac{\bar{c}q}{2V_a} + C_{X_{\delta_e}}(\alpha)\delta_e\right] \\ &\quad + \frac{\rho S_{\text{prop}} C_{\text{prop}}}{2\text{m}}\left[(k_{\text{motor}}\delta_t)^2 - V_a^2\right] + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{bp}{2V_a} + C_{Y_r}\frac{br}{2V_a}\right. \\ &\quad \left. + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right] + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \frac{\rho V_a^2 S}{2\text{m}}\left[C_Z(\alpha) + C_{Z_q}(\alpha)\frac{\bar{c}q}{2V_a} + C_{Z_{\delta_e}}(\alpha)\delta_e\right] + \eta_{\text{accel},z} \end{aligned}}$$

Accelerometer Models

or

$$\boxed{\begin{aligned} y_{\text{accel},x} &= \frac{f_x}{\mathsf{m}} + g\sin\theta + \eta_{\text{accel},x} \\ y_{\text{accel},y} &= \frac{f_y}{\mathsf{m}} - g\cos\theta\sin\phi + \eta_{\text{accel},y} \\ y_{\text{accel},z} &= \frac{f_z}{\mathsf{m}} - g\cos\theta\cos\phi + \eta_{\text{accel},z} \end{aligned}}$$

MEMS Rate Gyro



Point translating on a rotating rigid body experiences a coriolis acceleration:

$$\mathbf{a}_C = 2\mathbf{\Omega} \times \mathbf{v}$$

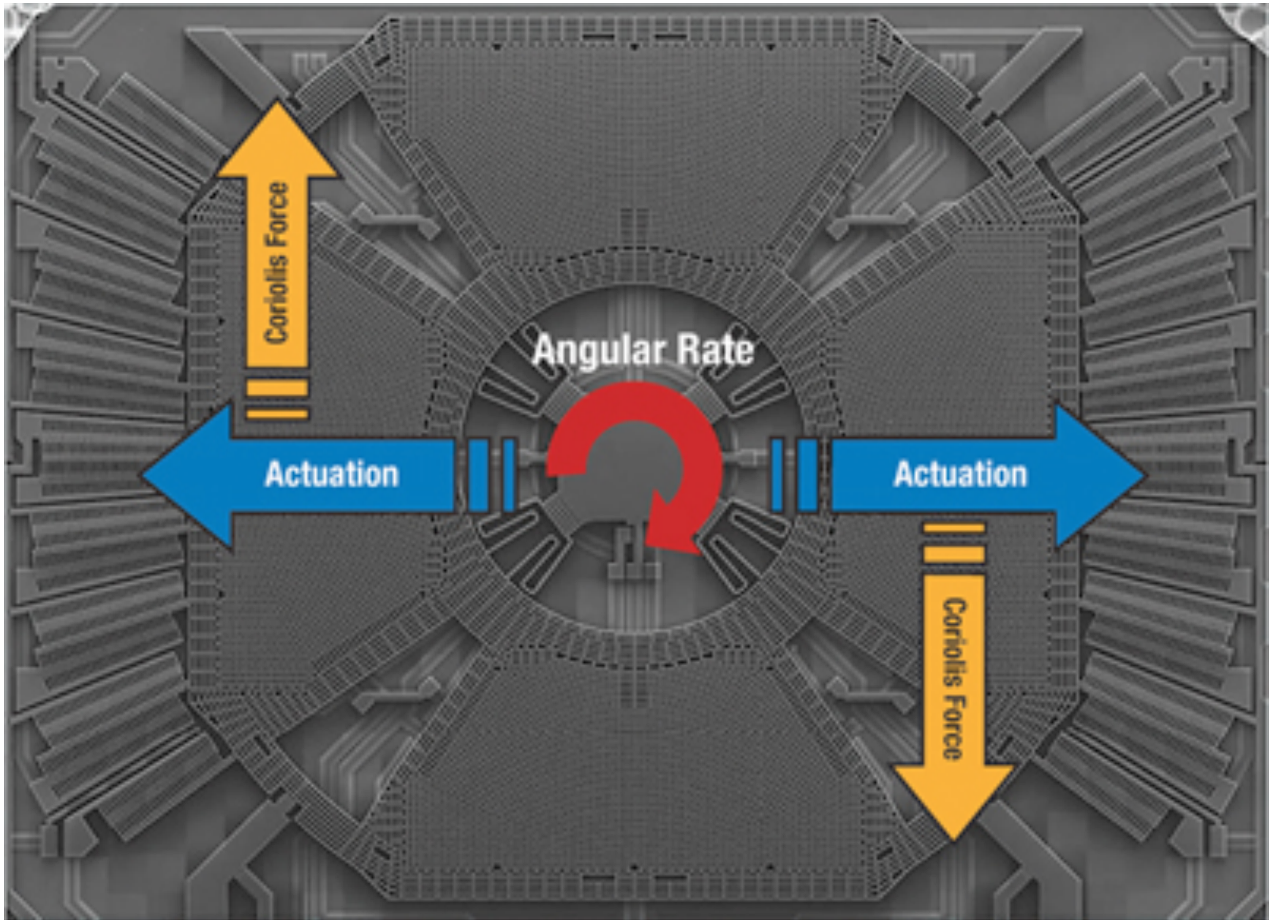
MEMS rate gyro – resonating proof mass:

$$|\mathbf{v}| = A\omega_n \sin(\omega_n t)$$

Sensor measures deflection of proof mass due to coriolis acceleration

$$\begin{aligned} V_{\text{gyro}} &= k_C |\mathbf{a}_C| \\ &= 2k_C |\mathbf{\Omega} \times \mathbf{v}| \\ &= \Omega |\mathbf{v}| \\ &= 2k_C \Omega |A\omega_n \sin(\omega_n t)| \\ &= 2k_C A\omega_n \Omega \\ &= K_C \Omega \end{aligned}$$

MEMS Rate Gyro



Rate Gyro Model

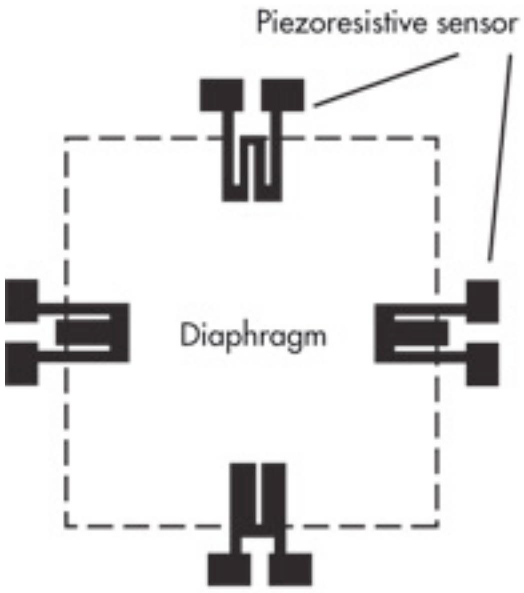
The manufacturing process implies that rate gyros will have a drift term, as well as zero mean Gaussian noise:

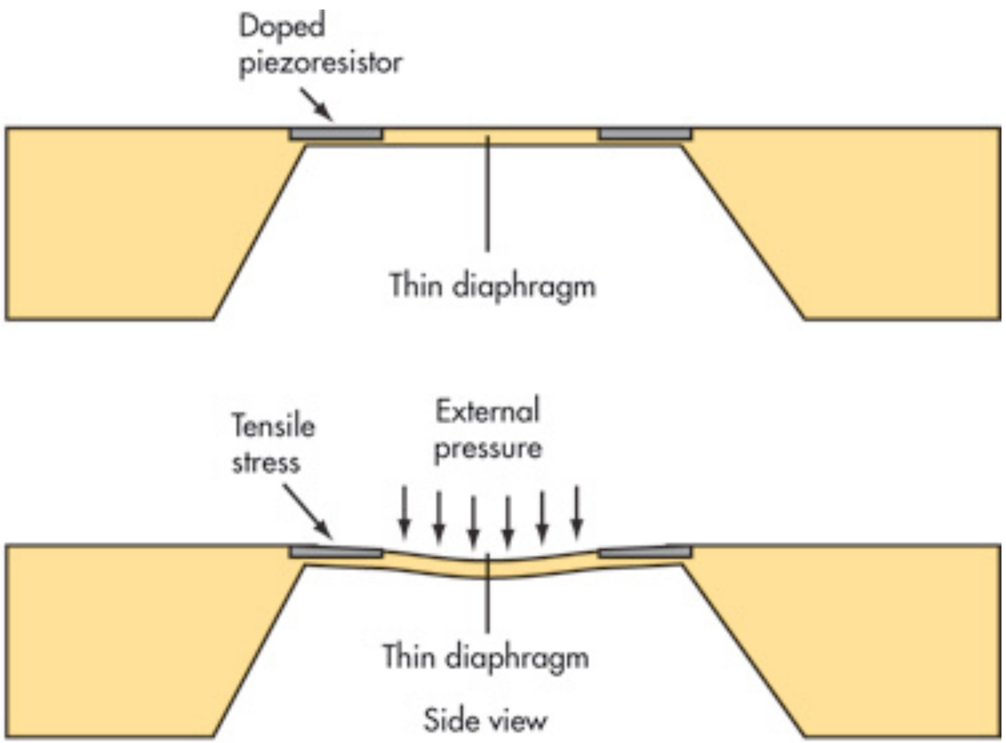
$$\Upsilon_{\text{gyro}} = k_{\text{gyro}}\Omega + \beta_{\text{gyro}} + \eta'_{\text{gyro}},$$

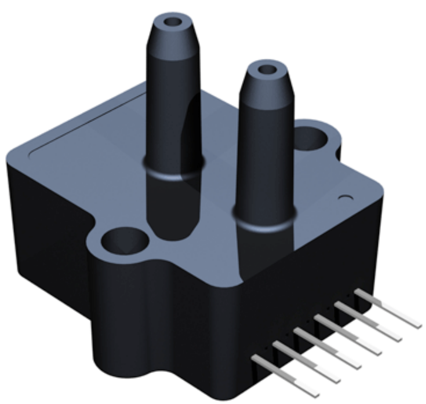
where Υ_{gyro} is in units of voltage. Calibrating to units of rad/s gives

$$\boxed{\begin{aligned} y_{\text{gyro},x} &= p + \beta_{\text{gyro},x} + \eta_{\text{gyro},x} \\ y_{\text{gyro},y} &= q + \beta_{\text{gyro},y} + \eta_{\text{gyro},y} \\ y_{\text{gyro},z} &= r + \beta_{\text{gyro},z} + \eta_{\text{gyro},z} \end{aligned}}$$

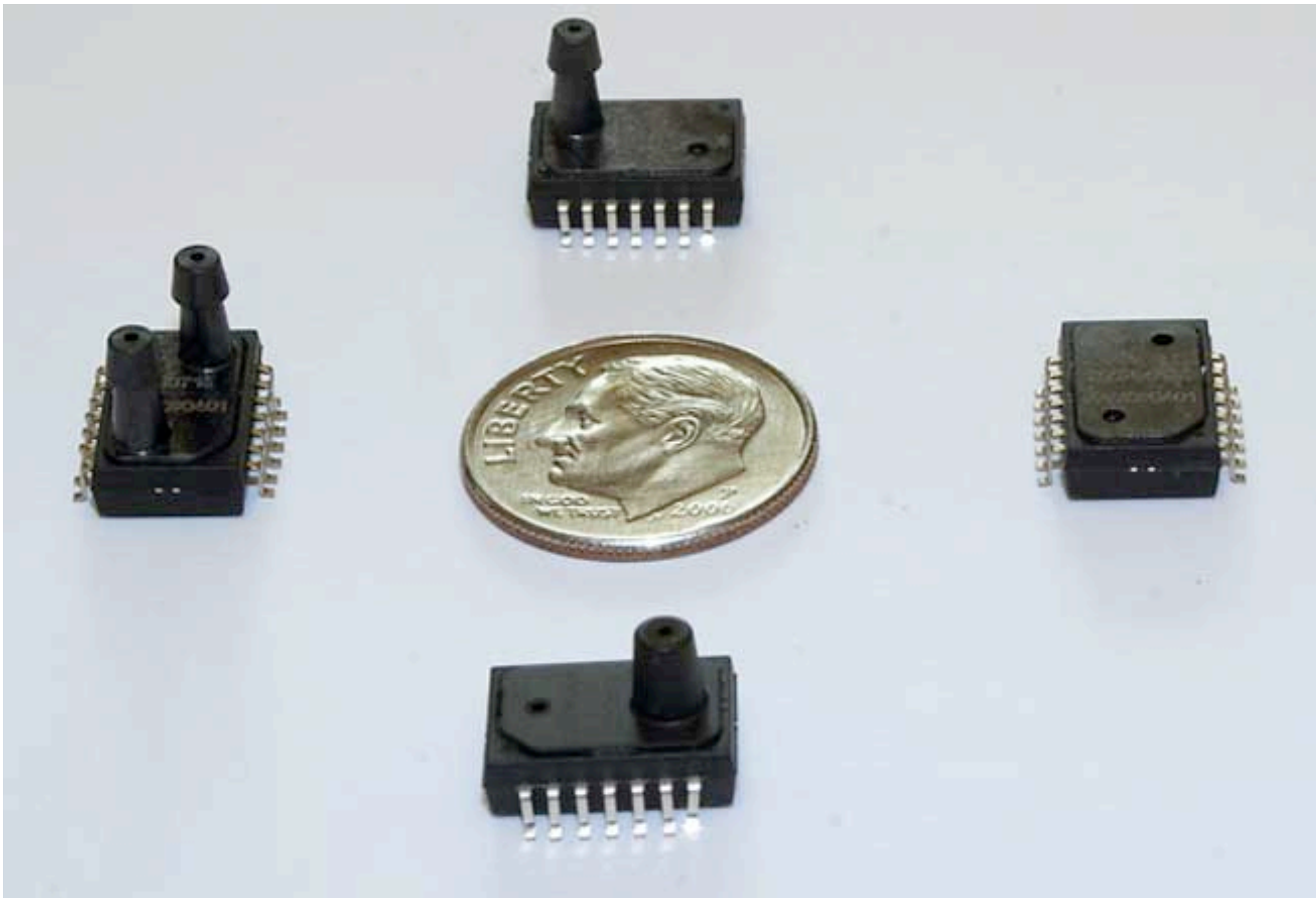
Pressure Measurement



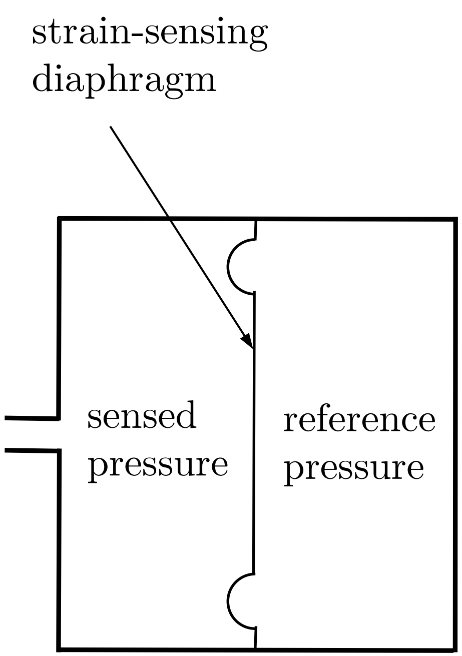




Pressure Measurement



Altitude Measurement



The basic equation of hydrostatics is

$$P_2 - P_1 = \rho g(z_2 - z_1)$$

Using the ground as reference, and assuming constant air density gives

$$\begin{aligned} P - P_{\text{ground}} &= -\rho g(h - h_{\text{ground}}) \\ &= -\rho g h_{\text{AGL}} \end{aligned}$$

Below 11,000 m, can use barometric formula:

$$P = P_0 \left[\frac{T_0}{T_0 + L_0 h_{\text{ASL}}} \right]^{\frac{gM}{RL_0}},$$

where P_0: standard pressure at sea level
T_0: standard temperature at sea level
L_0: rate of temperature decrease
g: gravitational constant
R: universal gas constant for air
M: standard molar mass of atmospheric air,

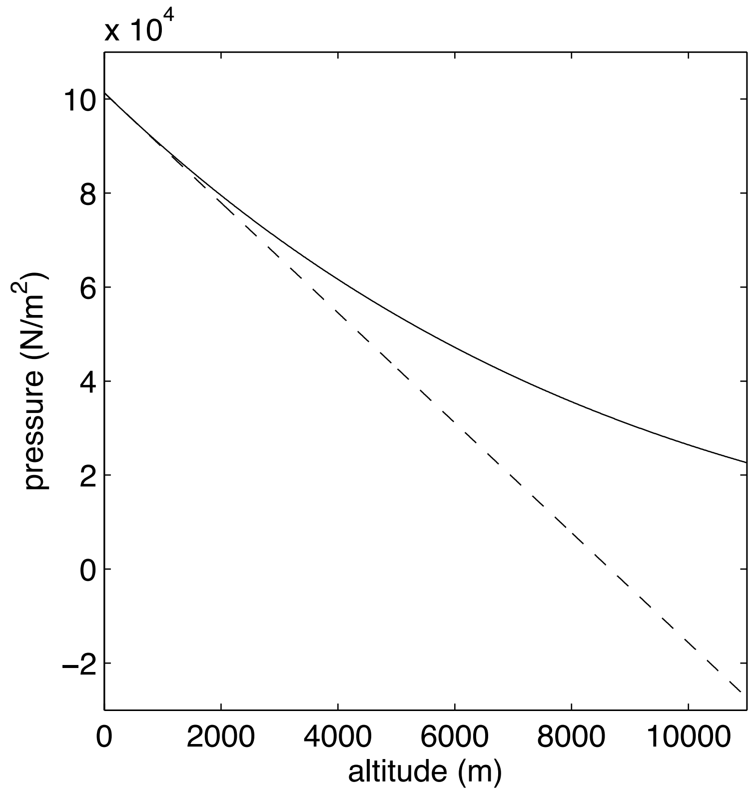
which takes into account change in density with altitude and temperature.

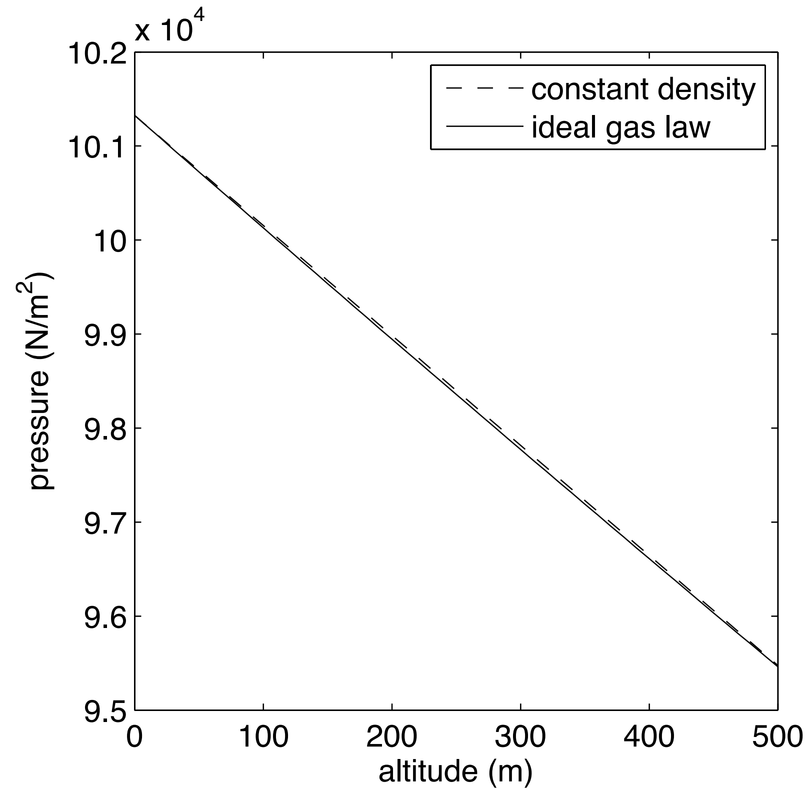
Altitude Measurement

We usually assume density is constant:

$$
\begin{aligned}
y_{\text{abs pres}} &= (P_{\text{ground}} - P) + \beta_{\text{abs pres}} + \eta_{\text{abs pres}} \\
&= \rho g h_{\text{AGL}} + \beta_{\text{abs pres}} + \eta_{\text{abs pres}}
\end{aligned}
$$

Is this valid?





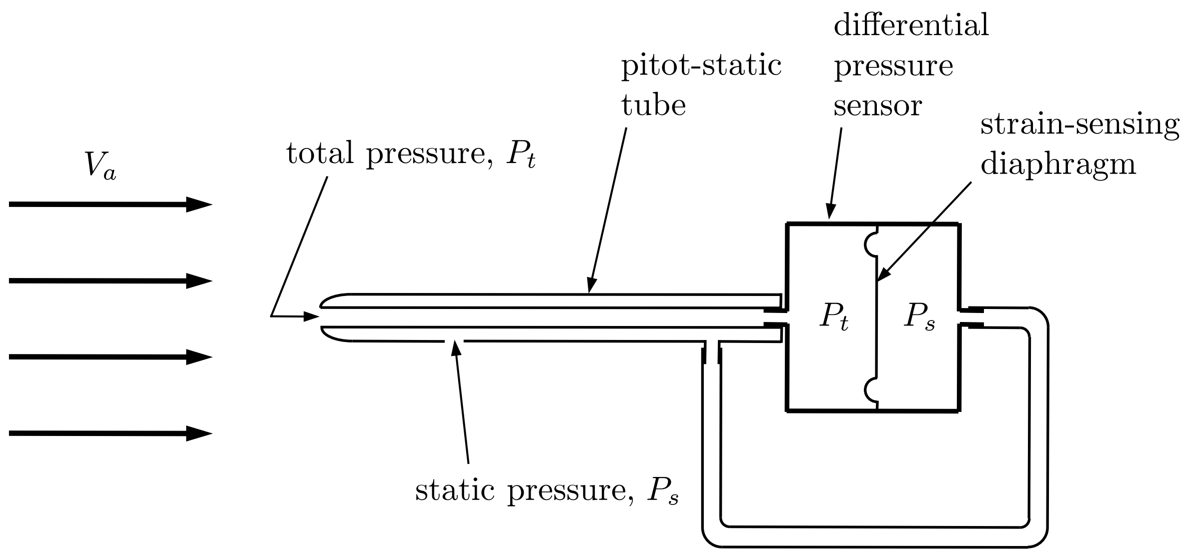
Airspeed Measurement

From Bernoulli's equation:

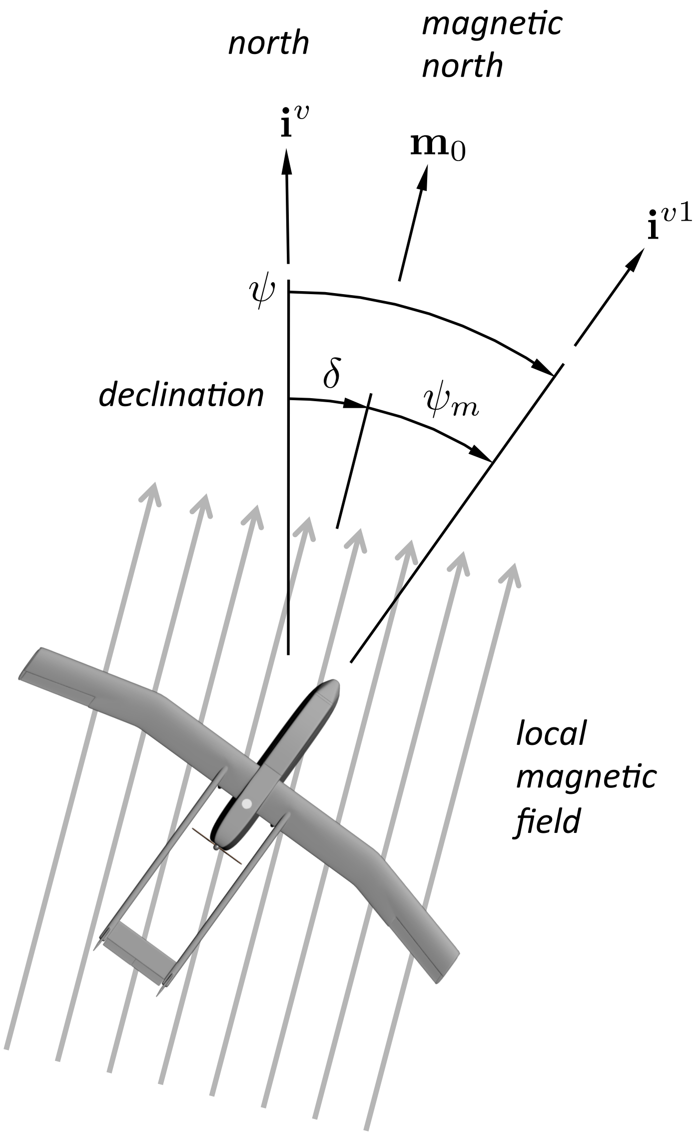
$$P_t = P_s + \frac{\rho V_a^2}{2} \qquad \text{or} \qquad \frac{\rho V_a^2}{2} = P_t - P_s$$

Pitot-static pressure sensor measures dynamic pressure:

$$y_{\text{diff pres}} = \frac{\rho V_a^2}{2} + \beta_{\text{diff pres}} + \eta_{\text{diff pres}}$$



Magnetometers & Digital Compasses



Heading is sum of magnetic declination angle and magnetic heading

$$\psi = \delta + \psi_m$$

Magnetic heading determined from measurements of body-frame components of magnetic field projected onto horizontal plane

$$\mathbf{m}_0^{v1} = \begin{pmatrix} m_{0x}^{v1} \\ m_{0y}^{v1} \\ m_{0z}^{v1} \end{pmatrix} = \mathcal{R}_b^{v1}(\phi, \theta)\mathbf{m}_0^b$$

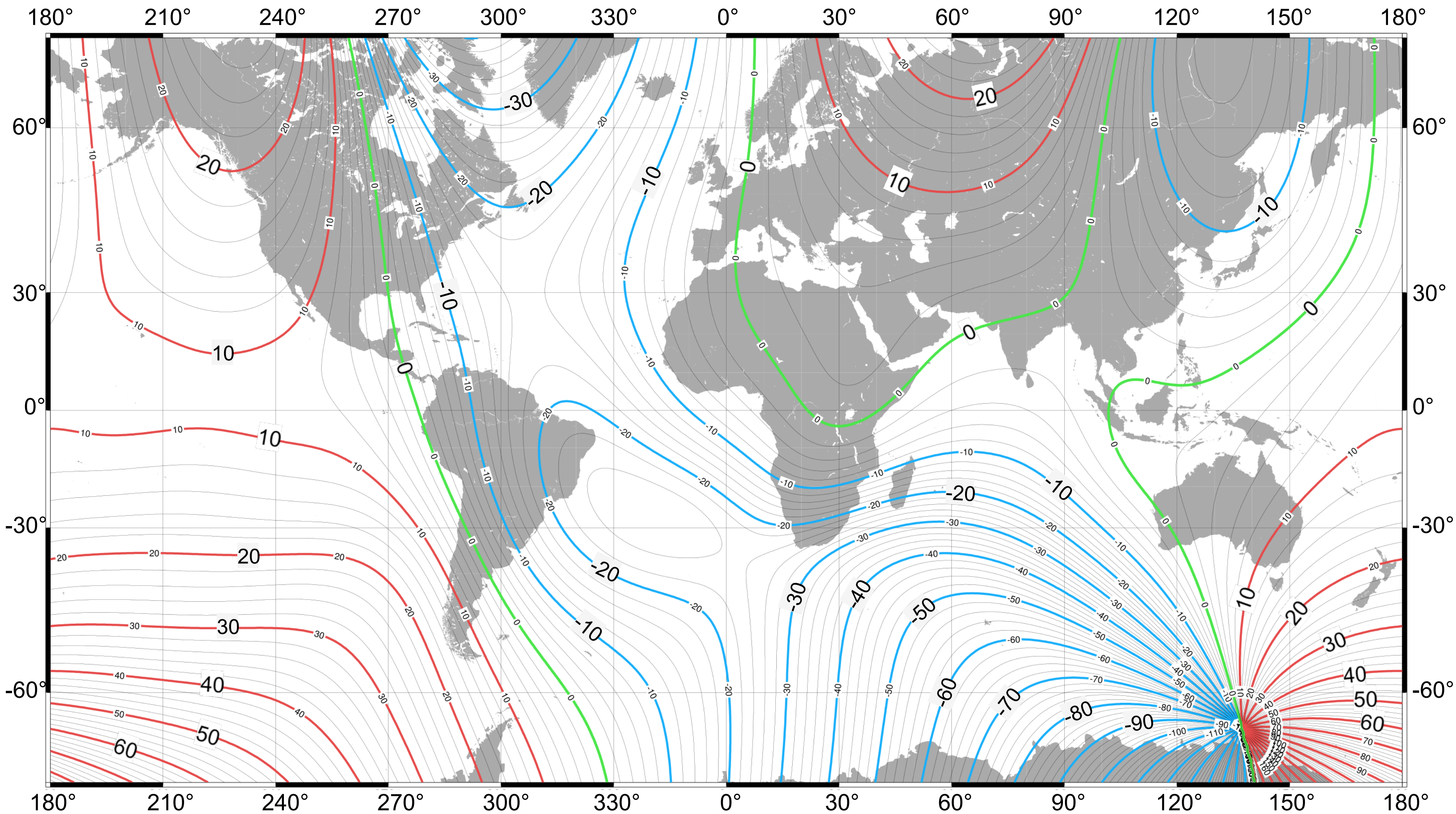
$$= \mathcal{R}_{v2}^{v1}(\theta)\mathcal{R}_b^{v2}(\phi)\mathbf{m}_0^b$$

$$\begin{pmatrix} m_{0x}^{v1} \\ m_{0y}^{v1} \\ m_{0z}^{v1} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta s_\phi & s_\theta c_\phi \\ 0 & c_\phi & -s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{pmatrix} \mathbf{m}_0^b$$

Solving for heading gives

$$\psi_m = -\text{atan2}(m_{0y}^{v1}, m_{0x}^{v1}).$$

Magnetic Declination Variation

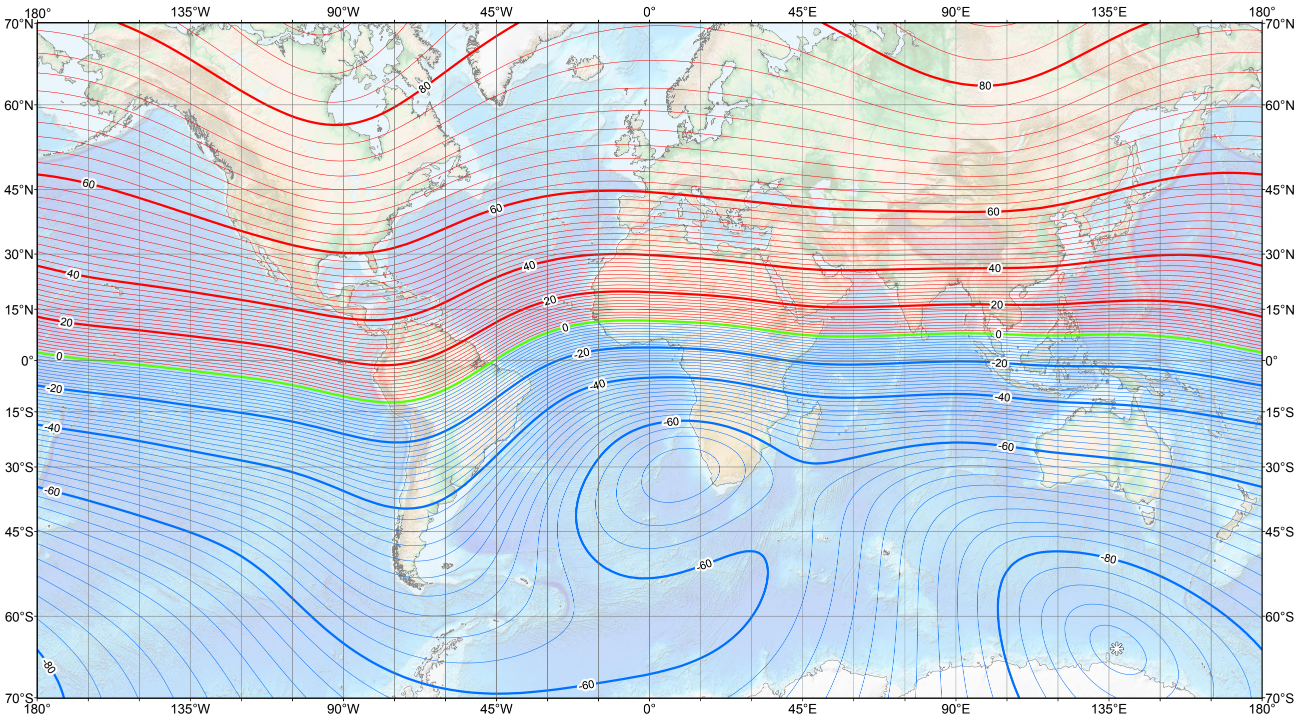


World Magnetic Model, National Geophysical Data Center

Magnetic Inclination

From Wikipedia: "Magnetic dip or magnetic inclination is the angle made by a compass needle with the horizontal at any point on the Earth's surface. Positive values of inclination indicate that the field is pointing downward, into the Earth, at the point of measurement.

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Inclination (I)

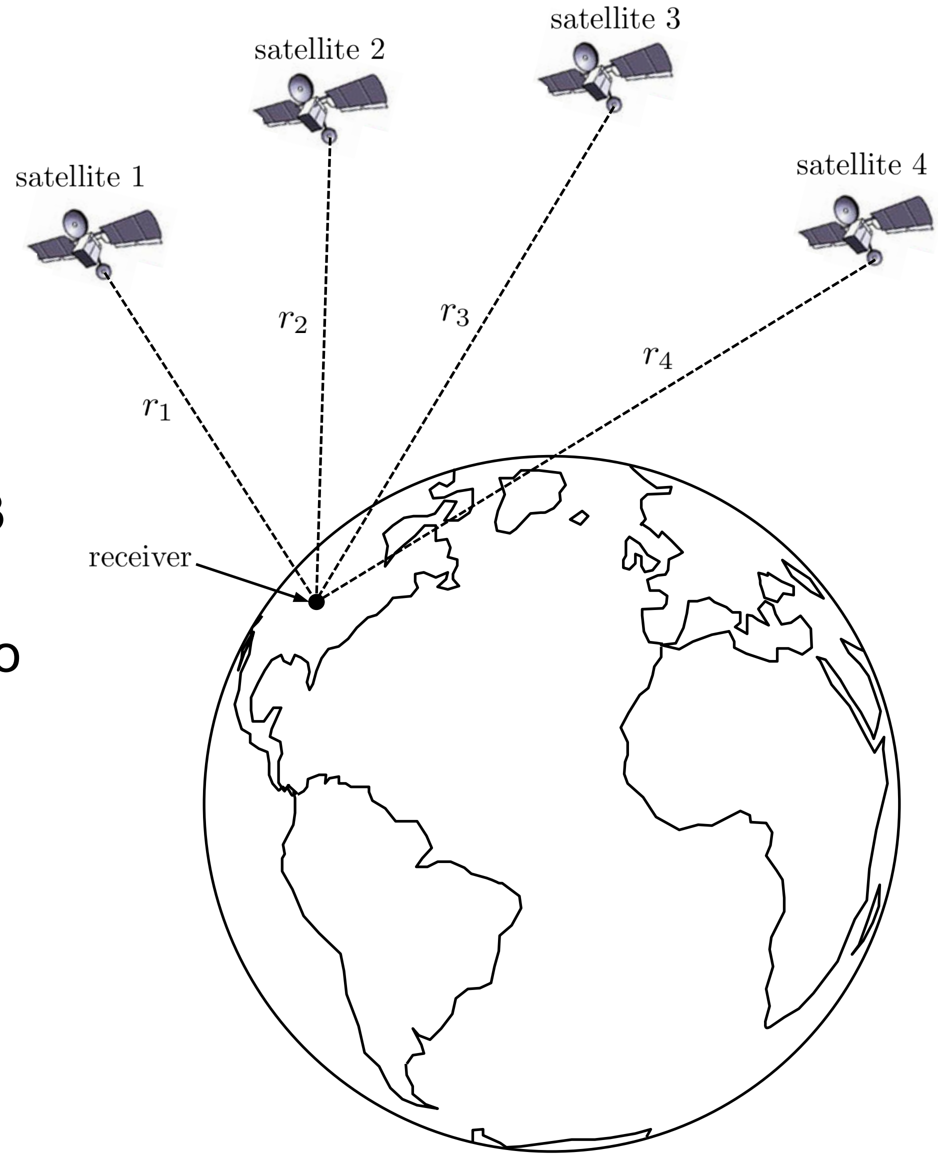


Main field inclination (I)
Contour interval: 2 degrees, red contours positive (down); blue negative (up); green zero line.
Mercator Projection.
: Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM/
Map reviewed by NGA/BGS
Published January 2010

Global Positioning System

- 24 satellites orbiting the earth
- Altitude 20,180 km
- Any point on Earth's surface can be seen by at least 4 satellites at all times
- Time of flight of radio signal from 4 satellites to receiver used to trilaterate location of receiver in 3 dimensions
- 4 range measurements needed to account for clock offset error
- 4 nonlinear equations in 4 unknowns results:
  - latitude
  - longitude
  - altitude
  - receiver clock time offset



GPS Error Sources

- Time of flight of radio signal from satellite to receiver used to calculate pseudorange
  - Called pseudorange to distinguish it from true range
- Numerous sources of error in time of flight measurement:
  - Ephemeris Data – errors in satellite location
  - Satellite Clock – due to clock drift
  - Ionosphere – upper atmosphere, free electrons slow transmission of GPS signal
  - Troposphere – lower atmosphere, weather (temperature and density) affect speed of light, GPS signal transmission
  - Multipath Reception – signals not following direct path
  - Receiver Measurement – limitations in accuracy of receiver timing
- Small timing errors can result in large position errors
  - 10 ns timing error → 3 m pseudorange error

GPS Error Characterization

- Cumulative effect of GPS pseudorange errors is described by user equivalent range error (UERE)
- UERE has two components
  - Bias
  - Random

1-σ, in meters

Error source	Bias	Random	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	2.0	0.7	2.1
Ionosphere	4.0	0.5	4.0
Troposphere monitoring	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver measurement	0.5	0.2	0.5
UERE, rms	5.1	1.4	5.3
Filtered UERE, rms	5.1	0.4	5.1

GPS Error Characterization

- Effect of satellite geometry on position calculation is expressed by dilution of precision (DOP)
- Satellites close together → high DOP
- Satellites far apart → low DOP
- DOP varies with time
- Horizontal DOP is smaller than vertical DOP
- Nominal HDOP = 1.3
- Nominal VDOP = 1.8

Total GPS Error (RMS)

Standard deviation of RMS error in the north-east plane:

$$
\begin{aligned}
E_{\text{n-e,rms}} &= \text{HDOP} \times \text{UERE}_{\text{rms}} \\
&= (1.3)(5.1 \text{ m}) \\
&= 6.6 \text{ m}
\end{aligned}
$$

Standard deviation of RMS altitude error:

$$
\begin{aligned}
E_{\text{h,rms}} &= \text{VDOP} \times \text{UERE}_{\text{rms}} \\
&= (1.8)(5.1 \text{ m}) \\
&= 9.2 \text{ m}
\end{aligned}
$$

GPS Error Model

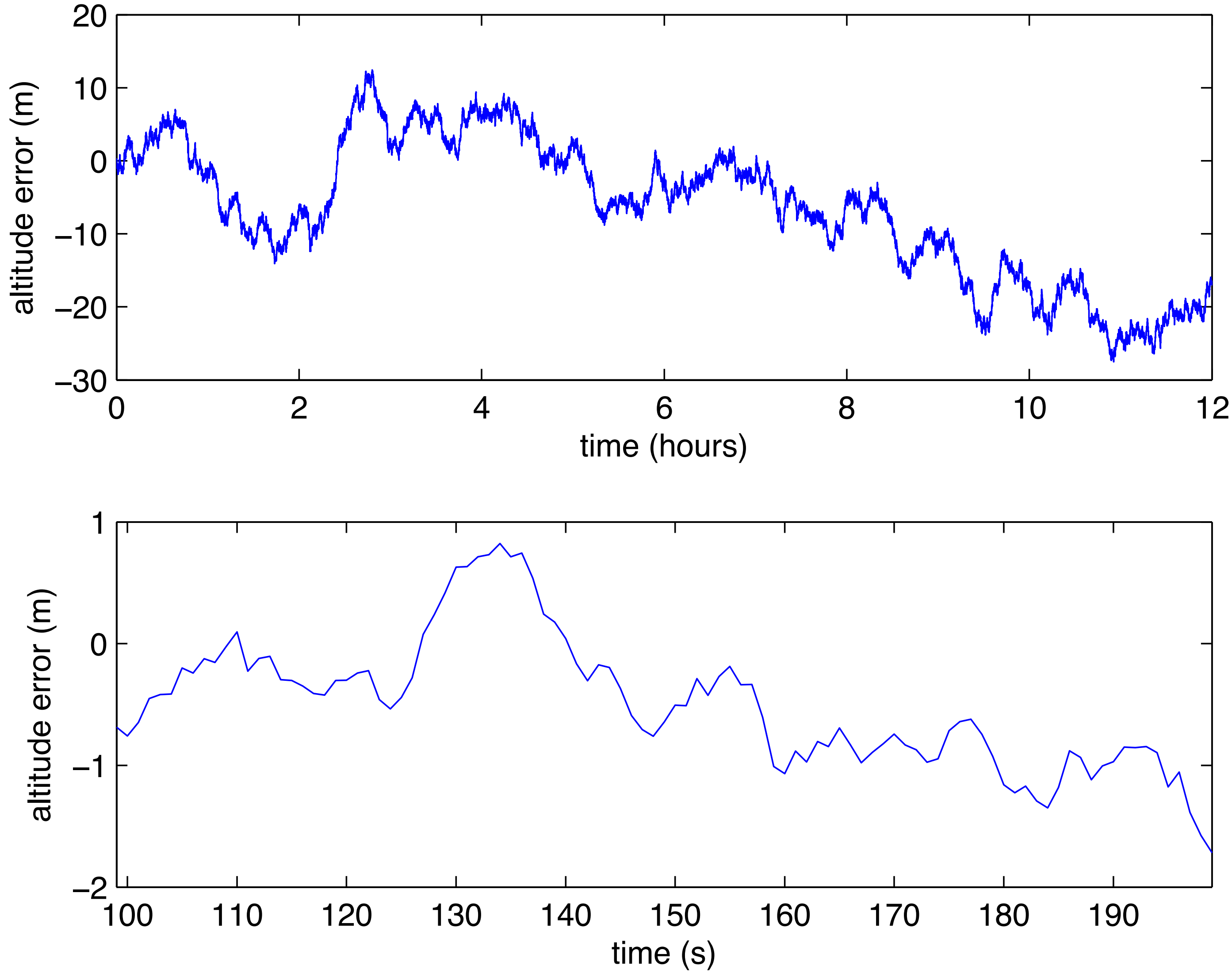
- Interested in transient behavior of errors – how does GPS error change with time
- We use Gauss-Markov error model proposed by Rankin

$$\nu[n+1] = e^{-k_{\mathrm{GPS}} T_s} \nu[n] + \eta_{\mathrm{GPS}}[n]$$

	Nominal 1-σ error (m)		Model Parameters		
Direction	Bias	Random	Std. Dev. η_{GPS} (m)	$1/k_{\mathrm{GPS}}$ (s)	T_s (s)
North	4.7	0.4	0.21	1100	1.0
East	4.7	0.4	0.21	1100	1.0
Altitude	9.2	0.7	0.40	1100	1.0

$$y_{\mathrm{GPS},n}[n] = p_n[n] + \nu_n[n]$$
$$y_{\mathrm{GPS},e}[n] = p_e[n] + \nu_e[n]$$
$$y_{\mathrm{GPS},h}[n] = -p_d[n] + \nu_h[n]$$

GPS Gauss Markov Process Error Model



Project

- Add sensor models to the simulation