TÉCNICO LISBOA

Unmanned Aerial Vehicles

M.Sc. in Aerospace Engineering

2020/2021 - Second Semester

Modeling and Identification of the Parrot AR.Drone

Laboratory handout

March 2021

1 Introduction

1.1 Objectives

The following goals are addressed in this laboratory:

1. Modeling of the kinematics and dynamics of a rigid-body.
2. Modeling of common and differential control inputs of a quadrotor.
3. Linearization of the quadrotor system dynamics about the hovering condition.
4. Analysis of inner and outer control loops for the height and vertical speed.
5. Identification of the height closed-loop control system.

1.2 Organization and timeline

This guide consists of a set of questions that explore different aspects related to modeling and identification of the Parrot AR.Drone.

There are two kinds of questions: theoretical questions, marked as (T), and laboratory questions, marked as (L). As a guideline, all theoretical questions should be solved before the laboratory session, and the simulations should be completed during that session.

A report in pdf format together with the matlab script files (`.m`), containing the code developed to answer the questions, must be submitted through fenix in the designated dates (check the course's planning). Please use the cover page available in the course's webpage as front page.

1.3 Academic ethics code

All members of the academic community of the University of Lisbon (faculty, researchers, staff members, students, and visitors) are required to uphold high ethical standards. Hence, the report submitted by each group of students must be original and correspond to their actual work.

2 Modeling

Let $\{I\}$ denote a local inertial reference coordinate frame, assumed in this work to be a North-East-Down (NED) coordinate frame centered at some point. Denote by $\{B\}$ a coordinate frame attached to the vehicle with origin at the vehicle's center of mass, usually referred to as the body-fixed reference frame. The kinematics of a rigid-body in 3-D are

$$\begin{cases} \dot{\mathbf{p}} = R(\boldsymbol{\lambda})\mathbf{v} \\ \dot{\boldsymbol{\lambda}} = Q(\boldsymbol{\lambda})\boldsymbol{\omega} \end{cases},$$

where

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$$

is the position of $\{B\}$ with respect to $\{I\}$,

$$\mathbf{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \in \mathbb{R}^3$$

is the linear velocity of $\{B\}$ with respect to $\{I\}$, expressed in $\{B\}$,

$$\boldsymbol{\lambda} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

is the vector of Z-Y-X Euler angles, where $\phi \in \mathbb{R}$, $\theta \in \,]-\pi/2, \pi/2[$, and $\psi \in \mathbb{R}$ are the roll, pitch, and yaw Euler angles, respectively,

$$\boldsymbol{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \in \mathbb{R}^3$$

is the angular velocity of $\{B\}$ with respect to $\{I\}$, expressed in $\{B\}$,

$$R(\boldsymbol{\lambda}) = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \in SO(3)$$

is the rotation matrix from $\{B\}$ to $\{I\}$, and

$$Q(\boldsymbol{\lambda}) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

The dynamics of a rigid-body in 3-D can be described by

$$\begin{cases} m\dot{\mathbf{v}} = -m\,S(\boldsymbol{\omega})\mathbf{v} + \mathbf{f} \\ J\dot{\boldsymbol{\omega}} = -S(\boldsymbol{\omega})J\boldsymbol{\omega} + \mathbf{n} \end{cases}, \tag{1}$$

where $S(.)$ is the skew-symmetric operator such that $S(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}$, $m \in \mathbb{R}^+$ is the mass, $J \in \mathbb{R}^{3\times 3}$ is the inertia tensor matrix, given by

$$J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix},$$

$\mathbf{f} \in \mathbb{R}^3$ and $\mathbf{n} \in \mathbb{R}^3$ are the vectors of external forces and torques, given by

$$\mathbf{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \quad \text{and} \quad \mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix},$$

respectively.

The Parrot AR.Drone is based on a classic quadrotor design, with four rotors, mounted symmetrically along two orthogonal axes, as depicted in Fig. 1, where the individual thrusts are denoted by $f_i \in \mathbb{R}$, $i = 1, \ldots, 4$. The corresponding individual torques are denoted by $n_i = cT_i \in \mathbb{R}$, $i = 1, \ldots, 4$, where $c \in \mathbb{R}$ is some scalar constant. In order to control all three angular degrees of freedom, two pairs of rotors are counter-rotating, as shown in the figure.

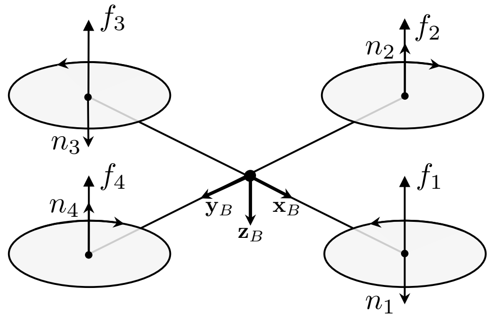

Figure 1: Simplified model of the Parrot AR.Drone

Define the vector of individual rotors thrusts

$$\mathbf{f}_T := \begin{bmatrix} f_1 \\ \vdots \\ f_4 \end{bmatrix} \in \mathbb{R}^4$$

and denote the gravitational force $\mathbf{f}_g \in \mathbb{R}^3$, expressed in $\{I\}$, by $\mathbf{f}_g = mg\mathbf{e}_3$, where $\mathbf{e}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

2.1. **(T)** Show that $\mathbf{f}$ and $\mathbf{n}$ in (1) can be written as a functions of $\mathbf{f}_T$ and $\mathbf{f}_g$, such that

$$\mathbf{f} = M\mathbf{f}_g + N\mathbf{f}_T \tag{2}$$

and

$$\mathbf{n} = P\mathbf{f}_T. \tag{3}$$

Determine the expressions for the matrices M, N, and P.

2.2. **(T)** The axes of the rotors of the Parrot AR.Drone are installed in a slightly different way from the one shown in Fig. 1, forming an angle of 45 degrees with the x and y body-fixed axes. Do the expressions for $\mathbf{f}$ and $\mathbf{n}$ change because of this change in configuration? If so, derive the new expression(s).

2.3. **(T)** Define a new control input

$$\mathbf{u} = \begin{bmatrix} T \\ \mathbf{n} \end{bmatrix} \in \mathbb{R}^4,$$

where the scalar T is given by $T = \sum_{i=1}^{4} f_i$ and $\mathbf{n} \in \mathbb{R}^3$ is given by (3), yielding

$$L\mathbf{f}_T = \mathbf{u},$$

where $L \in \mathbb{R}^{4\times4}$. Discuss the usefulness of considering this linear input transformation and write an explicit expression for the inverse transformation from $\mathbf{u}$ to $\mathbf{f}_T$.

2.4. **(T)** Define the alternative position vector ${}^B\mathbf{p} = R^T\mathbf{p}$ and let $\mathbf{x} = \begin{bmatrix} {}^B\mathbf{p}^T & \mathbf{v}^T & \boldsymbol{\lambda}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \in \mathbb{R}^{12}$ and $\mathbf{x}_1 = \begin{bmatrix} \mathbf{p}^T & \dot{\mathbf{p}}^T & \boldsymbol{\lambda}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \in \mathbb{R}^{12}$ denote two alternative state vectors for the system that describes a quadrotor. The nonlinear dynamics take the form $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$ and $\dot{\mathbf{x}}_1 = \mathbf{g}_1(\mathbf{x}_1, \mathbf{u})$, respectively. Determine the expressions for $\mathbf{g}(\mathbf{x}, \mathbf{u})$ and $\mathbf{g}_1(\mathbf{x}_1, \mathbf{u})$.

2.5. **(T)** The quadrotor is a flat system, meaning that the all the states and inputs can be expressed as functions of the so-called flat output $\mathbf{y}$ and its time derivatives. For the case of quadrotors, the flat ouput takes the form $\mathbf{y} = \begin{bmatrix} \mathbf{p}^T & \psi \end{bmatrix}^T \in \mathbb{R}^4$. Show that, in particular, u_1, ϕ, and θ can be written as functions of $\ddot{\mathbf{y}}$ and ψ. Give a physical interpretation for this result.

2.6. **(T)** Consider that, in equilibrium, the quadrotor is hovering at a given position $\mathbf{p}_0$ with an arbitrary but constant yaw angle ψ_0. The equilibrium point for the system $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$ is fully described by a fixed flat output $\mathbf{y}_0 = \begin{bmatrix} \mathbf{p}_0^T & \psi_0 \end{bmatrix}^T$. Let $\mathbf{x}_0 = \begin{bmatrix} {}^B\mathbf{p}_0^T & \mathbf{v}_0^T & \boldsymbol{\lambda}_0^T & \boldsymbol{\omega}_0^T \end{bmatrix}^T$ denote the equilibrium state and $\mathbf{u}_0$ denote the corresponding equilibrium input. Determine $\mathbf{x}_0$ and $\mathbf{u}_0$.

2.7. **(T)** Define the incremental variables $\delta\mathbf{x} = \begin{bmatrix} \delta\mathbf{p}^T & \delta\mathbf{v}^T & \delta\boldsymbol{\lambda}^T & \delta\boldsymbol{\omega}^T \end{bmatrix}^T$, where

$$
\begin{aligned}
\delta\mathbf{p} &= {}^B\mathbf{p} - R^T\mathbf{p}_0 = R^T(\mathbf{p} - \mathbf{p}_0) \\
\delta\mathbf{v} &= \mathbf{v} - \mathbf{v}_0 \\
\delta\boldsymbol{\lambda} &= \boldsymbol{\lambda} - \boldsymbol{\lambda}_0 \\
\delta\boldsymbol{\omega} &= \boldsymbol{\omega} - \boldsymbol{\omega}_0,
\end{aligned}
$$

and

$$\delta\mathbf{u} = \mathbf{u} - \mathbf{u}_0,$$

to be used in the linearization of the system dynamics. The linearized system dynamics are given by

$$\delta\dot{\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta\mathbf{u}.$$

Determine $\mathbf{A} \in \mathbb{R}^{12\times 12}$ and $\mathbf{B} \in \mathbb{R}^{12\times 4}$.

2.8. **(T)** Consider the individual elements of $\delta\mathbf{x}$ and $\delta\mathbf{u}$ as specified in

$$\delta\mathbf{x} = \begin{bmatrix} \delta x & \delta y & \delta z & \delta\phi & \delta\theta & \delta\psi & \delta u & \delta v & \delta w & \delta p & \delta q & \delta r \end{bmatrix}^T$$

and

$$\delta\mathbf{u} = \begin{bmatrix} \delta T & \delta n_x & \delta n_y & \delta n_z \end{bmatrix}^T,$$

respectively. Denote by

$$\mathbf{X}(s) = \begin{bmatrix} X(s) & Y(s) & Z(s) & \Phi(s) & \Theta(s) & \Psi(s) & U(s) & V(s) & W(s) & P(s) & Q(s) & R(s) \end{bmatrix}^T$$

and

$$\mathbf{U}(s) = \begin{bmatrix} T(s) & N_x(s) & N_y(s) & N_z(s) \end{bmatrix}^T$$

the Laplace transforms of $\delta\mathbf{x}$ and $\delta\mathbf{u}$, respectively. Determine the transfer functions

$$G_\phi(s) = \frac{\Phi(s)}{N_x(s)}, \quad G_\theta(s) = \frac{\Theta(s)}{N_y(s)}, \quad G_\psi(s) = \frac{\Psi(s)}{N_z(s)},$$

$$G_x(s) = \frac{X(s)}{N_y(s)}, \quad G_y(s) = \frac{Y(s)}{N_x(s)}, \quad \text{and} \quad G_z(s) = \frac{Z(s)}{T(s)}.$$

Note: To obtain the transfer functions, assume that the moment of inertia J is diagonal.

2.9. **(T)** Relate $G_\phi(s)$ with $G_y(s)$ and $G_\theta(s)$ with $G_x(s)$. Give a physical interpretation for the obtained result and discuss the effect of considering different equilibria ψ_0 for the yaw angle.

2.10. **(T)** For the sake of simplicity, let $h = -z$ denote the height of the quadrotor above the ground. Consider the inner-outer loop control scheme for the height depicted in Fig. 2 and the alternative control scheme shown in Fig. 3. In both diagrams, h_{ref} denotes the reference height.

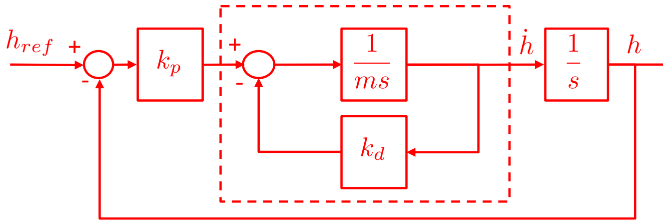

Figure 2: Height control scheme 1

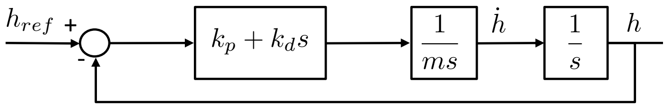

Figure 3: Height control scheme 2

Determine the closed-loop transfer functions for both control schemes and discuss the differences and possible advantages and disadvantages of each scheme.

2.11. **(T)** Considering the controller from Fig. 2, sketch the root-locus with respect to $k_p > 0$, assuming that $k_d > 0$ is a fixed but arbitrary. Discuss the effect that changing the control gains has on the response of the closed-loop system to step reference inputs.

3 Identification of the closed-loop height dynamics

The main goal of this section is to identify the closed-loop height dynamics using the simulation model and compare with the experimental data provided in `Height_x.mat` files.

Simulations

The ARDrone Simulink Development Kit provides the means for both simulation of the Parrot Ar.Drone and experiments. To begin simulations, run the script `start_here`. Then, choose option `(1) Simulation`. Finally, choose option `(2) Hover: Vehicle is held at constant position`. This will open the simulink model `ARDroneHoverSim.slx`.

You should first get familiarized with the several blocks, options, and scopes in `ARDroneHoverSim.slx`. In particular, you should verify that the implemented model is based on a linearization about the hovering condition, similar to the one derived in Section 2.

To obtain answers for the following questions, it is recommended that you save `ARDroneHoverSim.slx` with a different name, e.g. `ARDroneHoverSimHeight.slx`, and work with it. This new model should be modified in order to perform the required simulations for the identification of the height dynamics.

3.1. **(L)** Modify the Simulink model `ARDroneHoverSimHeight.slx` to obtain step responses using four different values for the proportional height controller gain. Define reference height signals according to the parameters detailed in Table 1. For each value of the gain, plot the simulated responses

Gain	Step at t=5 s	Cumulative step at t=15s
0.5	1.0 m	1.0 m
1.0	1.0 m	0.5 m
2.4	1.0 m	0.25 m
3.2	1.0 m	0.2 m

Table 1: Parameters for height reference inputs

together with the real responses provided in the `Height_x.mat` files. Comment the reason for considering steps of decreasing magnitude as the gain increases.

3.2. **(L)** Relate the obtained responses with the structure of the height control system described in Fig. 2. Discuss whether or not the theoretical, simulation, and experimental results are consistent. Hint: notice that the gain k_p in Fig. 2 can be identified with the proportional control gain used in the height controller. Also note that the root-locus sketch from Question 2.11. provides useful information.

3.3. **(L)** To further analyse the system, explore the simulink model to obtain plausible values for k_d or an alternative structure for the inner-loop system. For each value of k_p, plot the pole and zero map for the closed-loop system and compare with the root-locus sketch from Question 2.11.. Indicate possible reasons for the observed differences.