CIS2520 Data Structures
Fall 2011, Assignment 3

PART A (35%)

All functions below (f, f_1, f_2, g, g_1, g_2, h, √, log, etc.) are seen as functions from $\mathbb{Z}_+$ (the set of positive integers) to $\mathbb{R}_+$ (the set of positive real numbers). The domain of definition of a function f is denoted by D_f. The natural logarithm (logarithm to base e) is denoted by log.

Definitions

D1
We say that f is defined on a neighbourhood of infinity iff:
$\exists m\in \mathbb{Z}_+, m..+\infty \subseteq D_f$

D2
We say that f is less than or equal to g on a neighbourhood of infinity, and we write $f \underset{+\infty}{\leq} g$, iff:
$\exists m\in \mathbb{Z}_+, \forall n\in m..+\infty, f(n)\leq g(n)$

Note that f and g are then defined on a neighbourhood of infinity
(m..+∞ in the expression above).

D3
We say that f is O(g) iff:
$\exists\lambda\in \mathbb{R}_+, \exists m\in \mathbb{Z}_+, \forall n\in m..+\infty, f(n)\leq\lambda g(n)$

Note that f and g are then defined on a neighbourhood of infinity
(m..+∞ in the expression above).

D4
Let L be a real number. We say that the limit of f at +∞ is L, and we write $\lim\limits_{n\to+\infty} f(n)=L$, iff:
$\forall\varepsilon\in \mathbb{R}_+, \exists m\in \mathbb{Z}_+, \forall n\in m..+\infty, |f(n)-L|<\varepsilon$

In this case, we also say that the limit (exists and) is finite. Moreover, note that
f is then defined on a neighbourhood of infinity (m..+∞ in the expression above).

D5
We say that the limit of f at $+\infty$ is $+\infty$, and we write $\lim\limits_{n \to +\infty} f(n) = +\infty$, iff:
$\forall \varepsilon \in \mathbb{R}_+, \exists m \in \mathbb{Z}_+, \forall n \in m..+\infty, f(n) > \varepsilon$

In this case, we also say that the limit (exists and) is infinite. Moreover, note that f is then defined on a neighbourhood of infinity ($m..+\infty$ in the expression above).

Properties

P1
For any positive real numbers $\alpha \leq 1$, $\beta > 1$ and $\gamma > 1$ we have:
$1 \underset{+\infty}{\leq} \log_\beta n \underset{+\infty}{\leq} n^\alpha \underset{+\infty}{\leq} n.\log_\beta n \underset{+\infty}{\leq} n^\gamma \underset{+\infty}{\leq} \beta^n \underset{+\infty}{\leq} n! \underset{+\infty}{\leq} n^n$

P2
For any positive real number α we have: α is O(1).

P3
For any positive real numbers $\alpha > 1$ and $\beta > 1$ we have: $\log_\alpha$ is $O(\log_\beta)$.

P4
If f is defined on a neighbourhood of infinity then f is O(f).

P5
For any positive real number α,
if there exists a positive real number ε such that $\varepsilon \underset{+\infty}{\leq} f$ then $f+\alpha$ is O(f).

P6
If $f-g$ is defined on a neighbourhood of infinity then $f-g$ is O(f).

P7
If $f \underset{+\infty}{\leq} g$ then f is O(g).

P8
If f is O(g) and g is O(h) then f is O(h).

P9
If f is O(h) and g is O(h) then f+g is O(h).

P10
If f_1 is $O(g_1)$ and f_2 is $O(g_2)$ then f_1+f_2 is $O(g_1+g_2)$.

P11
If f_1 is $O(g_1)$ and f_2 is $O(g_2)$ then f_1+f_2 is $O(\max(g_1,g_2))$.

P12
If f_1 is $O(g_1)$ and f_2 is $O(g_2)$ then f_1f_2 is $O(g_1g_2)$.

P13
If the limit of f/g at $+\infty$ is finite then f is $O(g)$.

P14
If the limit of f/g at $+\infty$ is infinite then g is $O(f)$.

Questions

1) Show that we may have

(a) f_1 is $O(g_1)$, f_2 is $O(g_2)$, f_1-f_2 and g_1-g_2 are defined on a neighbourhood of infinity, but f_1-f_2 is not $O(g_1-g_2)$.

(b) f is $O(g)$,
but the limit of f/g at $+\infty$ does not exist.

2) Prove **(a)** P3, **(b)** P5, **(c)** P11, **(d)** P13.

3) Using the properties P1 to P14, show, step-by-step, that

(a) $2n^4+9(n^3\sqrt{n})-17$ is $O(n^4)$.

(b) $18\,n\log_2 n+19n+3$ is $O(n\log n)$.

PART B (65%)

Download **assign3.zip** from **Moodle**. It packs 9 files:

StudentType.h
StudentInterface.h
StudentImplementation.c

TreeType.h
TreeInterface.h
TreeImplementation.c

myProgram.c
test.txt
makefile

Complete **TreeImplementation.c** and **myProgram.c**. The latter is a simple test program: it uses the functions declared in **StudentInterface.h** and **TreeInterface.h** to create a binary search tree from the student data stored in **test.txt** and to sort these data according to the students' grades. The output of the program should be the following:

```
Ashley     35%
Tom        45%
Adam       45%
Liz        55%
Bob        60%
Karen      65%
Ann        70%
John       75%
Pete       80%
Mary       85%
Dave       90%
```

SUBMISSION

PART A
Turn in a hardcopy at the start of the lecture on Nov 15, i.e., no later than 2:31pm.

PART B
Place the 9 text files (and the 9 text files only)
in a folder **CIS2520_LastNameFirstName_A3**.
Zip it and upload it to **Moodle** by Nov 13, 11:55pm.