Who's the Best?

A Bayesian Hierarchical Model to Apportion Skill Contributions

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Stat 651 Project

Goals

1. Rank players by ability
2. Assess how significantly choice of faction affects outcome
3. Rank factions by relative advantage

Game	Player	Faction	Score	ScoreTransformed	ScoreProp
1	Kyle	Jol-nar	8	9	0.19148936
1	Tyrel	Hacan	9	11	0.23404255
1	Sam	Sol	10	14	0.29787234
1	Tyler	Letnev	9	11	0.23404255
1	Phil	Norr	2	2	0.04255319

- Conditionally independent only at game level
- 9 games total
- To get final score: take original, add how many players you beat, and divide by total points scored in game.

$$p(\underline{x_i}|\underline{\theta_p}, \underline{\theta_f}) \overset{ind}{\sim} \text{Dirichlet}(\underline{\theta_{pi}} + \underline{\theta_{fi}})$$

$$p(\theta_{pi}|\lambda) \overset{iid}{\sim} \text{Exp}(\frac{1}{3\lambda})$$

$$p(\theta_{fi}|\lambda) \overset{iid}{\sim} \text{Exp}(\frac{1}{3(1-\lambda)})$$

$$p(\lambda) \sim \text{Beta}(2, 4)$$

Dirichlet distribution enforces $\sum_{i=1}^{nPlayers} x_i = 1$ which is why we use proportion.

Computational Details

- Calculated log complete conditionals up to proportionality (no closed forms)
- Individual proposals for each parameter
- 4 chains, 50,000 iterations each, 5,000 removed for burn-in

Convergence Diagnostics

	n-Effective	$\hat{R}$		n-Effective	$\hat{R}$
Doug	14015	1.001	Letnev	5749	1.003
Jared	8167	1.000	Mentak	7421	1.003
Kyle	4500	1.001	Muaat	5820	1.002
Landen	6491	1.001	Naalu	6376	1.003
Phil	7406	1.001	Nekro	5650	1.002
Sam	5071	1.001	Norr	7471	1.001
Tyler	5236	1.001	Saar	4610	1.001
Tyrel	4780	1.001	Sol	5998	1.002
Arborec	5798	1.001	Winnu	5093	1.002
Ghosts	5838	1.002	Xxcha	7556	1.002
Hacan	4792	1.002	Yin	6935	1.001
Jol-nar	7085	1.002	Yssaril	4068	1.003
L1z1x	4362	1.002	λ	1594	1.004

Player Strength Estimates with 50% Probability Intervals
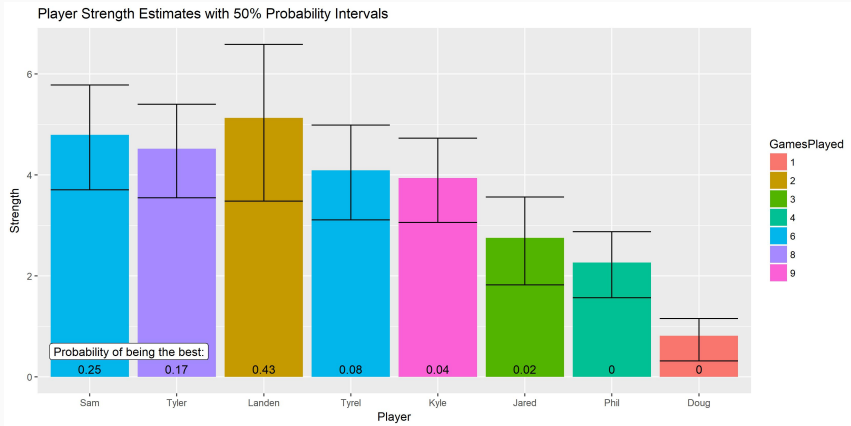
Probability Matrix

Probability that Player A is better than Player B

		Jared	Kyle	Landen	B Phil	Sam	Tyler	Tyrel
	Doug	0.07	0.01	0.02	0.09	0.00	0.00	0.01
	Jared	-	0.17	0.15	0.64	0.10	0.10	0.18
	Kyle	-	-	0.29	0.93	0.24	0.30	0.45
A	Landen	-	-	-	0.91	0.55	0.60	0.68
	Phil	-	-	-	-	0.04	0.03	0.08
	Sam	-	-	-	-	-	0.58	0.71
	Tyler	-	-	-	-	-	-	0.63
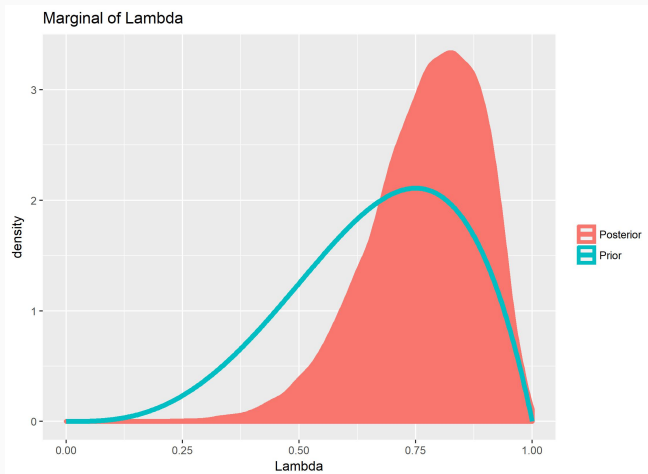
The probability of faction effects being larger than player effects dropped from 18.8% to 2.5%.

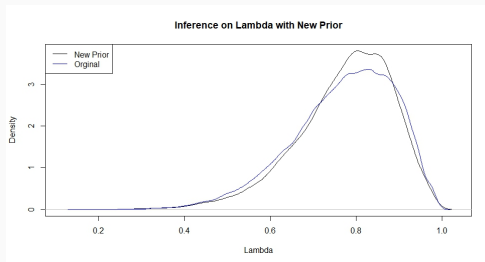Faction Strength Estimates with 50% Probability Intervals
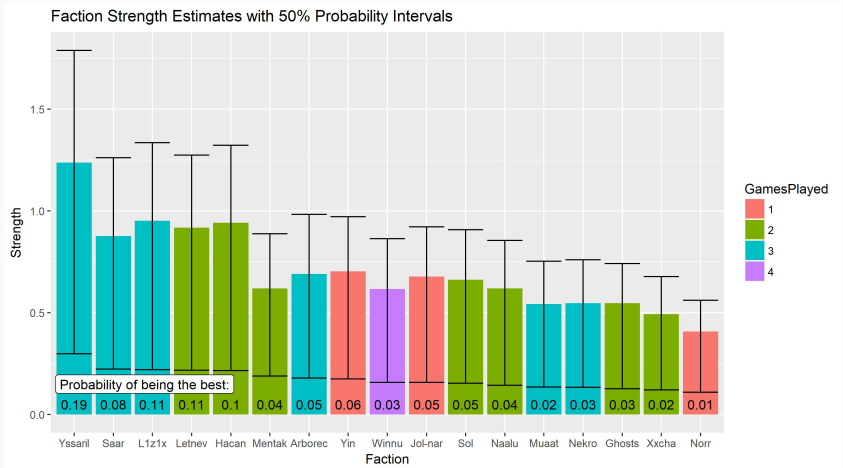
Alternate Lambda Prior

Lambda Under 2 Priors
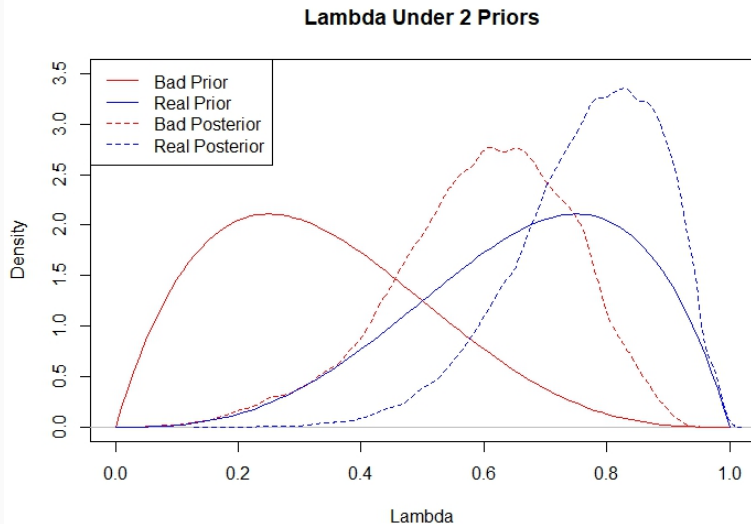
Our exponential prior of $1/(3\lambda)$ for player strengths and $1/(3*(1-\lambda))$ for faction strengths was arbitrary. What if we changed 3 to 10?

Inference on Lambda with New Prior

- No change to rank order of expected value of players
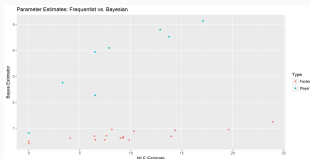- However, increased separation of players (because of higher expected variance between players)

Figure 1: Unconstrained MLEs for parameters, plotted against the Bayes Estimator.
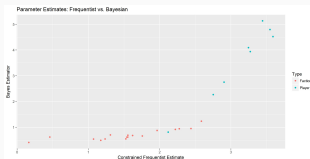

Figure 2: Constrained frequentist estimator, plotted against Bayes Estimator.

Weaknesses and Future Work

- In a large data setting, I would adjust my strength priors to estimate the variance of player and faction strengths separately.
- Stationarity in player skill
- Strategies evolve
- The exponential prior seems to unfairly punish or reward those with small samples
- Why additive strengths? A multiplicative model for the faction effect is equally reasonable.