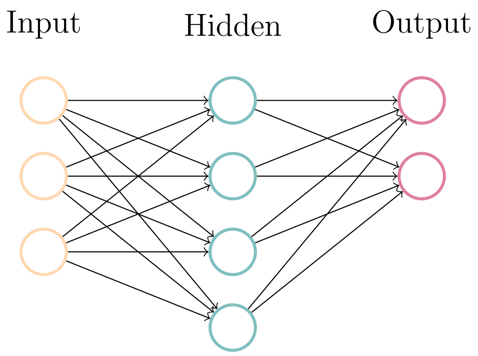
1 Week1

Introduction to Deep Learning.

1.1 What is a Neural Network?

In machine learning, a neural network (also artificial neural network or neural net, abbreviated ANN or NN) is a model inspired by the neuronal organization found in the biological neural networks in animal brains. (from wikipedia)

An artificial neural network is an interconnected group of nodes, inspired by a simplification of neurons in a brain. Here, each circular node represents an artificial neuron and an arrow represents a connection from the output of one artificial neuron to the input of another.



1.2 Supervised Learning with Neural Networks

Standard NN, Convolutional NN, Recurrent NN.
Structured Data vs. Unstructured Data.

1.3 Why is Deep Learning taking off?

Scale drives deep learning progress.

1.4 About this Course

1. Neural Networks and Deep Learning
2. Improving Deep Neural Networks: Hyperparameter tuning, Regularization and Optimization
3. Structuring your Machine Learning project
4. Convolutional NeuralNetworks
5. Natural Language Processing: Building sequence models

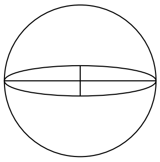
1.5 Outline of this Course

- Week1. Introduction
- Week2. Basics of Neural Network programming
- Week3. One hidden layer Neural Networks
- Week4. Deep Neural Networks

2 Week2

Basics of Neural Network Programming

How do I write an equation in LaTeX?



In 1902, Einstein created this equation: $E = mc^2$

And Newton came up with this one: $\sum F = ma$

$$5 + 5 = 10 \tag{1}$$

$$\begin{aligned} A &= \frac{5\pi r^2}{2} \\ A &= \frac{1}{2}\pi r^2 \end{aligned} \tag{2}$$

2.1 Neural Network Notations

General comments:
superscript (i) will denote the i^{th} training example.
superscript $[l]$ will denote the l^{th} layer.

Sizes:

- m: number of examples in the dataset
- n_x: input size
- n_y: output size
- $n_h^{[l]}$: number of hidden units of the $l^t h$ layer.
  In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h^{[number of layer+1]}$
- L: number of layers in the network

Objects:

- $X \in \mathbb{R}^{n_x \times m}$ is the input matrix
- $x^{(i)} \in \mathbb{R}^{n_x}$ is the i^{th} example represented as a column vector
- $Y \in \mathbb{R}^{n_y \times m}$ is the label matrix
- $y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example
- $W^{[l]} \in \mathbb{R}^{\sharp \text{ of units in next layer} \times \sharp \text{ of units in the previous layer}}$ is the weight matrix, superscript $[l]$ indicates the layer
-

Common forward propagation equation examples:

-
-

Examples of cost functions:

-
-

2.2 Binary Classification

Use matrix without using for loops.
Computation using Forward propagation and Backward propagation.
Logistic regression is an algorithm for binary classification.

m training examples $\{(X^{(1)}, Y^{(1)}), (X^{(2)}, Y^{(2)}), \ldots, (X^{(m)}, Y^{(m)})\}$
where $X^{(i)} \in \mathbb{R}^{n_x}$ and $Y^{(i)} \in \{0, 1\}$ for $i \in [1, m]$

$$X = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ X^{(1)} & X^{(2)} & \ldots & X^{(m)} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \in \mathbb{R}^{n_x \times m}$$

$$X.shape = (n_x, m)$$

$$Y = [Y^{(1)}, Y^{(2)}, \ldots, Y^{(m)}] \in \mathbb{R}^{1 \times m}$$

$$Y.shape = (1, m)$$

2.3 Logistic Regression

Given x, want $\hat{y} = P(y = 1|x)$ where $x \in \mathbb{R}^{n_x}$

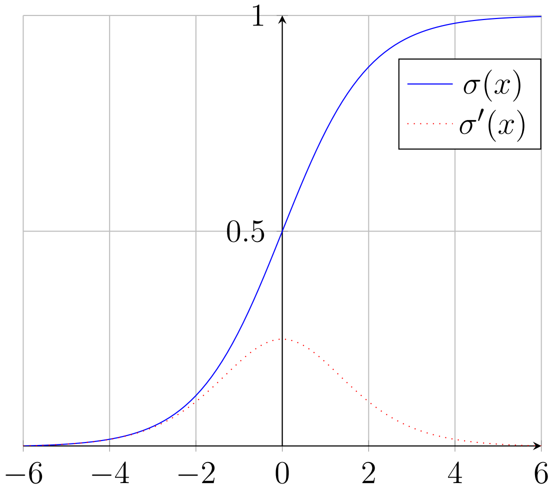
$(\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b) = \sigma(z^{(i)})$ for $i = 1, 2, \ldots, m_{train})$

$P(y|x) = P(y = 1|x) + P(y = 0|x) = \hat{y} + (1 - \hat{y}) = \hat{y}^y(1 - \hat{y})^{(1-y)}$

Parameters: $w \in \mathbb{R}^{n_x}$ a n_x dimensional vector, $b \in \mathbb{R}$ a real number.

Output $\hat{y} = \sigma(w^T x + b) = \sigma(z) = \sigma(z) = \frac{1}{1+e^{-z}}$

Drawing a sigmoid function and its derivative in tikz



2.4 Logistic Regression Cost Function

To train the parameter w and b of a Logistic Regression Model, we need a cost function.

$\hat{y} = \sigma(w^T X + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

$\hat{y}^{(i)} = \sigma(w^T X^{(i)} + b)$ where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

Given $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

Loss(error) function (for a **single** training Example):
$\boxed{\mathcal{L}(\hat{y}, y)} = -(y \log \hat{y} + (1-y) \log{(1-\hat{y})})$

Cost function (for the **entire** training Examples; $i = 1, 2, \ldots, m$):
$\boxed{\mathcal{J}(w, b)} = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} [y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log{(1-\hat{y}^{(i)})}]$

The loss function computes the error for a single training example; the cost function is the average of the loss functions of the entire training set.

In training logistic regression model, we will try to find w and b such that they minimize the Cost function $\mathcal{J}(w, b)$.

Logistic Regression can be seen as a very small Neural Network.

Explanation of Logistic Regression's Cost Function:

$\log P(y|x) = y \log \hat{y} + (1-y)\log(1-\hat{y}) = -\mathcal{L}(\hat{y}, y)$ tells that in order to make probabiliy large, we have to minimize the Loss function.

$\log P(\text{labels in training set}) = \log \prod_{i=1}^{m} P(y^{(i)}|x^{(i)})$

$= \sum_{i=1}^{m} P(y^{(i)}|x^{(i)}) = -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

(Maximum Likelihood Estimation: choose parameters to maximize it.)

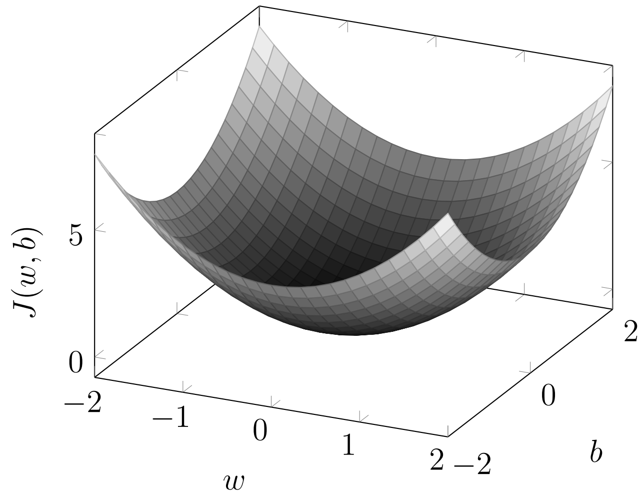
Cost: $\mathcal{J}(w, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Minimize Cost in order to carry out the Maximum Likelihood Estimation with the logistic regression model under the assumption that our training examples were iid (identically independently distributed).

2.5 Gradient Descent

$$\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})]$$

Want to find w and b that minimize the Cost function $\mathcal{J}(w,b)$.



$\mathcal{J}(w,b)$ is a convex function with a single local optimum.
No matter where you initialize the point, you should get to the same point (Global optimum).

Repeat:

$w := w - \alpha\frac{\partial\mathcal{J}(w,b)}{\partial w} = w - \alpha dw$

$b := b - \alpha\frac{\partial\mathcal{J}(w,b)}{\partial b} = b - \alpha db$

where α is the learning rate.

2.6 Derivatives

derivatives; slope

Given $f(a) = 3a$

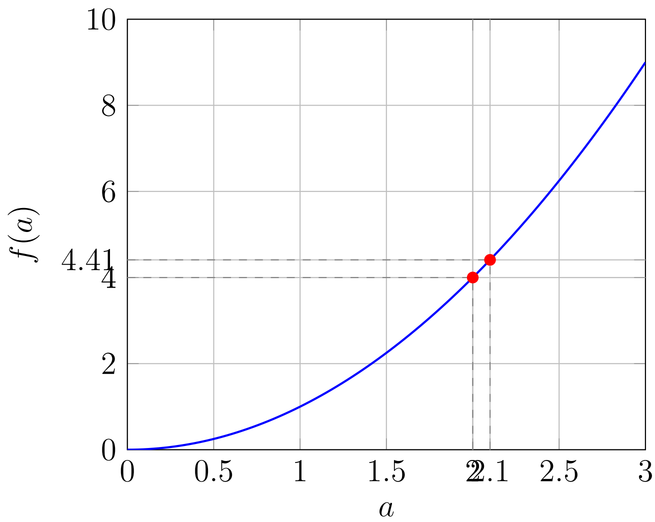
$\epsilon = .001, a = 2 + \epsilon$

$\frac{f(a)-f(a+\epsilon)}{\epsilon}$

make ϵ close to zero $\rightarrow$ derivatives.

$\frac{df(a)}{da} = \frac{d}{da}f(a)$

2.7 More Derivative Examples



$a = 2,\ f(a) = 4$

$a = 2.001,\ f(a) = 4.004001$

$\frac{d}{da}f(a) = 4$, when $a = 2$

$\frac{d}{da}f(a) = 10$, when $a = 5$

$\frac{d}{da}f(a) = \frac{d}{da}a^2 = 2a$

Given a nudge $\epsilon = 0.001$ to a, the $f(a)$ goes up $2 * a$.

2.8 Computation Graph

- Forward propagation step(forward pulse) : compute output of the network.
- Backward pulse: compute the gradients or derivatives.

$J(a, b, c) = 3(a + bc)$

$u = bc$

$v = a + u$

$J = 3v$

In order to compute derivatices, you go backward propagation.

One step of backward propagation on a computation graph yields derivative of final output variable.

2.9 Computing derivatives.

$u = bc$

$v = a + u$

$J = 3v$

Given $a = 5, b = 3, c = 2$, then $v = 11, J = 33$

We want to see how much J changes if we change the values of a, b, c, u, v for $J = 3v$

If we increase v to 11.001, then $J = 33.003$.

$\frac{dJ}{dv} = 3$

$a = 5 \rightarrow a = 5.001$

$v = 11 \rightarrow a = 11.001$

$J = 33 \rightarrow a = 33.003$

By Chain Rule:

$\frac{dJ}{da} = 3 = \frac{dJ}{dv}\frac{dv}{da} = 3 \times 1$

$\frac{dFinalOutputVar}{dvar}$ where var can be $a, b, c, ...$

We can simply denote $\frac{dJ}{dv} = dv$, $\frac{dJ}{da} = da$, etc with respect to J.

Similarly, $\frac{dJ}{du} = \frac{dJ}{dv}\frac{dv}{du} = 3 \times 1$

$\frac{dJ}{db} = \frac{dJ}{du}\frac{du}{db} = 3 \times 2 = 6$, where $u = bc = 2b$, and $\frac{du}{db} = 2$, given $a = 5, b = 3, c = 2$.

$\frac{dJ}{dc} = \frac{dJ}{du}\frac{du}{dc} = 3 \times 3 = 9$, where $u = bc = 3c$, and $\frac{du}{dc} = 3$, given $a = 5, b = 3, c = 2$.

$$\frac{dJ}{da} = 3$$

$$\frac{dJ}{du} = 3$$

$$\frac{dJ}{db} = 6$$

$$\frac{dJ}{dc} = 9$$

The coding convention $dvar$ represents: The derivative of a final output variable with respect to various intermediate quantities.

2.10 Logistic Regression Gradient Descent

Compute derivatives using Computation Graph(a bit overkill?) to implement/derive gradient descent for Logistic Regression.

Logistic regression recap (for a single i^{th} training example, $x^{(i)}$):
$z^{(i)} = w^T x^{(i)} + b$
$\hat{y}^{(i)} = a^{(i)} = \sigma(z^{(i)})$
$\mathcal{L}(a^{(i)}, y^{(i)}) = -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$

$\mathcal{J}(w, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} [y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$

Computation graph, given: x_1, w_1, x_2, w_2, b
$\boxed{z^{(i)} = w_1 x_1 + w_2 x_2 + b} \rightarrow \boxed{\hat{y}^{(i)} = a^{(i)} = \sigma(z^{(i)})} \rightarrow \boxed{\mathcal{L}(a^{(i)}, y^{(i)})}$

, where $x^{(i)} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Modify w and b to reduce the loss $\mathcal{L}(a^{(i)}, y^{(i)})$. In order to find such w and b, we **compute the derivatives with respect to $\mathcal{L}(a^{(i)}, y^{(i)})$.**

$\boxed{da^{(i)}} = \frac{d\mathcal{L}(a^{(i)}, y^{(i)})}{da} = -\frac{d}{da}(y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})) = -\frac{y^{(i)}}{a^{(i)}} + \frac{1-y^{(i)}}{1-a^{(i)}}$

$\boxed{dz^{(i)}} = \frac{d\mathcal{L}(a^{(i)}, y^{(i)})}{dz} = \frac{d\mathcal{L}(a^{(i)}, y^{(i)})}{da} \frac{da}{dz} = (-\frac{y^{(i)}}{a^{(i)}} + \frac{1-y^{(i)}}{1-a^{(i)}}) \frac{da}{dz} = a^{(i)} - y^{(i)}$
, where $\frac{da}{dz} = \frac{d}{dz}(\frac{1}{1+e^{-z^{(i)}}}) = \frac{e^{-z^{(i)}}}{(1+e^{-z^{(i)}})^2} = \frac{1}{1+e^{-z^{(i)}}}(1 - \frac{1}{1+e^{-z^{(i)}}}) = a^{(i)}(1 - a^{(i)})$

We first do Forward Propagation with initial (randomly initialized) w, b. Given incorrect prediction of y, do Backward Propagation using gradient descent and update the paramers w, b.

Go backward to compute :

$\boxed{dw_1} = \frac{\partial \mathcal{L}}{\partial w_1} = \frac{d\mathcal{L}(a,y)}{dw_1} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{dw_1} = x_1 dz$

$\boxed{dw_2} = \frac{\partial \mathcal{L}}{\partial w_2} = \frac{d\mathcal{L}(a,y)}{dw_2} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{dw_2} = x_2 dz$

$\boxed{\text{db}} = \frac{\partial \mathcal{L}}{\partial b} = \frac{d\mathcal{L}(a,y)}{db} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{db} = dz$

Compute dz to compute dw_1, dw_2, db and update with gradient descent:

$w_1 := w_1 - \alpha dw_1$

$w_2 := w_2 - \alpha dw_2$

$b := b - \alpha db$

2.11 Gradient Descent on m Examples

$\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(a^{(i)}, y^{(i)})$

, where $a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$

$dw_1 = \frac{\partial}{\partial w_1}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} dw_1^{(i)} = \frac{1}{m}\sum_{i=1}^{m} x_1{}^{(i)} dz^{(i)}$

$dw_2 = \frac{\partial}{\partial w_2}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial w_2}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} dw_2^{(i)} = \frac{1}{m}\sum_{i=1}^{m} x_2{}^{(i)} dz^{(i)}$

$db = \frac{\partial}{\partial b}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial b}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} db^{(i)} = \frac{1}{m}\sum_{i=1}^{m} dz^{(i)}$

Logistic regression on m examples:

$J = 0$; $dw_1 = 0$; $dw_2 = 0$; $db = 0$

for $i = 1$ to m:
  $z^{(i)} = w^T x^{(i)} + b$
  $a^{(i)} = \sigma(z^{(i)})$
  $\mathcal{J} += -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$
  $dz^{(i)} = a^{(i)} - y^{(i)}$

  * The value of dw_1, dw_2, db in the code is cumulative:
  $dw_1 += x_1^{(i)} dz^{(i)}$
  $dw_2 += x_2^{(i)} dz^{(i)}$
  $\ldots$
  $dw_{n_x} += x_{n_x}^{(i)} dz^{(i)}$

  $db += dz^{(i)}$

$J/ = m$, $dw_1/ = m$, $dw_2/ = m$, $db/ = m$

After the calculations for all m examples, we update(1 step of gradient descent):
$w_1 := w_1 - \alpha dw_1$
$w_2 := w_2 - \alpha dw_2$
$\ldots$
$w_{n_x} := w_{n_x} - \alpha dw_{n_x}$
$b := b - \alpha db$

We have to do multiple steps of above gradient descent.

It has two weaknesses: two for-loops (one for m training examples and another for features: $w_{(i)}$ where i can be big.) $\rightarrow$ $\boxed{\text{Vectorization!}}$

In the context of logistic regression, forward and backward propagation are used to learn the model parameters. Here's a brief overview:

Forward Propagation: In forward propagation, you pass the input data through the model to get the prediction. For a batch of m training examples, each of dimension n, the forward propagation is:

1. Compute the linear combination of the input features and weights: $z = w^T X + b$.
2. Apply the sigmoid function to z to get the predicted probabilities: $a = \sigma(z)$.
3. Compute the cost function $\mathcal{J}$, which is the average of the loss function $\mathcal{L}$ over all training examples.

Backward Propagation: Backward propagation is used to update the model parameters (weights and bias) using the gradient of the cost function. It involves the following steps:

1. Compute the derivative of the cost function $\mathcal{J}$ with respect to the predicted probabilities.
2. Compute the derivative of the predicted probabilities with respect to z.
3. Compute the derivative of z with respect to the weights and bias.
4. Update the weights and bias using the computed derivatives. The goal of these steps is to minimize the cost function $\mathcal{J}$ by adjusting the model parameters. This process is **repeated for several iterations** or until the model's performance is satisfactory.

2.12 Vectorization

What is vectorization?

$z = w^T + b$, where $w \in \mathcal{R}^{n_x}$ and $x \in \mathcal{R}^{n_x}$

```
import numpy as np

z = np.dot(w,x) + b
```

In Jupiter notebook:

```
import time

# 1. Vectorized version
a = np.random.rand(1000000)
b = np.random.rand(1000000)

tic = time.time()
c = np.dot(a,b)
toc = time.time()

print("1. Vectorized version:" + str(1000*(toc-tic))+ "ms")

# 2. For loop
c = 0
tic = time.time()
for i in range(1000000):
    c += a[i]*b[i]
toc = time.time()

print("2. For loop:" + str(1000*(toc-tic))+ "ms")
```

CPU and GPU has SIMD (single instruction multiple data). If you use built-in functions such as numpy's. It enables numpy to take better advantage of parallelization.

2.13 More Vectorization Examples

Whenever possible, avoid explicit for-loops

$u = Av$

$u_i = \sum_j A_{ij} v_j$ for $i = 1, \ldots, n$

1. Non-vectorized:

```
import numpy as np

u = np.zeros((n,1))
for i in range(n):
    for j in range(m):
        u[i] += A[i][j]*v[j]
```

2. Vectorized:

```
import numpy as np

u = np.dot(A,v)
```

Vectors and matrix valued functions.

say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$u = \begin{bmatrix} e^{v_1} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
import numpy as np
u = np.zeros((n,1))

# 1. for-loop
for i in range(n):
    u[i] = math.exp(v[i])

# 2. Vectorized
u = np.exp(v)
u = np.log(v)
u = np.abs(v) # absolute value
u = np.maximum(v,0)
u = v**2
u = 1/v
```

Logistic regression derivatives

$J = 0$; $dw_1 = 0$; $dw_2 = 0$; $db = 0$

for $i = 1$ to m:
$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$
$\mathcal{J} += -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$
$dz^{(i)} = a^{(i)} - y^{(i)}$

* The value of dw_1, dw_2, db in the code is cumulative:
$dw_1 += x_1^{(i)} dz^{(i)}$
$dw_2 += x_2^{(i)} dz^{(i)}$
$\ldots$
$dw_{n_x} += x_{n_x}^{(i)} dz^{(i)}$

$dw_2 += x_2^{(i)} dz^{(i)}$
$db += dz^{(i)}$

$J/ = m$, $dw_1/ = m$, $dw_2/ = m$, $\ldots$, $dw_{n_x}/ = m$, $db/ = m$

2.14 Vectorizing Logistic Regression

$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$ for $i = 1, \dots, m$

$$X = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \in \mathbb{R}^{n_x \times m}$$

$$w \in \mathbb{R}^{n_x \times 1}$$

$$b = \begin{bmatrix} b & b & \dots & b \end{bmatrix}$$

$$Z = w^T X + b = \begin{bmatrix} z^{(1)} & z^{(2)} & \dots & z^{(m)} \end{bmatrix} = \begin{bmatrix} w^T x^{(1)} + b & w^T x^{(2)} + b & \dots & w^T x^{(m)} + b \end{bmatrix}$$

, where $Z \in \mathbb{R}^{1 \times m}$

$$A = \begin{bmatrix} a^{(1)} & a^{(2)} & \dots & a^{(m)} \end{bmatrix} = \begin{bmatrix} \sigma(z^{(1)}) & \sigma(z^{(2)}) & \dots & \sigma(z^{(m)}) \end{bmatrix} = \sigma(Z)$$

```python
import numpy as np

# Broadcasting: even though b is in 1xR, it spans as a vector
Z = np.dot(w.T, x) + b
```

2.15 Vectorizing Logistic Regression's Gradient Output

$dz^{(i)} = a^{(i)} - y^{(i)}$
$dZ = \left[dz^{(1)}, \quad \dots \quad, dz^{(m)}\right]$
$A = \left[a^{(1)}, \quad \dots \quad, a^{(m)}\right]$
$Y = \left[y^{(1)}, \quad \dots \quad, y^{(m)}\right]$
$dZ = A - Y = \left[a^{(1)} - y^{(1)}, \quad \dots \quad, a^{(m)} - y^{(m)}\right]$

$J = 0;\ dw_1 = 0;\ dw_2 = 0;\ db = 0$

for $i = 1$ to m:
$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$
$\mathcal{J} += -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$
$dz^{(i)} = a^{(i)} - y^{(i)}$

* The value of dw_1, dw_2, $\dots$, dw_{n_x}, db in the code is cumulative:
$dw_1 += x_1^{(i)} dz^{(i)}$
$dw_2 += x_2^{(i)} dz^{(i)}$
$\dots$
$dw_{n_x} += x_{n_x}^{(i)} dz^{(i)}$

$db += dz^{(i)}$

$J /= m,\ dw_1 /= m,\ dw_2 /= m,\ \dots,\ dw_{n_x} /= m,\ db /= m$

$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)} = \frac{1}{m} \text{np.sum}(dz)$

$$dw = \frac{1}{m} X dz^T = \frac{1}{m} \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \dots & x_1^{(m)} \\ x_2^{(1)} & x_2^{(2)} & \dots & x_2^{(m)} \\ \vdots & \vdots & \dots & \vdots \\ x_{n_x}^{(1)} & x_{n_x}^{(2)} & \dots & x_{n_x}^{(m)} \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ X^{(1)} & X^{(2)} & \dots & X^{(m)} \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix} = \frac{1}{m} \left[X^{(1)} dz^{(1)} + \dots + X^{(m)} dz^{(m)}\right]$$

Vectorize:

$dw += x^{(i)} dz^{(i)}$

$Z = w^T + b = np.dot(w.T, X) + b$

$A = \sigma(Z)$

$dZ = A - Y$

$dw = \frac{1}{m} X dZ^T$

$db = \frac{1}{m}\ \mathrm{np.sum}(dZ)$

Single Iteration of Gradient Descent for Logistic Regression:

$w := w - \alpha dw$
$b := b - \alpha db$

2.16 Broadcasting in Python

Calories from Carbs, Proteins, Fats in 100g of different foods:

$$\begin{bmatrix} & Apples & Beef & Eggs & Potatoes \\ Carb & 56.0 & 0.0 & 4.4 & 68.0 \\ Protein & 1.2 & 104.0 & 52.0 & 8.0 \\ Fat & 1.8 & 135.0 & 99.0 & 0.9 \end{bmatrix}$$

Calculate the percentage of calories for Carb, Potein, Fat:

```
import numpy as np

A = np.array[[56.0, 0.0, 4.4, 68.0],
    [1.2, 104.0, 52.0, 8.0],
    [1.8, 135.0, 99.0, 0.9]])
print(A)

# sum vertically(axis=0)
cal = A.sum(axis=0)
print(cal)

# compute percentage
percentage = 100* A/cal.reshape(1,4)
print(percentage)
```

General Principle:

(m, n) matrix operation $\rightarrow$ (1,n),(1,m) $\rightarrow$ (m,n), (m,n)

(m, n) + real number $\mathbb{R}$ $\rightarrow$ (m+$\mathbb{R}$,n+$\mathbb{R}$)

2.17 A Note on Python/Numpy Vectors

```python
import numpy as np

# Don't use this (rank 1 array):
# a = np.random.randn(5)

# Use this:
a = np.random.randn(5,1)
a = np.random.randn(1,5)
assert(a.shape == (5,1))
```

2.18 Explanation of Logistic Regression Cost Function(Optional)

Logistic regression cost function (1 example):

If $y = 1 : P(y|x) = \hat{y}$
If $y = 0 : P(y|x) = 1 - \hat{y}$

$P(y|x) = \hat{y}^y (1 - \hat{y})^{(1-y)}$

$logP(y|x) = y \log \hat{y} + (1 - y) \log(1 - \hat{y})$

Logistic regression cost function (m examples):

$\log P(\text{labels in training set}) = \log \prod_{i=1}^{m} P(y^{(i)}|x^{(i)})$

$= \sum_{i=1}^{m} \log P(y^{(i)}|x^{(i)}) = -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Cost(minimize):

$\mathcal{J}(w, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Minimizing the loss corresponds with maximizing $\log P(y|x)$.

2.19 Week2. Assignment

1. Python+Basics+With+Numpy+v3.ipynb
2. Logistic+Regression+with+a+Neural+Network+mindset+v5.ipynb