1 Week1

Introduction to Deep Learning.

1.1 What is a Neural Network?

1.2 Supervised Learning with Neural Networks

1.3 Why is Deep Learning taking off?

1.4 About this Course

1. Neural Networks and Deep Learning
2. Improving Deep Neural Networks: Hyperparameter tuning, Regularization and Optimization
3. Structuring your Machine Learning project
4. Convolutional NeuralNetworks
5. Natural Language Processing: Building sequence models

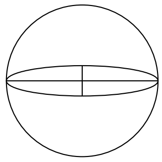
1.5 Outline of this Course

- Week1. Introduction
- Week2. Basics of Neural Network programming
- Week3. One hidden layer Neural Networks
- Week4. Deep Neural Networks

2 Week2

Basics of Neural Network Programming

How do I write an equation in LaTeX?



In 1902, Einstein created this equation: $E = mc^2$

And Newton came up with this one: $\sum F = ma$

$$5 + 5 = 10 \tag{1}$$

$$\begin{aligned} A &= \frac{5\pi r^2}{2} \\ A &= \frac{1}{2}\pi r^2 \end{aligned} \tag{2}$$

2.1 Neural Network Notations

General comments:
superscript (i) will denote the i^{th} training example.
superscript $[l]$ will denote the l^{th} layer.

Sizes:

- m: number of examples in the dataset

- n_x: input size

- n_y: output size

- $n_h^{[l]}$: number of hidden units of the l^th layer.
  In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h^{[number of layer+1]}$

- L: number of layers in the network

Objects:

- $X \in \mathbb{R}^{n_x \times m}$ is the input matrix

- $x^{(i)} \in \mathbb{R}^{n_x} is the i^{th} example represented as a column vector$

- $Y \in \mathbb{R}^{n_y \times m}$ is the label matrix

- $y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example

- $W^{[l]} \in \mathbb{R}^{number of units in next layer \times number of units in the previous layer}$ is the weight matrix, superscript $[l]$ indicates the layer

-

Common forward propagation equation examples:

-

-

Examples of cost functions:

-

-

2.2 Binary Classification

Use matrix without using for loops.
Computation using Forward propagation and Backward propagation.
Logistic regression is an algorithm for binary classification.

m training examples $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$
where $x^{(i)} \in \mathbb{R}^{n_x}$ and $y^{(i)} \in \{0, 1\}$ for $i \in [1, m]$

$$X = \begin{bmatrix} \vdots & \vdots & \vdots \\ X^{(1)} & X^{(1)} & X^{(m)} \\ \vdots & \vdots & \vdots \end{bmatrix} \in \mathbb{R}^{n_x \times m}$$

$$X.shape = (n_x, m)$$

$$Y = [Y^{(1)}, Y^{(2)}, \ldots, Y^{(m)}] \in \mathbb{R}^{1 \times m}$$

$$Y.shape = (1, m)$$

2.3 Logistic Regression

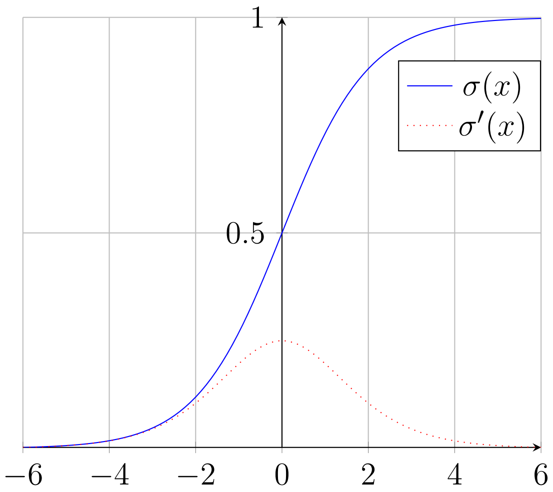
Given x, want $\hat{y} = P(y = 1|x)$ where $x \in \mathbb{R}^{n_x}$

Parameters: $w \in \mathbb{R}^{n_x}$ a n_x dimensional vector, $b \in \mathbb{R}$ a real number.

Output $\hat{y} = \sigma(w^T x + b) = \sigma(z)$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Drawing a sigmoid function and its derivative in tikz



2.4 Logistic Regression Cost Function

To train the parameter w and b of a Logistic Regression Model, we need a cost function.

$\hat{y} = \sigma(w^T X + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

$\hat{y}^{(i)} = \sigma(w^T X^{(i)} + b)$ where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

Given $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

Loss(error) function (for a single training Example):
$\mathcal{L}(\hat{y}, y) = -(y \log \hat{y} + (1-y) \log(1-\hat{y}))$

Cost function (for the entire training Examples):

$$\mathcal{J}(w, b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} [y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})]$$

The loss function computes the error for a single training example; the cost function is the average of the loss functions of the entire training set.

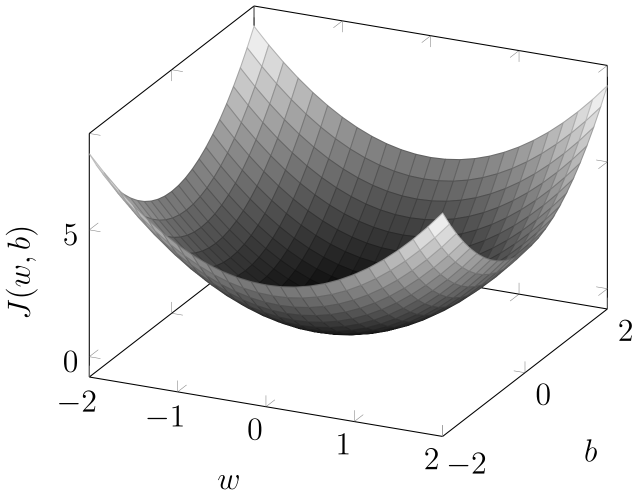
In training logistic regression model, we will try to find w and b such that they minimize the Cost function $\mathcal{J}(w, b)$.

Logistic Regression can be seen as a very small Neural Network.

2.5 Gradient Descent

$$\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})]$$

Want to find w and b that minimize the Cost function $\mathcal{J}(w,b)$.



$\mathcal{J}(w,b)$ is a convex function with a single local optimum.
No matter where you initialize the point, you should get to the same point (Global optimum).

Repeat:

$w := w - \alpha\frac{\partial\mathcal{J}(w,b)}{\partial w}$

$w := w - \alpha dw$

$b := b - \alpha\frac{\partial\mathcal{J}(w,b)}{\partial b}$

$b := b - \alpha db$

where α is the learning rate.

2.6 Derivatives

derivatives; slope

Given $f(a) = 3a$

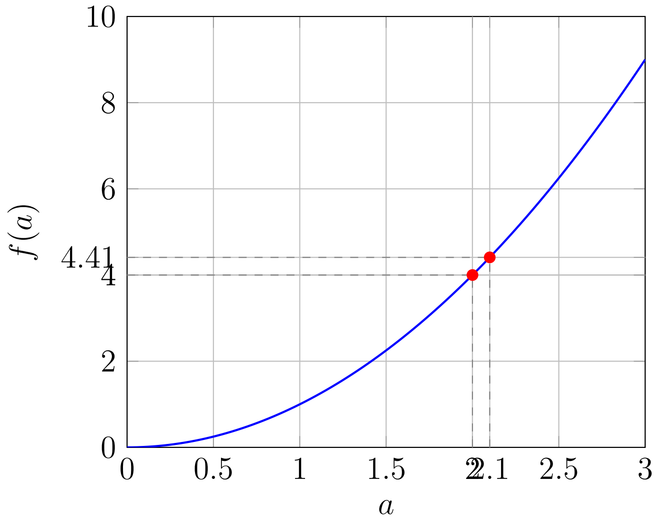
$\epsilon = .001, a = 2 + \epsilon$

$\frac{f(a)-f(a+\epsilon)}{\epsilon}$

make ϵ close to zero $\rightarrow$ derivatives.

$\frac{df(a)}{da} = \frac{d}{da}f(a)$

2.7 More Derivative Examples



$a = 2,\ f(a) = 4$

$a = 2.001,\ f(a) = 4.004001$

$\frac{d}{da} f(a) = 4$, when $a = 2$

$\frac{d}{da} f(a) = 10$, when $a = 5$

$\frac{d}{da} f(a) = \frac{d}{da} a^2 = 2a$

Given a nudge $\epsilon = 0.001$ to a, the $f(a)$ goes up $2 * a$.

2.8 Computation Graph

- Forward propagation step(forward pulse) : compute output of the network.
- Backward pulse: compute the gradients or derivatives.

$J(a, b, c) = 3(a + bc)$
$u = bc$
$v = a + u$
$J = 3v$

In order to compute derivatices, you go backward propagation.

One step of backward propagation on a computation graph yields derivative of final output variable.

2.9 Computing derivatives.

$u = bc$

$v = a + u$

$J = 3v$

Given $a = 5, b = 3, c = 2$, then $v = 11, J = 33$

We want to see how much J changes if we change the values of a, b, c, u, v for $J = 3v$

If we increase v to 11.001, then $J = 33.003$.

$\frac{dJ}{dv} = 3$

$a = 5 \rightarrow a = 5.001$

$v = 11 \rightarrow a = 11.001$

$J = 33 \rightarrow a = 33.003$

By Chain Rule:

$\frac{dJ}{da} = 3 = \frac{dJ}{dv}\frac{dv}{da} = 3 \times 1$

$\frac{dFinalOutputVar}{dvar}$ where var can be $a, b, c, ...$

We can simply denote $\frac{dJ}{dv} = dv$, $\frac{dJ}{da} = da$, etc with respect to J.

Similarly, $\frac{dJ}{du} = \frac{dJ}{dv}\frac{dv}{du} = 3 \times 1$

$\frac{dJ}{db} = \frac{dJ}{du}\frac{du}{db} = 3 \times 2 = 6$, where $u = bc = 2b$, and $\frac{du}{db} = 2$, given $a = 5, b = 3, c = 2$.

$\frac{dJ}{dc} = \frac{dJ}{du}\frac{du}{dc} = 3 \times 3 = 9$, where $u = bc = 3c$, and $\frac{du}{dc} = 3$, given $a = 5, b = 3, c = 2$.

$$\frac{dJ}{da} = 3$$

$$\frac{dJ}{du} = 3$$

$$\frac{dJ}{db} = 6$$

$$\frac{dJ}{dc} = 9$$

The coding convention $dvar$ represents: The derivative of a final output variable with respect to various intermediate quantities.

2.10 Logistic Regression Gradient Descent

Compute derivatives using Computation Graph(a bit overkill?) to implement/derive gradient descent for Logistic Regression.

Logistic regression recap
$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

$\mathcal{L}(a, y) = -(ylog(a) + (1-y)log(1-a))$

Computation graph:

Given: x_1, w_1, b_1, x_2, w_2, b_2

$\boxed{z = w_1 x_1 + w_2 x_2 + b} \rightarrow \boxed{\hat{y} = a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$

Modify w and b to reduce the loss $\mathcal{L}(a, y)$

In order to find such w and b, we compute the derivatives with respect to ?.

$da = \frac{d\mathcal{L}(a,y)}{da} = -\frac{d}{da}(ylog(a) + (1-y)log(1-a)) = -\frac{y}{a} + \frac{1-y}{1-a}$

$\boxed{\text{dz}} = \frac{d\mathcal{L}(a,y)}{dz} = \frac{d\mathcal{L}(a,y)}{da}\frac{da}{dz} = (-\frac{y}{a} + \frac{1-y}{1-a})\frac{da}{dz} = a - y$
, where $\frac{da}{dz} = \frac{d}{dz}(\frac{1}{1+e^{-z}}) = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}(1 - \frac{1}{1+e^{-z}}) = a(1-a)$

Go backward to compute :

$\boxed{dw_1} = \frac{\partial \mathcal{L}}{\partial w_1} = \frac{d\mathcal{L}(a,y)}{dw_1} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{dw_1} = x_1 dz$

$\boxed{dw_2} = \frac{\partial \mathcal{L}}{\partial w_2} = \frac{d\mathcal{L}(a,y)}{dw_2} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{dw_2} = x_2 dz$

$\boxed{\text{db}} = \frac{\partial \mathcal{L}}{\partial b} = \frac{d\mathcal{L}(a,y)}{db} = \frac{d\mathcal{L}(a,y)}{dz}\frac{dz}{db} = dz$

Compute dz to compute dw_1, dw_2, db and do the update with gradient descent:

$w_1 := w_1 - \alpha dw_1$

$w_2 := w_2 - \alpha dw_2$

$b := b - \alpha db$

2.11 Gradient Descent on m Examples

$\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(a^{(i)}, y^{(i)})$

, where $a^{(i)} = \hat{y^{(i)}} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$

$\frac{\partial}{\partial w_1}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} dw_1^{(i)}$

$\frac{\partial}{\partial w_2}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial w_2}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} dw_2^{(i)}$

$\frac{\partial}{\partial b}\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\frac{\partial}{\partial b}\mathcal{L}(a^{(i)}, y^{(i)}) = \frac{1}{m}\sum_{i=1}^{m} db^{(i)}$

Logistic regression on m examples:

$J = 0$; $dw_1 = 0$; $dw_2 = 0$; $db = 0$

for $i = 1$ to m:
$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$
$\mathcal{J} += -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$
$dz^{(i)} = a^{(i)} - y^{(i)}$

* The value of dw_1, dw_2, db in the code is cumulative:
$dw_1 += x_1^{(i)} dz^{(i)}$
$dw_2 += x_2^{(i)} dz^{(i)}$
$db += dz^{(i)}$

$J /= m$, $dw_1 /= m$, $dw_2 /= m$, $db /= m$

Finally, after finishing calculations for all m examples, we update(implment one step of gradient descent):

$w_1 := w_1 - \alpha dw_1$
$w_2 := w_2 - \alpha dw_2$
$b := b - \alpha db$

We have to multiple steps of above gradient descent.

It has two weaknesses: two for-loops (one for m training examples and another for features: $w_{(i)}$ where i can be big.) $\rightarrow$ $\boxed{\text{Vectorization!}}$

2.12 Vectorization

What is vectorization?

$$z = w^T + b, \text{ where } w \in \mathcal{R}^{n_x} \text{ and } x \in \mathcal{R}^{n_x}$$

```
import numpy as np

z = np.dot(w,x) + b
```

In Jupiter notebook:

```
import time

# 1. Vectorized version
a = np.random.rand(1000000)
b = np.random.rand(1000000)

tic = time.time()
c = np.dot(a,b)
toc = time.time()

print("1. Vectorized version:" + str(1000*(toc-tic))+ "ms")

# 2. For loop
c = 0
tic = time.time()
for i in range(1000000):
    c += a[i]*b[i]
toc = time.time()

print("2. For loop:" + str(1000*(toc-tic))+ "ms")
```

CPU and GPU has SIMD (single instruction multiple data). If you use built-in functions such as numpy's. It enables numpy to take better advantage of parallelization.

2.13 More Vectorization Examples

Whenever possible, avoid explicit for-loops

$u = Av$

$u_i = \sum_j A_{ij} v_j$ for $i = 1, \dots, n$

1. Non-vectorized:

```
import numpy as np

u = np.zeros((n,1))
for i in range(n):
    for j in range(m):
        u[i] += A[i][j]*v[j]
```

2. Vectorized:

```
import numpy as np

u = np.dot(A,v)
```

Vectors and matrix valued functions.

say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$u = \begin{bmatrix} e^{v_1} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
import numpy as np
u = np.zeros((n,1))

# 1. for-loop
for i in range(n):
    u[i] = math.exp(v[i])

# 2. Vectorized
u = np.exp(v)
u = np.log(v)
u = np.abs(v) # absolute value
u = np.maximum(v,0)
u = v**2
u = 1/v
```

Logistic regression derivatives

$J = 0$; $dw_1 = 0$; $dw_2 = 0$; $db = 0$

for $i = 1$ to m:
$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$
$\mathcal{J}+ = -[y^{(i)} log(a^{(i)}) + (1 - y^{(i)}) log(1 - a^{(i)})]$
$dz^{(i)} = a^{(i)} - y^{(i)}$

* The value of dw_1, dw_2, db in the code is cumulative:
$dw_1+ = x_1^{(i)} dz^{(i)}$
$dw_2+ = x_2^{(i)} dz^{(i)}$
$\ldots$
$dw_{n_x}+ = x_{n_x}^{(i)} dz^{(i)}$

$dw_2+ = x_2^{(i)} dz^{(i)}$
$db+ = dz^{(i)}$

$J/ = m$, $dw_1/ = m$, $dw_2/ = m$, $db/ = m$

2.14 Vectorizing Logistic Regression

$z^{(i)} = w^T x^{(i)} + b$
$a^{(i)} = \sigma(z^{(i)})$ for $i = 1, \dots, m$

$$X = \begin{bmatrix} \vdots & \vdots & \dots & \vdots \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ \vdots & \vdots & \dots & \vdots \end{bmatrix} \in \mathbb{R}^{n_x \times m}$$

$$w \in \mathbb{R}^{n_x \times 1}$$

$$b = \begin{bmatrix} b & b & \dots & b \end{bmatrix}$$

$$Z = w^T X + b = \begin{bmatrix} z^{(1)} & z^{(2)} & \dots & z^{(m)} \end{bmatrix} = \begin{bmatrix} w^T x^{(1)} + b & w^T x^{(2)} + b & \dots & w^T x^{(m)} + b \end{bmatrix}$$

, where $Z \in \mathbb{R}^{1 \times m}$

$$A = \begin{bmatrix} a^{(1)} & a^{(2)} & \dots & a^{(m)} \end{bmatrix} = \begin{bmatrix} \sigma(z^{(1)}) & \sigma(z^{(2)}) & \dots & \sigma(z^{(m)}) \end{bmatrix} = \sigma(Z)$$

```python
import numpy as np

# Broadcasting: even though b is in 1xR, it spans as a vector
Z = np.dot(w.T, x) + b
```