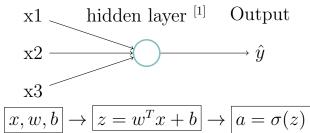
1 Week3

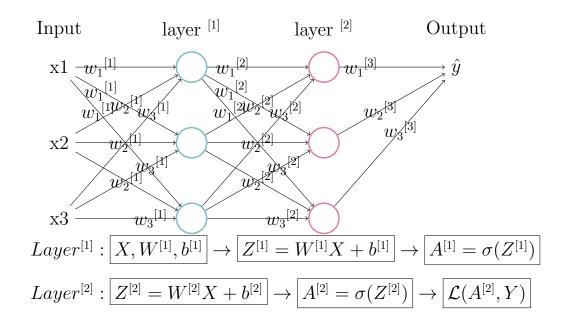
Shallow Neural Network.

1.1 Neural Network Overview

What is a Neural Network?

Input





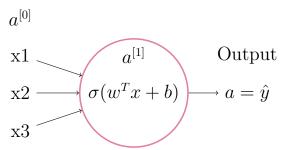
1.2 Neural Network Representations

Values of the input features (activation): $X = a^{[0]}$

The following is the 2-Layer Neural Network:

$$a^{[0]} \qquad a^{[1]} \qquad a^{[2]} \qquad x^{[2]} \qquad x^{[1]} \qquad a^{[2]} \qquad x^{[2]} \qquad x^{[1]} \qquad a^{[1]} \qquad a^{[2]} \qquad x^{[2]} \qquad x^{[1]} \qquad a^{[1]} \qquad a^{[2]} \qquad a^{[2]} \qquad a^{[1]} \qquad a^{[2]} \qquad a^{[2]} \qquad a^{[2]} \qquad a^{[1]} \qquad a^{[2]} \qquad a^{[1]} \qquad a^{[2]} \qquad a^{[$$

1.3 Computing Neural Network Output



Each circle(node) represents 2 steps of calculation:

$$z = w^T x + b$$

$$a = \sigma(z)$$

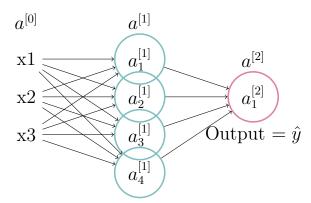
- 1. The weighted sum of the inputs is calculated.
- 2. The bias is added.
- 3. The result is fed to an activation function.
- 4. Specific neuron is activated.

$$z_1^{[1]} = w_1^{[1]^T} X + b_1^{[1]} \rightarrow a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]^T} X + b_2^{[1]} \rightarrow a_2^{[1]} = \sigma(z_2^{[1]}), \text{ where } a_i^{[l]} \stackrel{\leftarrow \text{layer}}{\leftarrow \text{node in layer}}$$

$$z^{[1]} = \begin{bmatrix} w_1^{[1]T}X + b_1^{[1]} \\ w_2^{[1]T}X + b_2^{[1]} \\ w_3^{[1]T}X + b_3^{[1]} \\ w_4^{[1]T}X + b_4^{[1]} \end{bmatrix} \rightarrow \begin{bmatrix} \sigma(w_1^{[1]T}X + b_1^{[1]}) \\ \sigma(w_2^{[1]T}X + b_2^{[1]}) \\ \sigma(w_3^{[1]T}X + b_3^{[1]}) \\ \sigma(w_4^{[1]T}X + b_4^{[1]}) \end{bmatrix}$$

$$a^{[1]} = \sigma(z^{[1]}) = \begin{bmatrix} \sigma(z_1^{[1]}) \\ \sigma(z_2^{[1]}) \\ \sigma(z_3^{[1]}) \\ \sigma(z_4^{[1]}) \end{bmatrix} = \sigma[\begin{bmatrix} \dots & w_1^{[1]^T} & \dots \\ \dots & w_2^{[1]^T} & \dots \\ \dots & w_3^{[1]^T} & \dots \\ \dots & w_4^{[1]^T} & \dots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}]$$



Given input x:

$$\begin{split} z^{[1]} &= W^{[1]} a^{[0]} + b^{[1]}, \text{ where } _{(4,1)=(4,3)(3,1)+(4,1)} \\ a^{[1]} &= \sigma(z^{[1]}) \\ z^{[2]} &= W^{[2]} a^{[1]} + b^{[2]}, \text{ where } _{(1,1)=(1,4)(4,1)+(1,1)} \\ a^{[2]} &= \sigma(z^{[2]}) \end{split}$$

1.4 Vectorizing Across Multiple Examples

$$x^{(i)} \rightarrow a^{[2](i)} = \hat{y}^{(i)}$$
 for $i = 1$ to m :
$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

For m examples:

$$X = \begin{bmatrix} \begin{vmatrix} & & & & & & & \\ X^{(1)} & X^{(2)} & \dots & X^{(m)} \\ & & & & & & \end{bmatrix}$$

$$Z^{[1]} = \begin{bmatrix} \begin{vmatrix} & & & & & & \\ z^{1} & z^{[1](2)} & \dots & z^{[1](m)} \\ & & & & & & \end{bmatrix} = W^{[1]^T}X + b^{[1]}$$

$$A^{[1]} = \begin{bmatrix} \begin{vmatrix} & & & & & \\ a^{1} & a^{[1](2)} & \dots & a^{[1](m)} \\ & & & & & & \end{bmatrix} = \sigma(Z^{[1]}) = \sigma(W^{[1]^T}X + b^{[1]})$$

 \updownarrow : across hidden units in the i^{th} training example; \sharp of units/node

 \leftrightarrow : across m training examples

1.5 Justification for vectorized implementation

Given m=3, let

$$\begin{split} W^{[1]}X^{(1)} &= \begin{bmatrix} | \\ A \\ | \end{bmatrix}, \ W^{[1]}X^{(2)} &= \begin{bmatrix} | \\ B \\ | \end{bmatrix}, \ W^{[1]}X^{(3)} &= \begin{bmatrix} | \\ C \\ | \end{bmatrix} \\ \text{then, } Z^{[1]} &= W^{[1]} \begin{bmatrix} | & | & | & | \\ X^{(1)} & X^{(2)} & X^{(3)} \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A & B & C \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ Z^{1} & Z^{[1](2)} & Z^{[1](3)} \\ | & | & | \end{bmatrix} \end{split}$$

1. Using for-loops:

for
$$i = 1$$
 to m :

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

2. Using Matrix(without for-loop):

Given X:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

1.6 Activation Functions

There're other activation functions other than sigmoid and you can use different activation functions for different layers;

$$\sigma(z) \rightarrow g(z) = tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

tanh always works better on hidden layers than sigmoid function, except for output layer (use sigmoid for output layer) where it requires to output be either 0 or 1 (binary classification)

Use different activation functions $g^{[l]}(z^{[i]})$, where $l=1,2,\ldots,L$ (number of layers in the network)

Both activation functions have down side; If z is either very small or large, the gradient(slope of the function) is very small which will slow down the gradient descent(backward propagation/learning speed).

ReLU function is a popular choice for the activation function; a = max(0, z) that does not have above downsides.

In case of Binary Classification, use sigmoid for the output layer, and ReLu for other hidden layer's activation function. Or Leaky ReLU (a = max(.01z, z)) recently.(not commonly used in practice)

1.7 Why Non-linear Activation Functions

Linear Activation through Hidden layers become meaningless. Use g(z) as non-linear function.

Using linear activation for hidden layers and sigmoid for the output layer will simply become a Logistic Regression problem..

1.8 Derivatives of Activation Functions.

sigmoid activation function $g(z) = \frac{1}{1+e^{-z}} = a$

$$g'(z) = \frac{d}{dz}g(z) = g(z)(1 - g(z)) = a(1 - a)$$

tanh activation function $g(z) = tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}} = a$

$$g'(z) = 1 - (tanh(z))^2 = 1 - a^2$$

ReLU activation function g(z) = max(0, z)

$$g'(z) = 0 \text{ if } z < 0$$

$$g'(z) = 1 \text{ if } z \ge 0$$

LeakyReLU activation function g(z) = max(.01z, z)

$$g'(z) = .01 \text{ if } z < 0$$

$$g'(z) = 1 \text{ if } z \ge 0$$

1.9 Gradient Descent For Neural Networks

Backward propagation.

Parameters:

 $w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}$ matrices with dimensions:

$$(n^{[1]},n^{[0]}), (n^{[1]},1), (n^{[2]},n^{[1]}), (n^{[2]},1),$$
 where

 $n_x=n^{[0]}=$ num of input features, $n^{[1]}=$ num of hidden units, $n^{[2]}=$ 1=num of output units

Cost Function:

$$J(w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}, y)$$
, where $\hat{y} = a^{[2]}$

Gradien Descent:

Repeat:

Compute predicts $\hat{y}^{(i)}$ for i=1,...,m $dw^{[1]} = \frac{\partial J}{\partial w^{[1]}}, \ db^{[1]} = \frac{\partial J}{\partial b^{[1]}}$ $w^{[1]} := w^{[1]} - \alpha dw^{[1]}$ $b^{[1]} := b^{[1]} - \alpha db^{[1]}$

 $w^{[2]} := w^{[2]} - \alpha dw^{[2]}$ $b^{[2]} := b^{[2]} - \alpha db^{[2]}$

Formulas for computing derivatives:

Forward propagtion:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]}=g^{[2]}(Z^{[2]})=\sigma(Z^{[2]}),$$
 sigmoid for binary classification.

Backward propagtion:

$$dZ^{[2]} = A^{[2]} - Y = [y^{(1)}, \dots, y^{(m)}]$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} \text{ np.sum}(dZ^{[2]}, axis = 1, keepdims = True)$$

, where keepdims ensures $(n^{[2]}, 1)$ instead of (n, 1)

$$dZ^{[1]} = W^{[2]^T} dZ^{[2]} * g^{[1]^{'}} (Z^{[1]}); \text{ elementwise product in }_{(n^{[1]},m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]}=\frac{1}{m} \text{ np.sum}(dZ^{[1]}, axis=1, keepdims=True)$$

1.10 BackPropagation Intuition

Logistic Regression Recap

Forward Propagation in Logistic Regression:

$$x \longrightarrow x \longrightarrow z = w^T x + b \rightarrow a = \sigma(z) \longrightarrow \mathcal{L}(a, y)$$

Backward Propagation in Logistic Regression:

$$da = \frac{d}{da}\mathcal{L}(a, y) = \frac{d}{da}[-y\log a - (1 - y)\log(1 - a)] = -\frac{y}{a} + \frac{1 - y}{1 - a} = \frac{a - y}{a(1 - a)}$$
$$dz = \frac{d}{dz}\mathcal{L}(a, y) = \frac{d}{da}\mathcal{L}(a, y)\frac{dz}{da} = da \cdot \frac{dz}{da} = da \cdot g'(z)$$

, where a=g(z) and therefore $\frac{dz}{da}=g'(z)$ In case of logistic regression: $g(z)=\sigma(z)$, therefore g'(z)=g(z)(1-g(z))

$$dw = \frac{d}{dz}\mathcal{L}(a, y)\frac{dz}{dw} = dz \cdot x$$

$$db = \frac{d}{dz}\mathcal{L}(a,y)\frac{dz}{db} = dz$$

Neural network gradients:

Compute forward:

$$W^{[1]} \longrightarrow z^{[1]} = W^{[1]}x + b^{[1]} \longrightarrow a^{[1]} = \sigma(z^{[1]}) \longrightarrow \cdots$$

$$b^{[1]} \longrightarrow dz^{[1]} \longrightarrow da^{[1]}$$

$$W^{[2]} \longrightarrow z^{[2]} = W^{[2]}a^{[1]} + b^{[2]} \longrightarrow a^{[2]} = \sigma(z^{[2]}) \longrightarrow \mathcal{L}(a^{[2]}, y)$$

$$b^{[2]} \longrightarrow dz^{[2]}, dw^{[2]}, db^{[2]} \longrightarrow da^{[2]}$$
Compute backward $(n_x = n^{[0]}, n^{[1]}, n^{[2]} = 1)$

$$da^{[2]} \rightarrow dz^{[2]} \rightarrow dw^{[2]}, db^{[2]} \rightarrow da^{[1]} \rightarrow dz^{[1]} \rightarrow dw^{[1]}, db^{[1]}$$

$$dz^{[2]} = a^{[2]} - y \quad \text{(logistic regression: activation for output layer is sigmoid)} \quad a^{[2]} = \sigma(z^{[2]})$$

$$dw^{[2]} = dz^{[2]}a^{[1]} \quad \text{(similar to logistic regression problem)} \quad dw = dz \cdot x$$

$$db^{[2]} = dz^{[2]}$$

Summary of gradient descent

For a simple training example: (from the previouse slide)

$$\begin{split} dz^{[2]} &= a^{[2]} - y \\ dW^{[2]} &= dz^{[2]} - a^{[1]^T} \\ db^{[2]} &= dz^{[2]} \\ dz^{[1]} &= W^{[2]^T} dz^{[2]} * g^{[1]'}(z^{[1]}) \end{split}$$

$$dW^{[1]} = dz^{[1]}x^T$$

$$db^{[1]} = dz^{[1]}$$

For m training examples:

$$dZ^{[2]} = A^{[2]} - Y$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

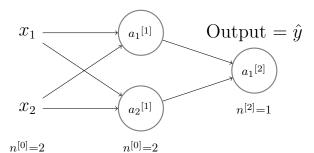
$$dZ^{[1]} = W^{[2]^T} dZ^{[2]} * g^{[1]'}(Z^{[1]}), \text{ elementwise in }_{(n^{[1]},m)} \text{ dimension}.$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

1.11 Random Initialization

What happens if you initialize weights to zero?



 $w^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ gives $w^{[2]} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ and hidden units in [1] are identical. So having more than one hidden units become meaningless.

$$a_1^{[1]} = a_2^{[1]}$$

 $dz_1^{[1]} = dz_2^{[1]}$

Random Initialization

$$egin{aligned} w^{[1]} &= np.random.randn((2,2))*.01 \\ b^{[1]} &= np.zeros((2,1)) \\ w^{[2]} &= np.random.randn((1,2))*.01 \\ b^{[2]} &= 0 \end{aligned}$$

(big multiplier will make slope of the gradient descent is very small, so choose it as a small value.)