Lecture 03
Pruning and Sparsity

Part I

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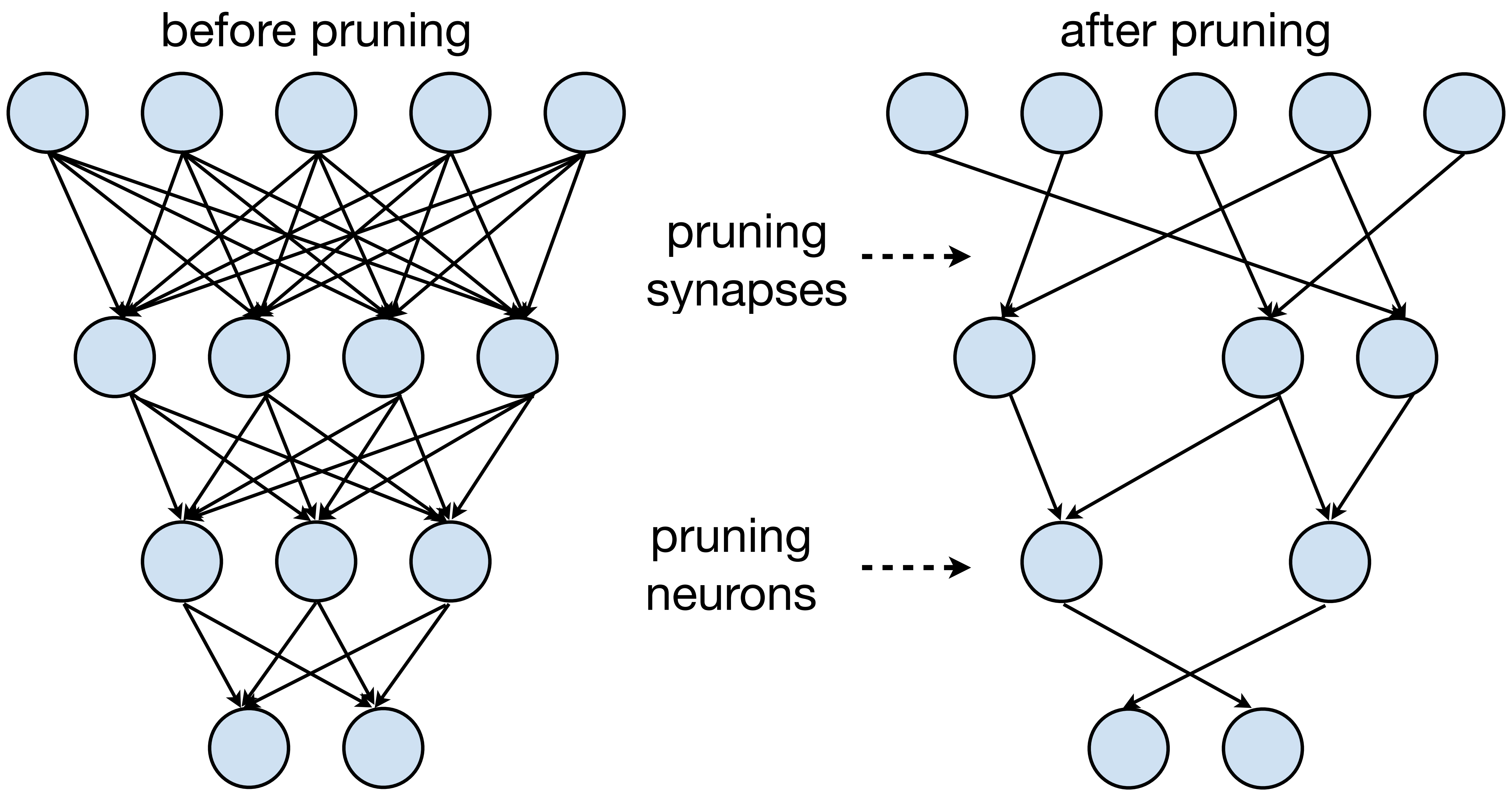
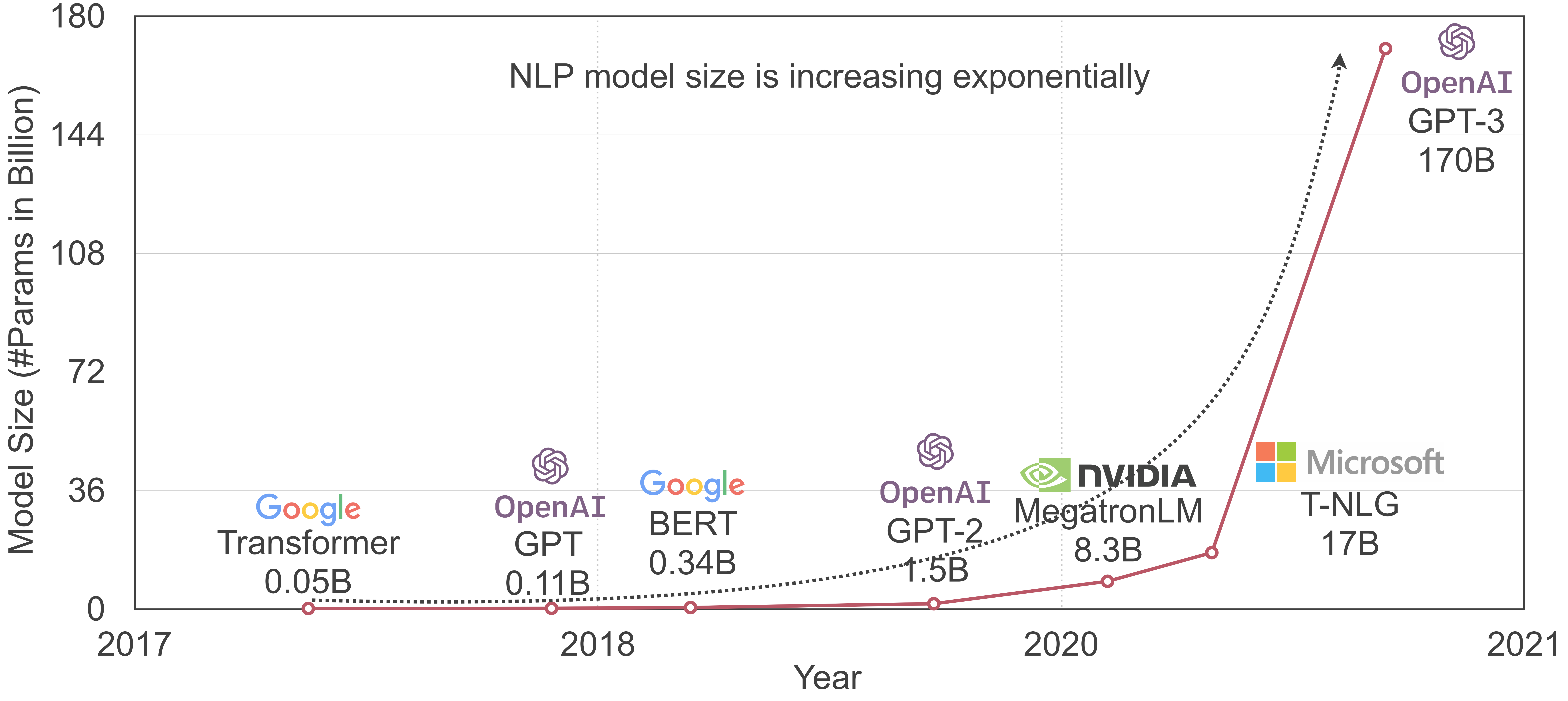
Lecture Plan

Today we will:

1. Introduce ***neural network pruning*** which can reduce the parameter counts of neural networks by more than 90%, decreasing the storage requirements and improving computation efficiency of neural networks.
2. Go through all steps of pruning, and introduce different ***granularities*** and ***criteria*** of neural network pruning.



Today's AI is too BIG!



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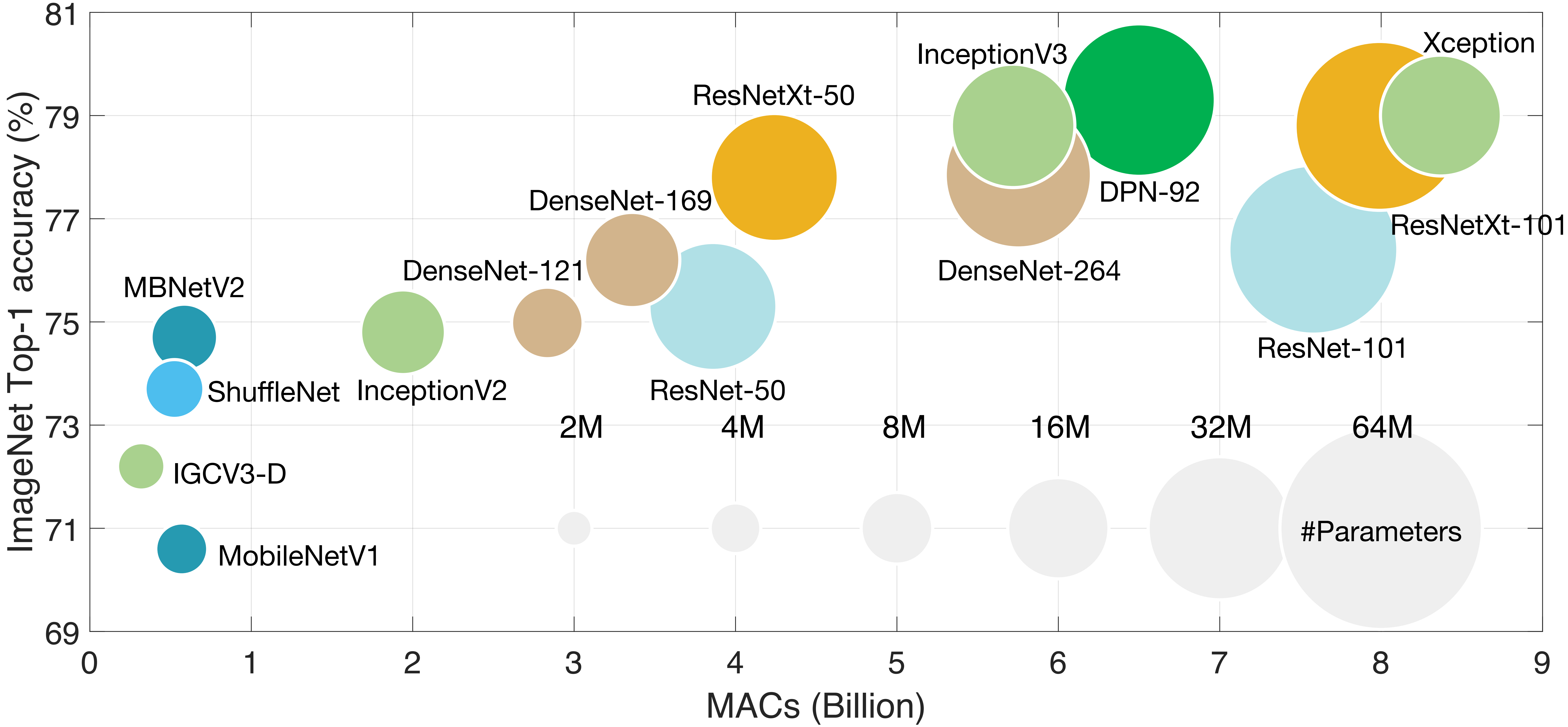


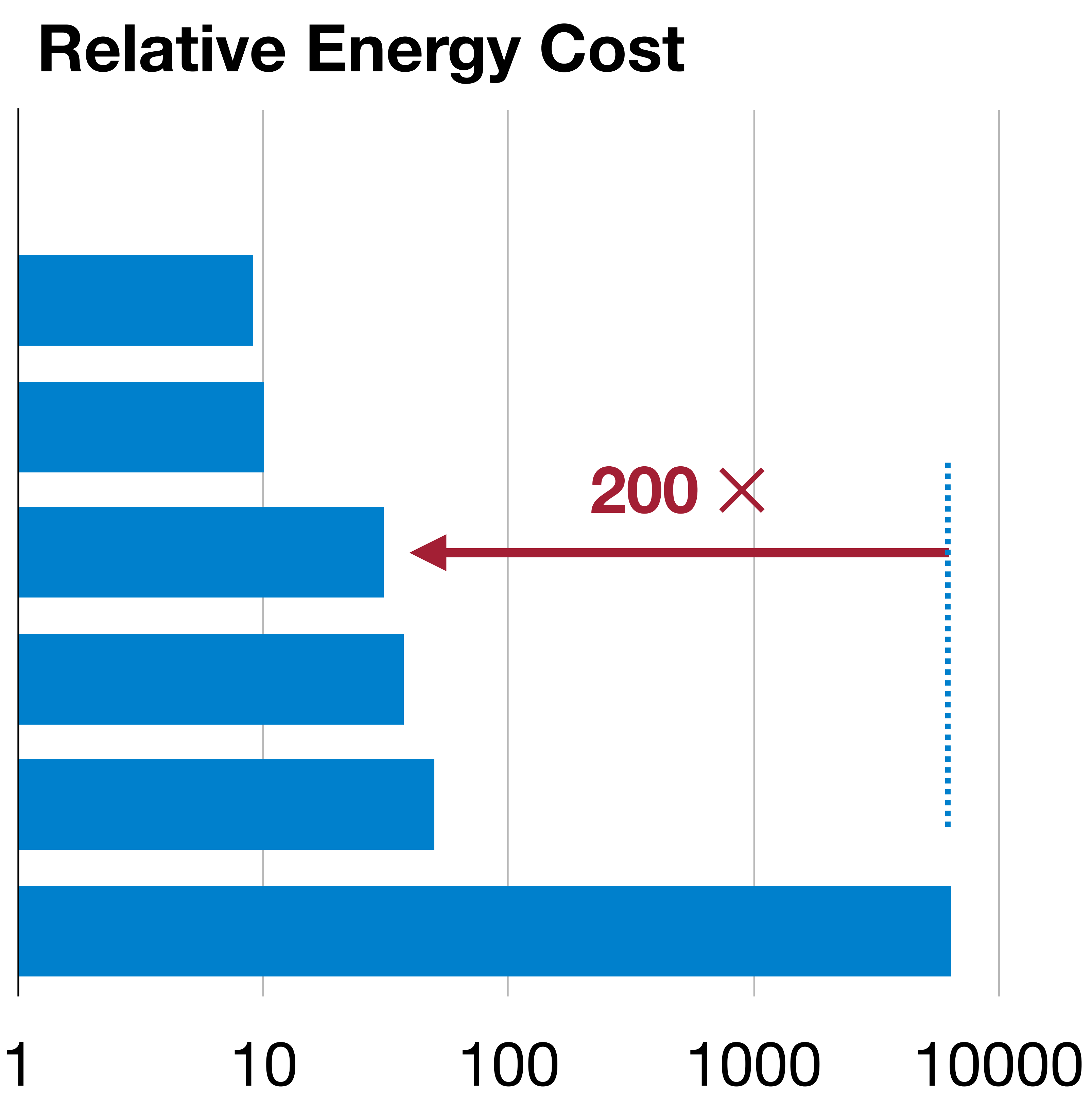
Model Compression and Hardware Acceleration for Neural Networks: A Comprehensive Survey [Deng *et al.*, IEEE 2020]

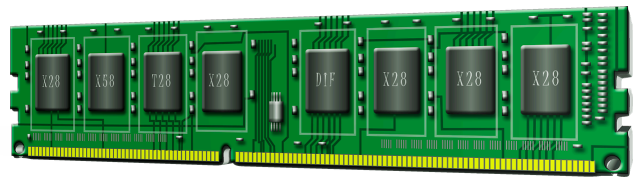
Memory is Expensive

Data Movement → More Memory Reference → More Energy

Operation	Energy [pJ]
32 bit int ADD	0.1
32 bit float ADD	0.9
32 bit Register File	1
32 bit int MULT	3.1
32 bit float MULT	3.7
32 bit SRAM Cache	5
32 bit DRAM Memory	640

Rough Energy Cost For Various Operations in 45nm 0.9V



1  = 200 ×+

This image is in the public domain

Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

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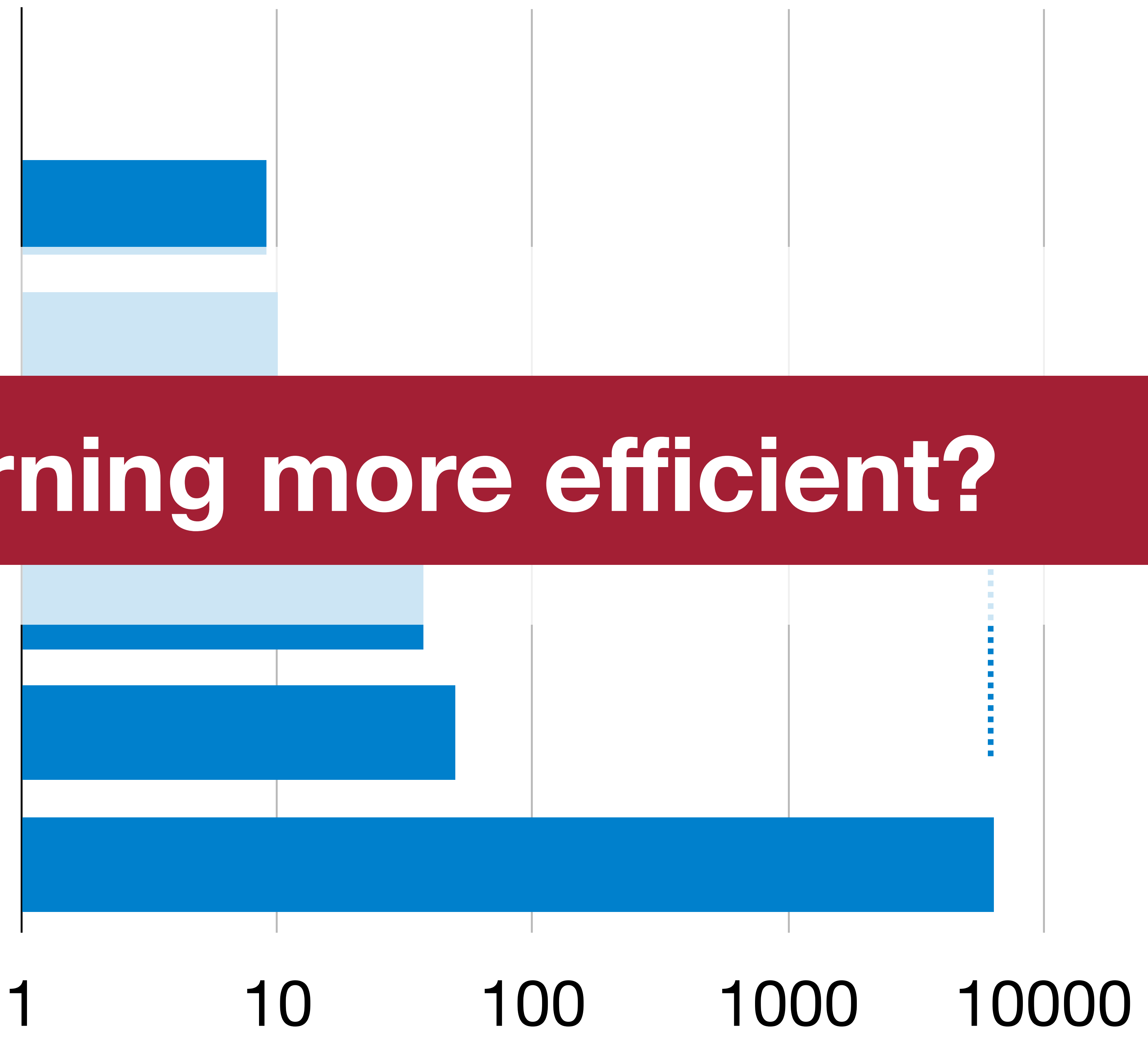
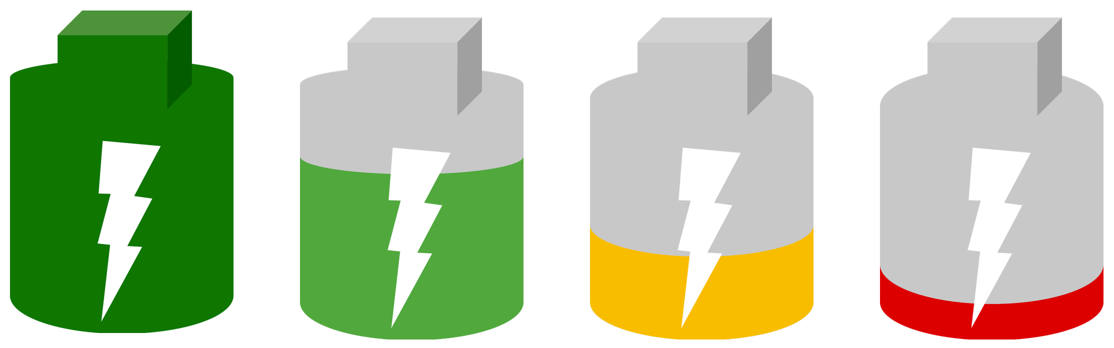
Relative Energy Cost



How should we make deep learning more efficient?



Battery images are in the public domain
Image 1, image 2, image 2, image 4

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Neural Network Pruning

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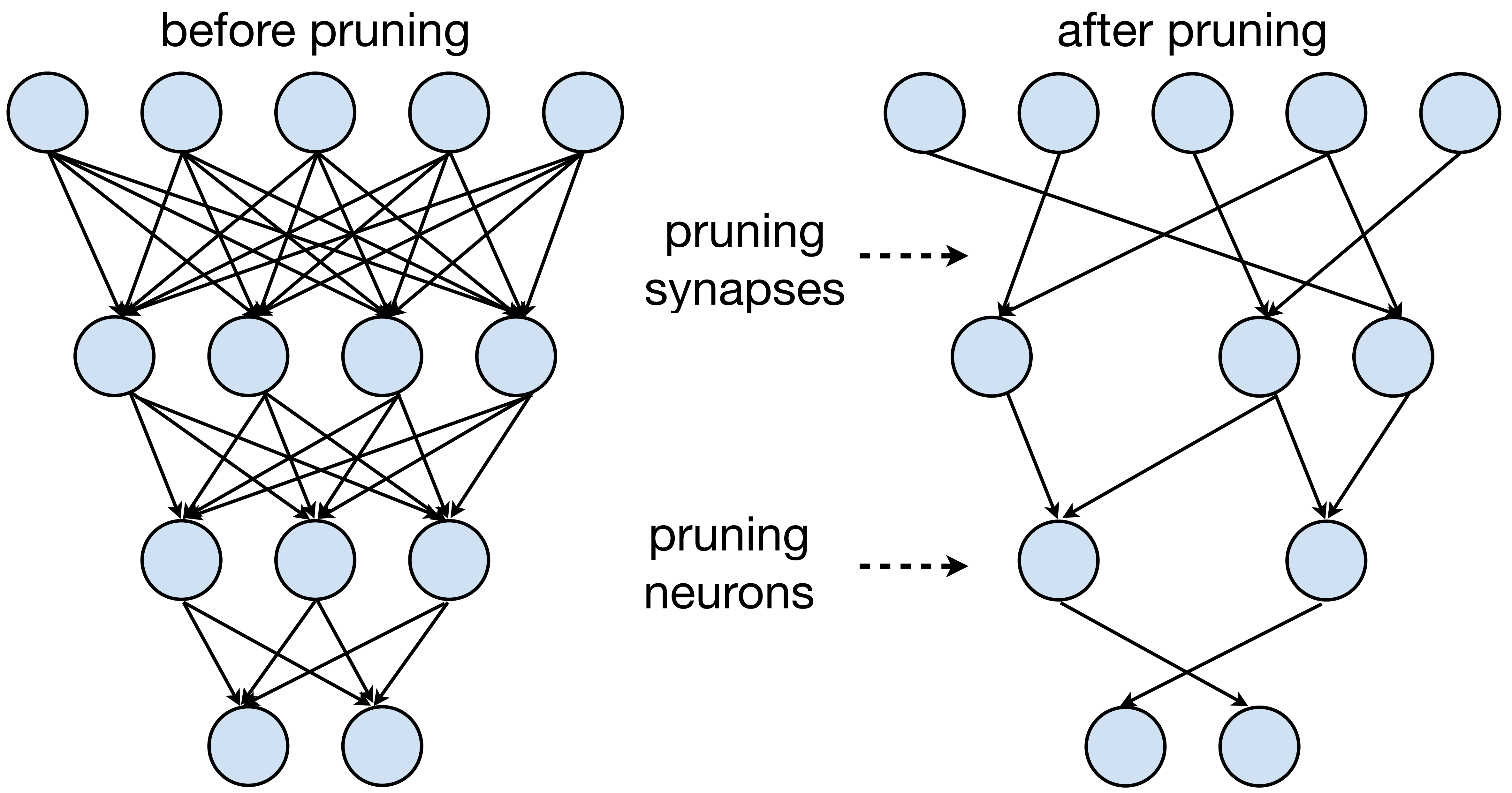
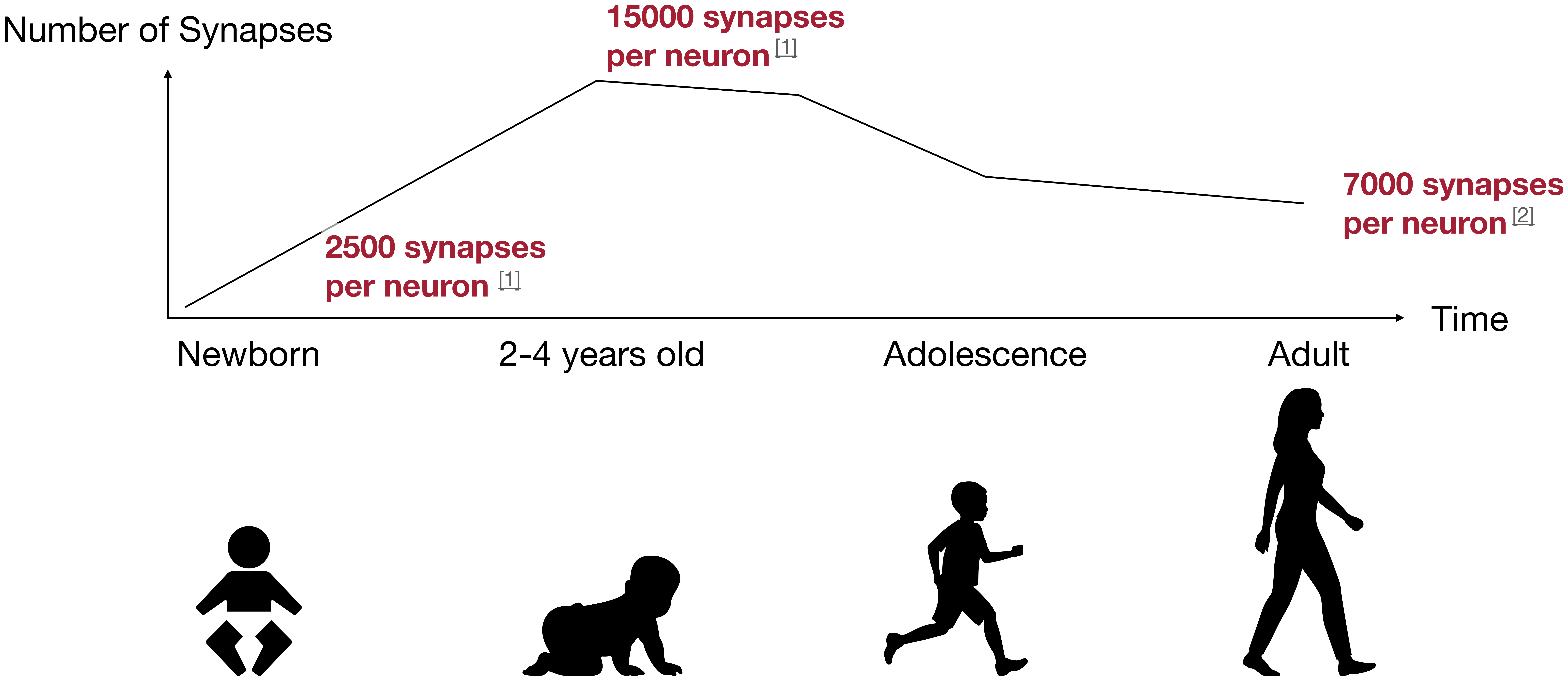


Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Pruning Happens in Human Brain

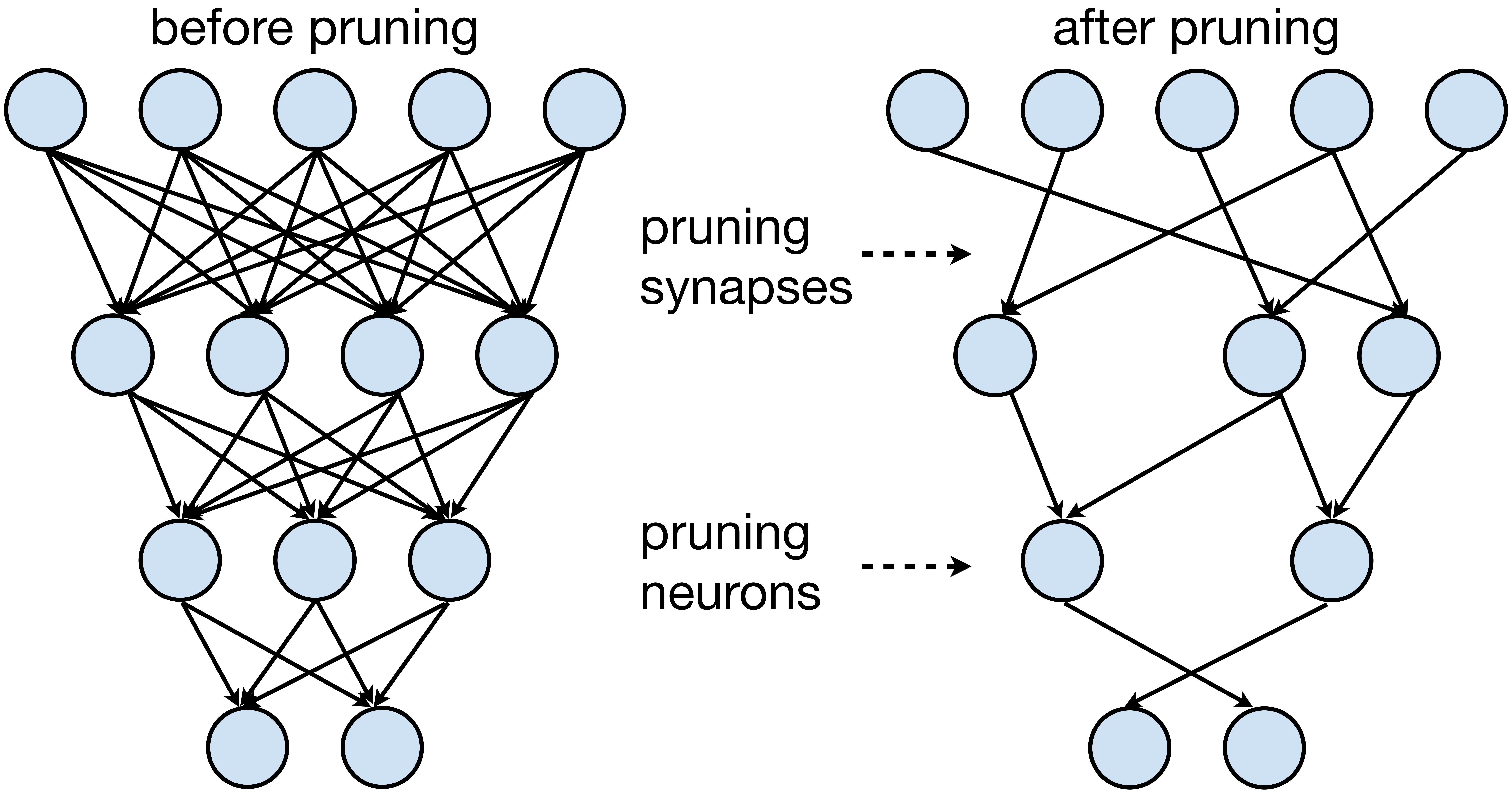


Do We Have Brain to Spare? [Drachman DA, Neurology 2004]
Peter Huttenlocher (1931–2013) [Walsh, C. A., Nature 2013]

Data Source: 1, 2
Slide Inspiration: Alila Medical Media

Neural Network Pruning

Make neural network smaller by removing synapses and neurons



Optimal Brain Damage [LeCun *et al.,* NeurIPS 1989]
Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Neural Network Pruning

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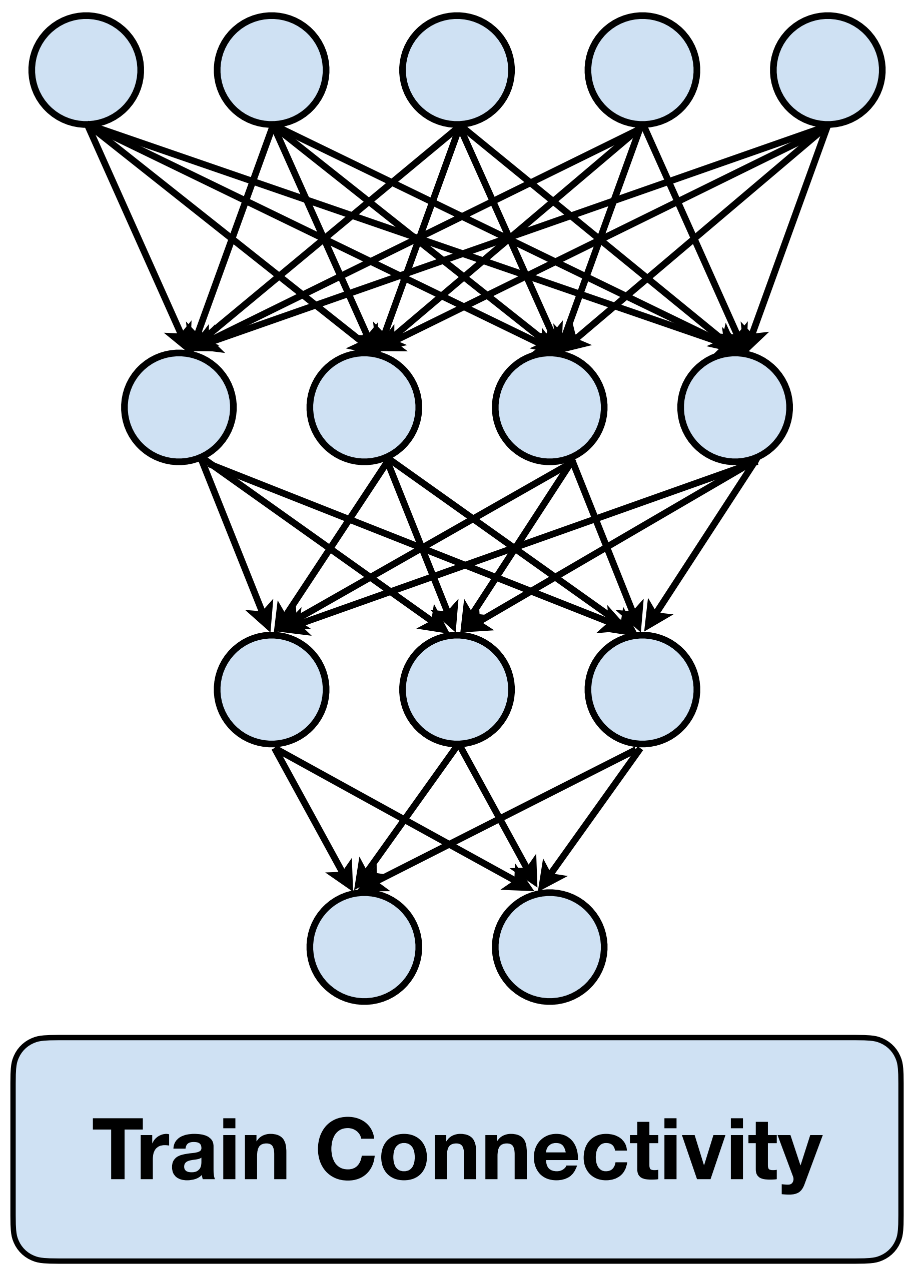

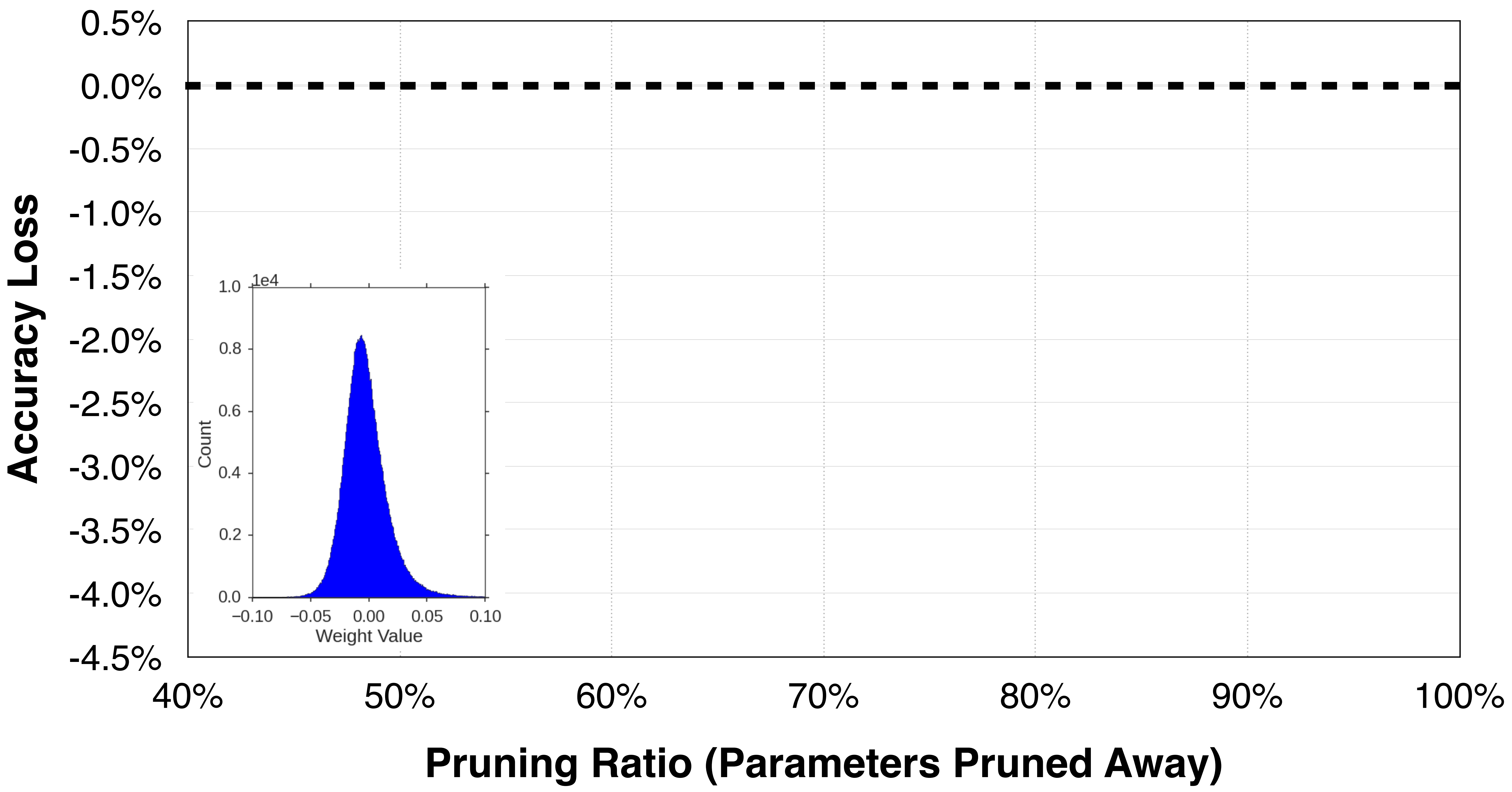

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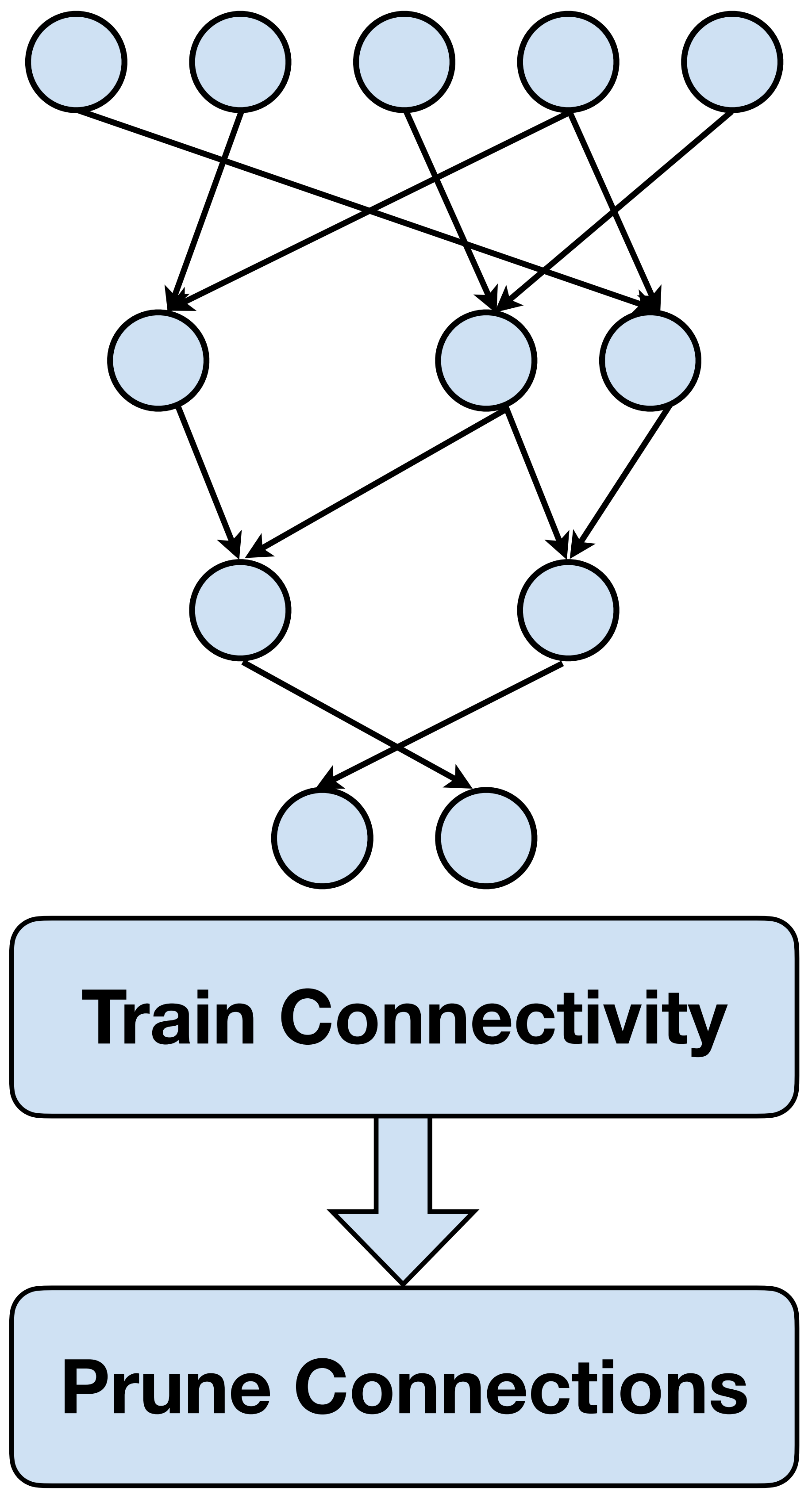

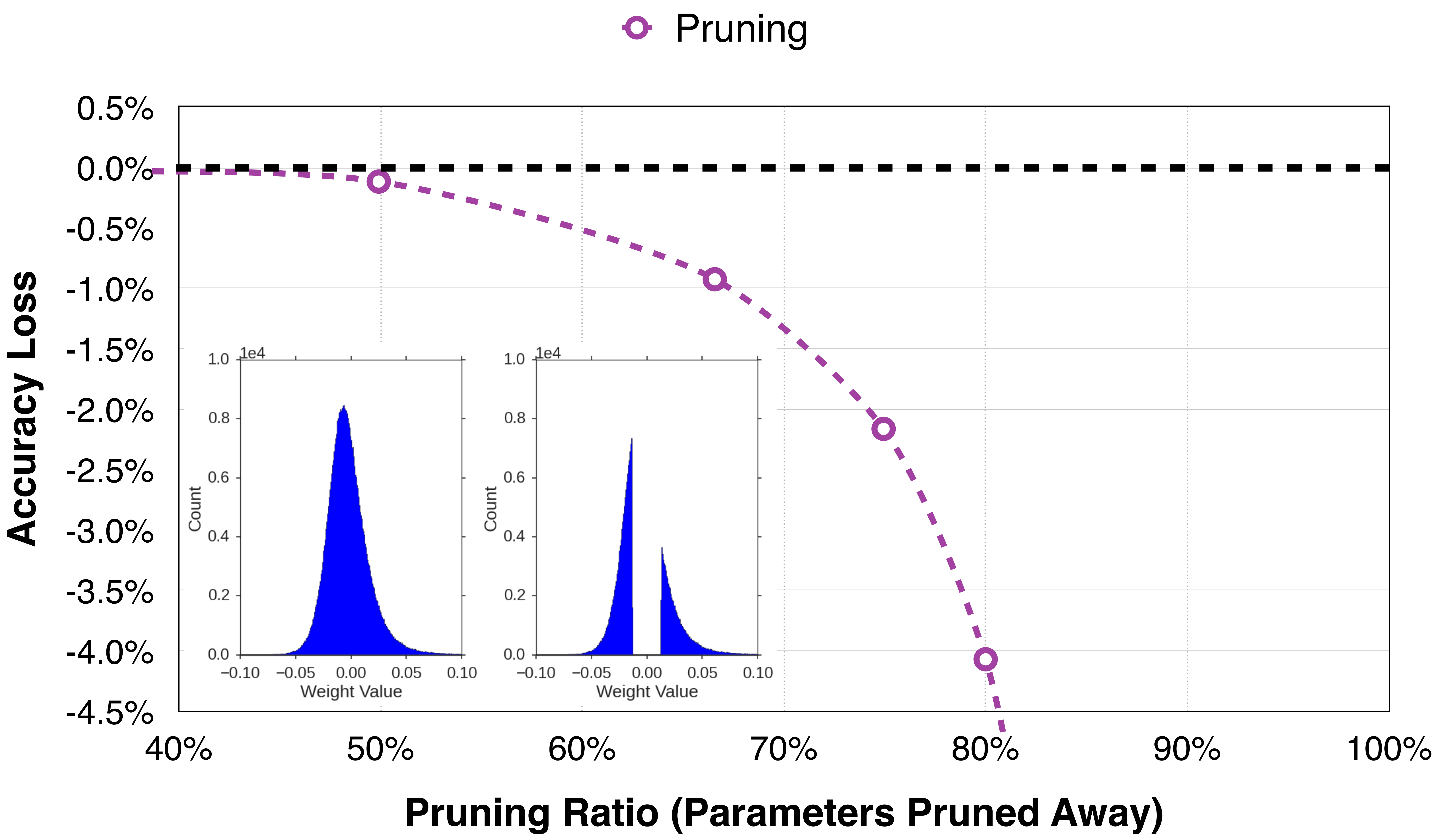

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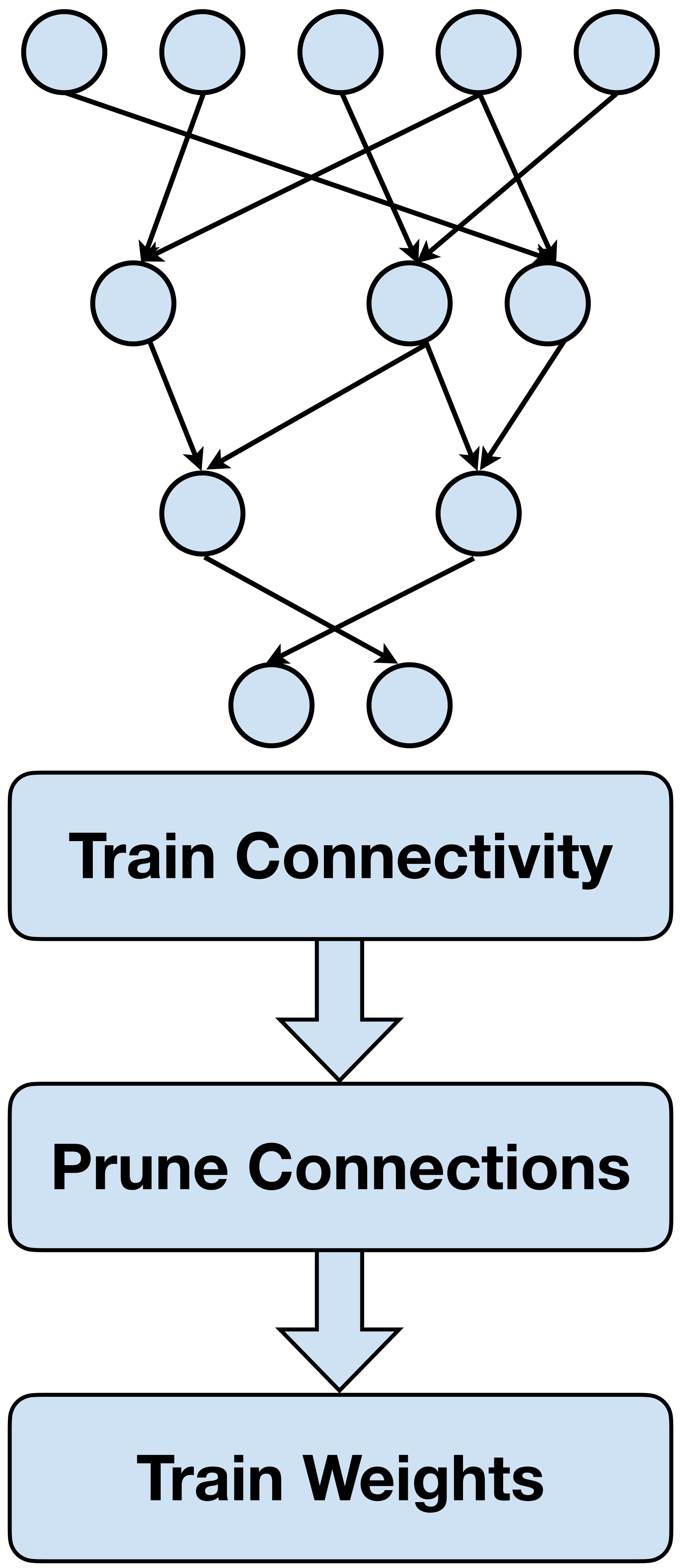

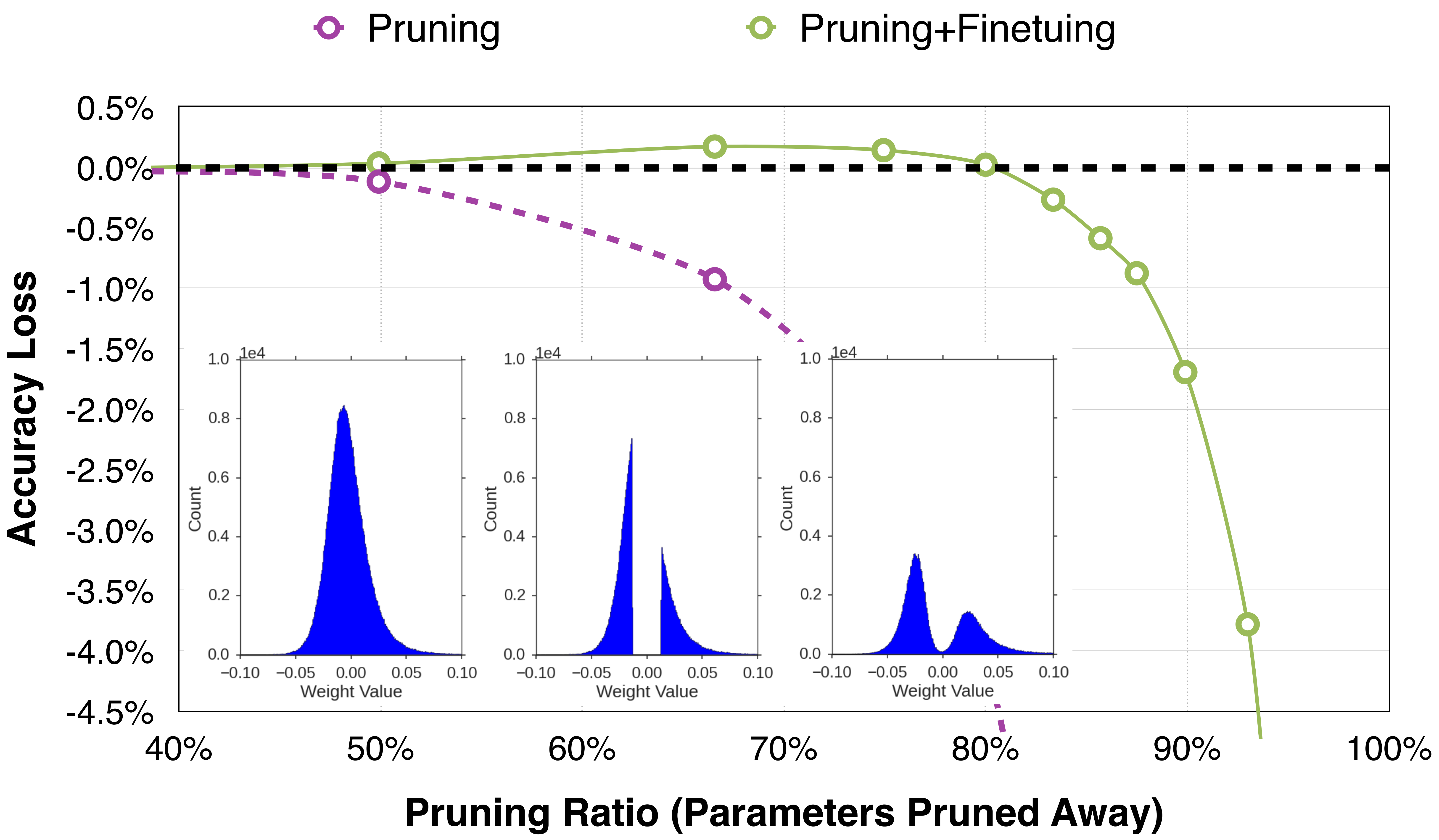

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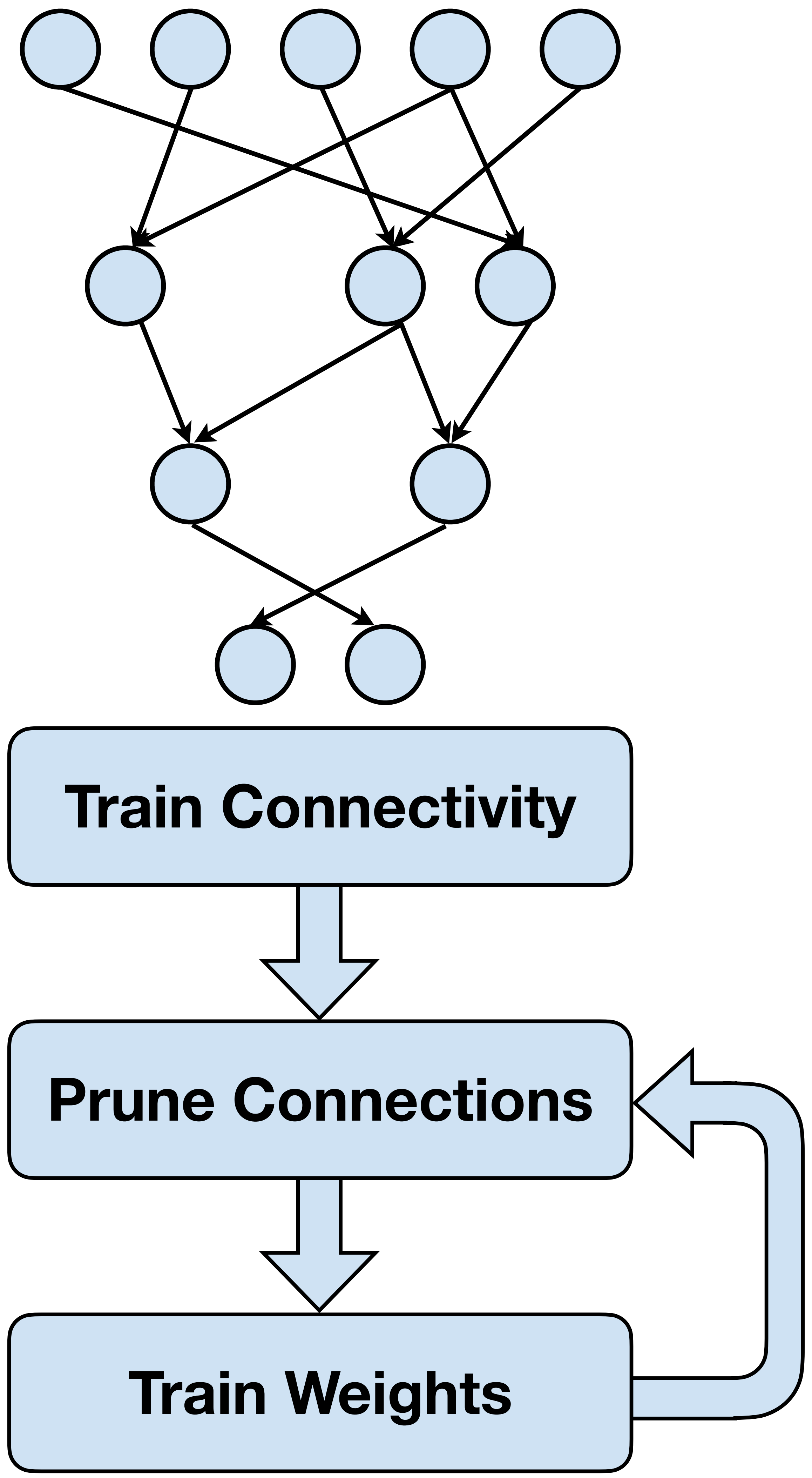
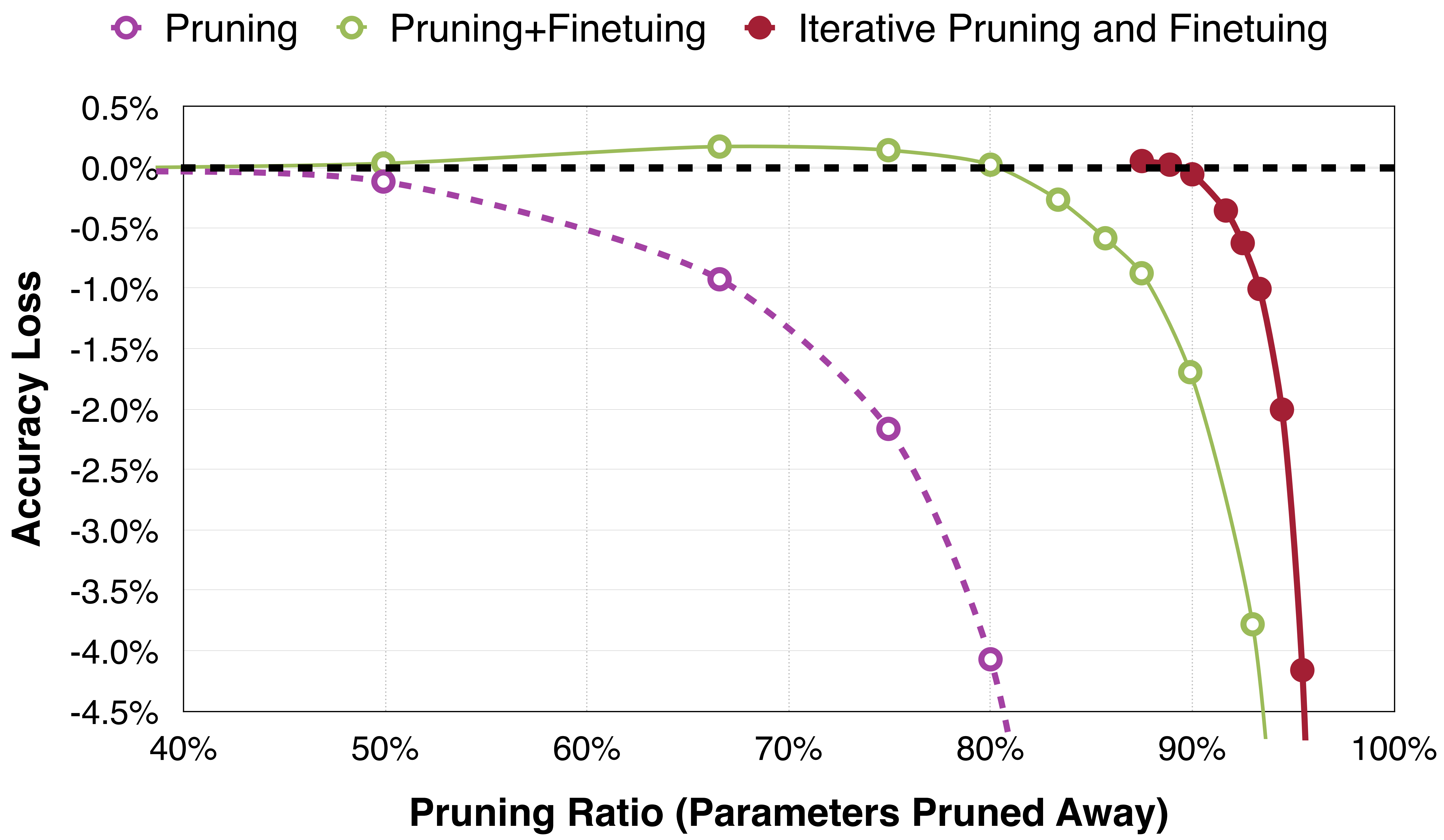




Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Neural Network Pruning

Make neural network smaller by removing synapses and neurons

Neural Network	#Parameters			MACs
	Before Pruning	After Pruning	Reduction	Reduction
AlexNet	61 M	6.7 M	**9 ×**	3 ×
VGG-16	138 M	10.3 M	**12 ×**	5 ×
GoogleNet	7 M	2.0 M	**3.5 ×**	5 ×
ResNet50	26 M	7.47 M	**3.4 ×**	6.3 ×
SqueezeNet	1 M	0.38 M	**3.2 ×**	3.5 ×

Efficient Methods and Hardware for Deep Learning [Han S., Stanford University]

Neural Network Pruning

Pruning the NeuralTalk LSTM does not hurt image caption quality.

Baseline: a basketball player in a white uniform is playing with a ball .

Pruned 90%: a basketball player in a white uniform is playing with a basketball.

Baseline: a brown dog is running through a grassy field.

Pruned 90%: a brown dog is running through a grassy area.

Baseline: a man is riding a surfboard on a wave.

Pruned 90%: a man in a wetsuit is riding a wave on a beach.

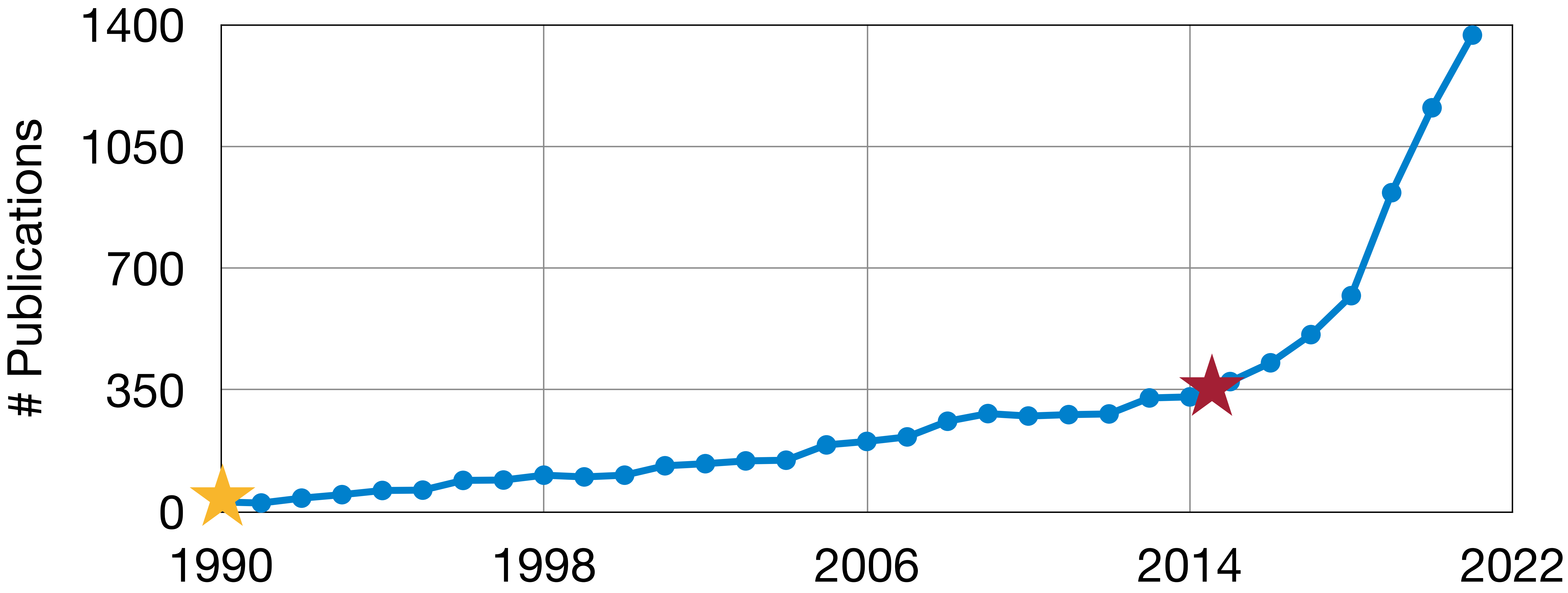
Baseline: a soccer player in red is running in the field.

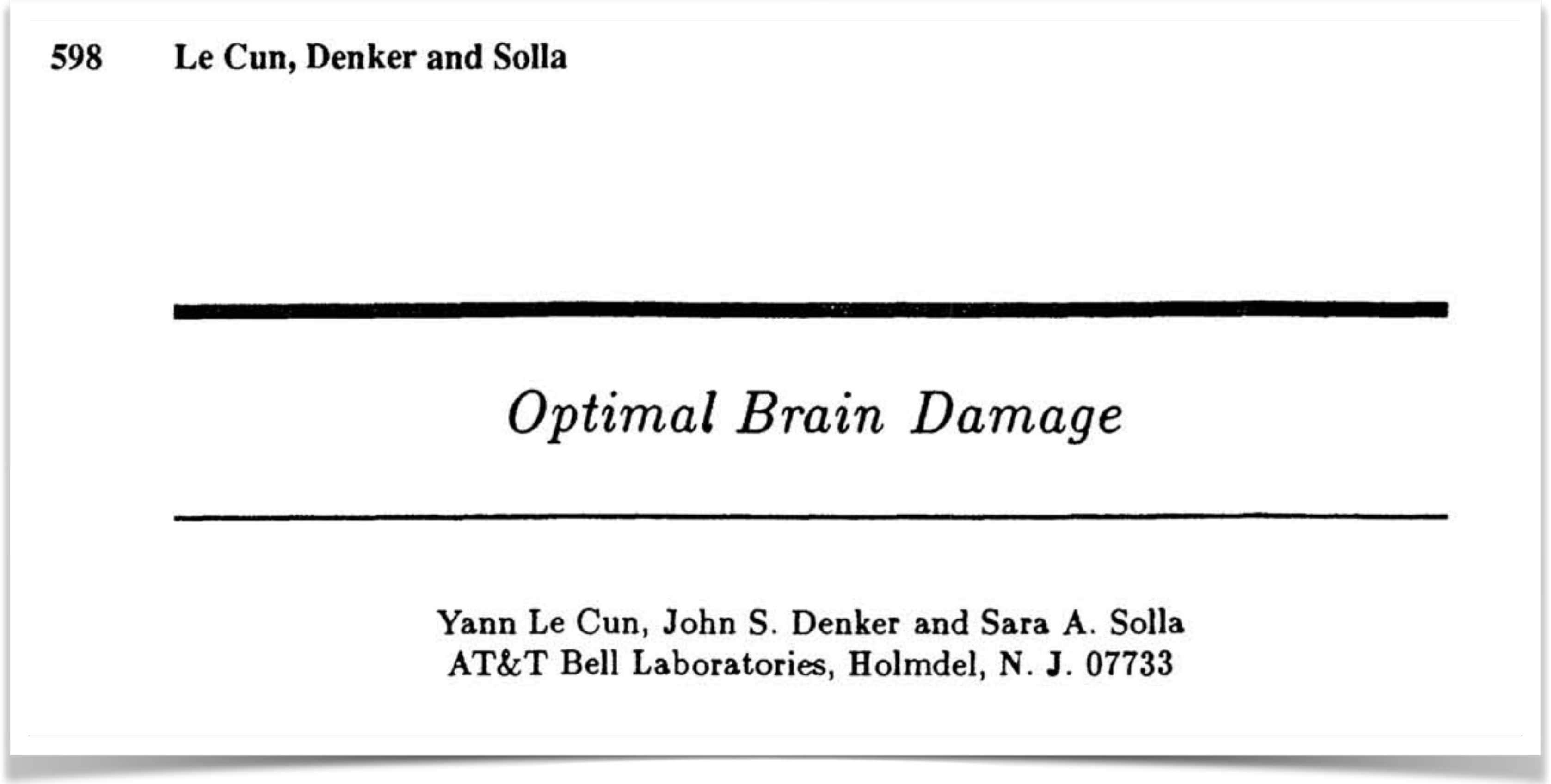
Pruned 95%: a man in a red shirt and black and white black shirt is running through a field.

Efficient Methods and Hardware for Deep Learning [Han S., Stanford University]

Neural Network Pruning

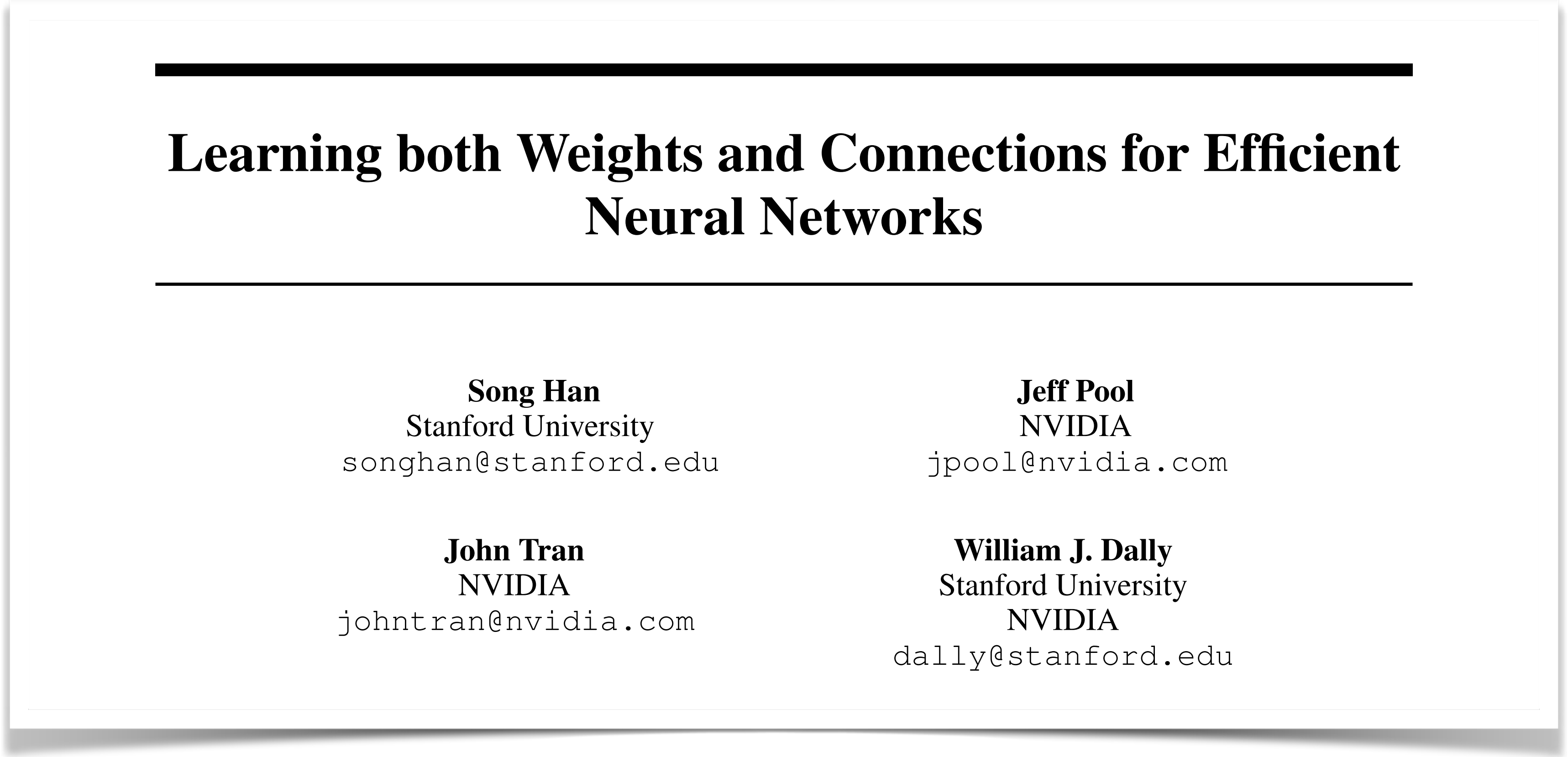
Make neural network smaller by removing synapses and neurons




598 Le Cun, Denker and Solla

Optimal Brain Damage

Yann Le Cun, John S. Denker and Sara A. Solla
AT&T Bell Laboratories, Holmdel, N. J. 07733


Learning both Weights and Connections for Efficient Neural Networks

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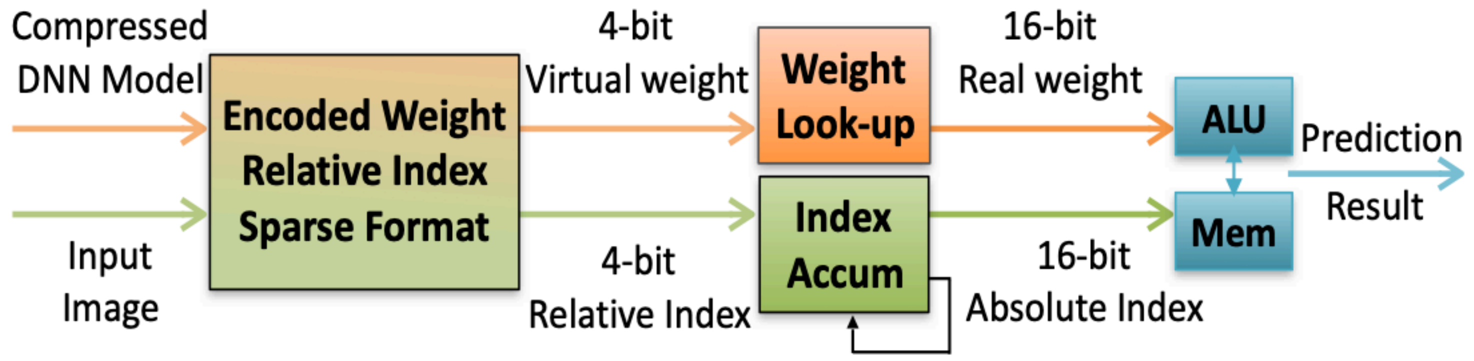
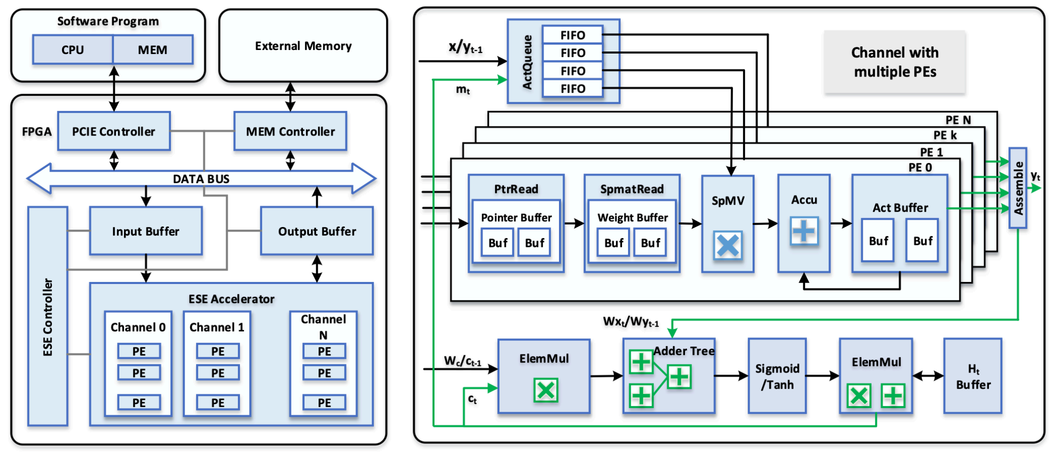
Souce: Dimension.ai

Pruning in the Industry

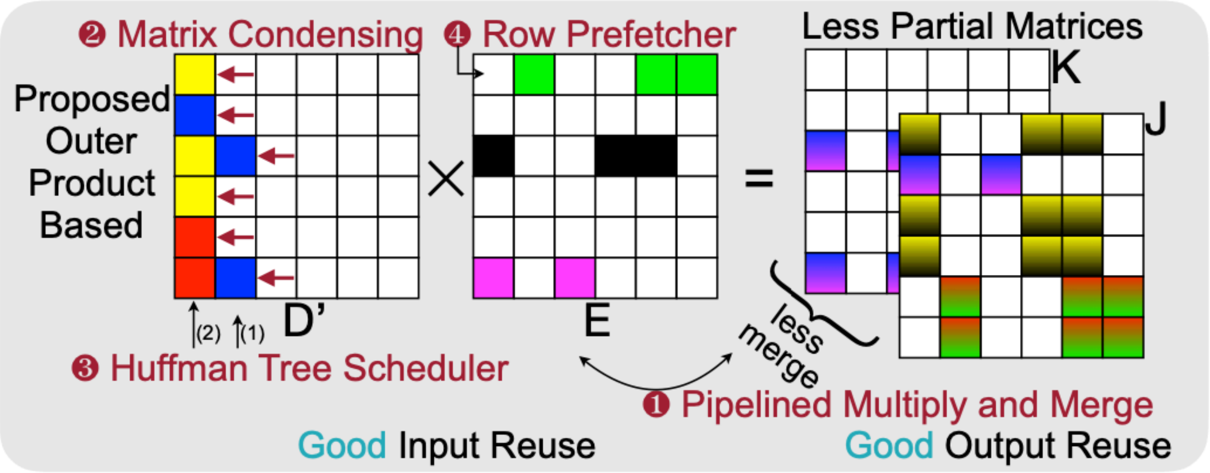
Hardware support for sparsity



EIE [Han *et al.*, ISCA 2016]

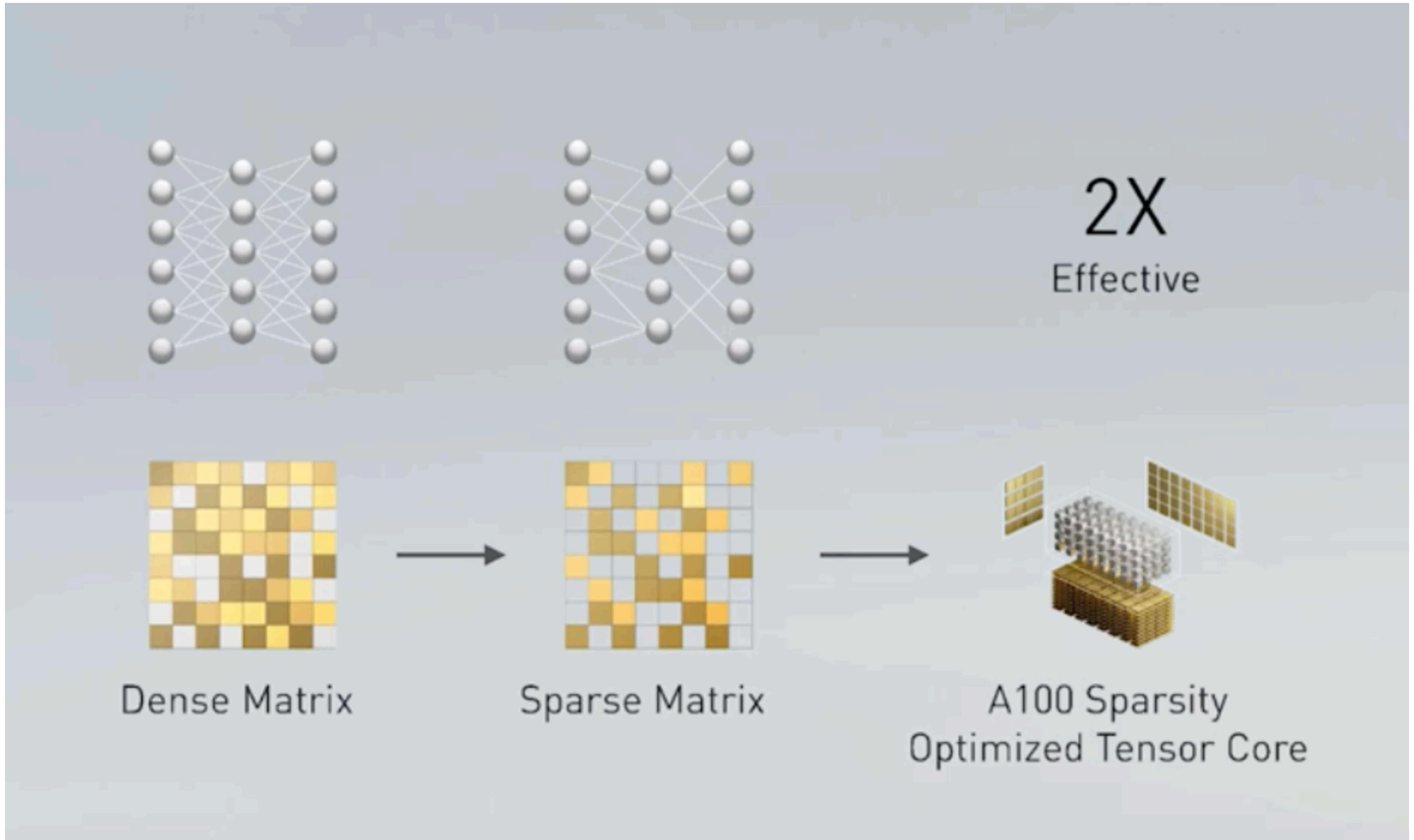


ESE [Han *et al.*, FPGA 2017]



SpArch [Zhang *et al.*, HPCA 2020]
SpAtten [Wang *et al.*, HPCA 2021]

NVIDIA



2:4 sparsity in A100 GPU
2X peak performance, 1.5X measured BERT speedup

Pruning in the Industry

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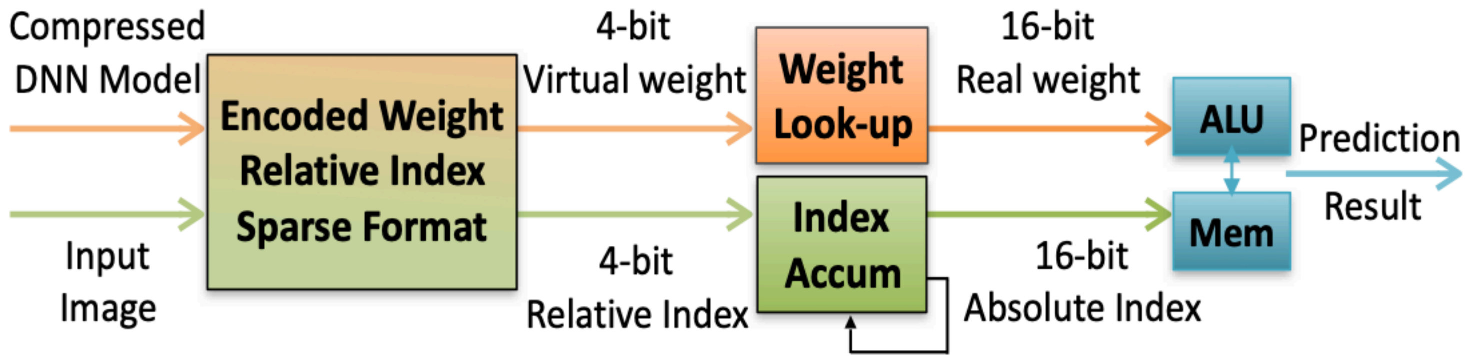

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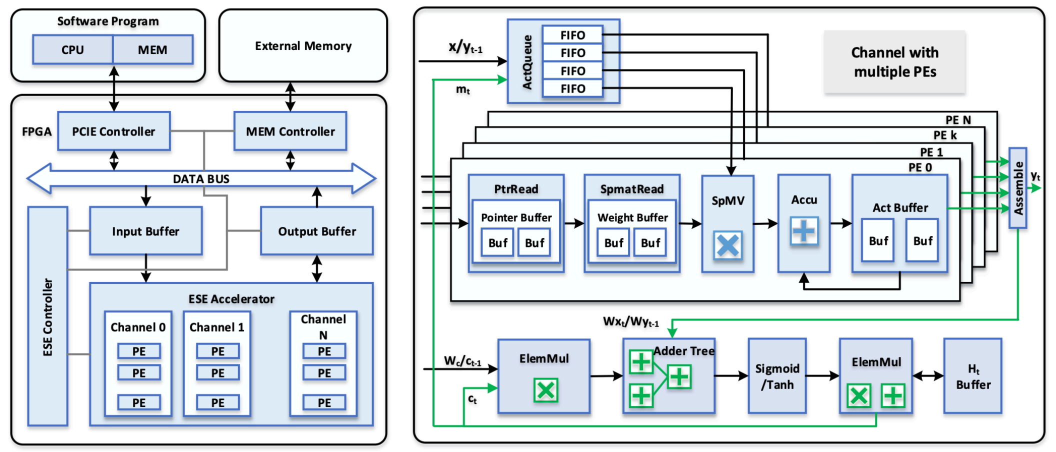

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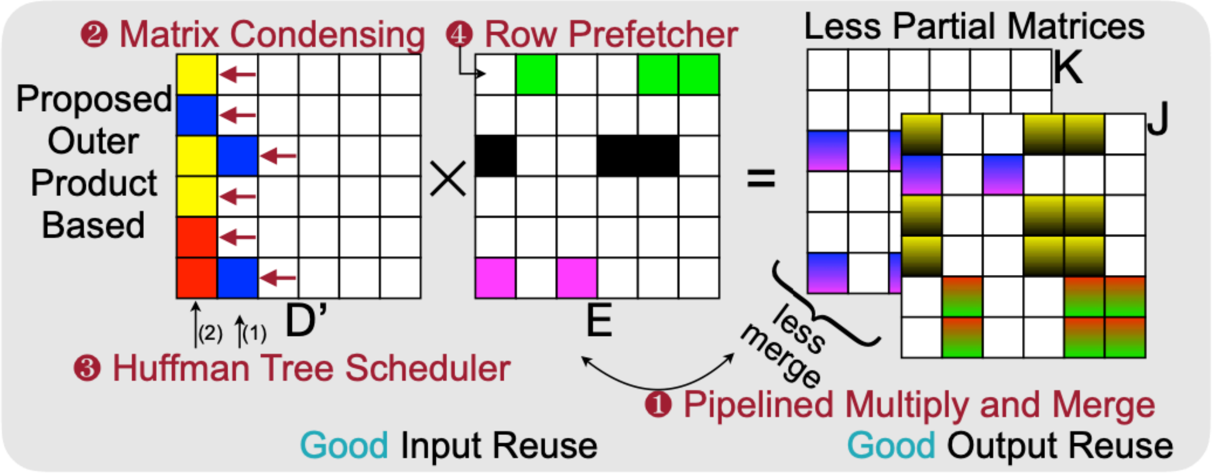

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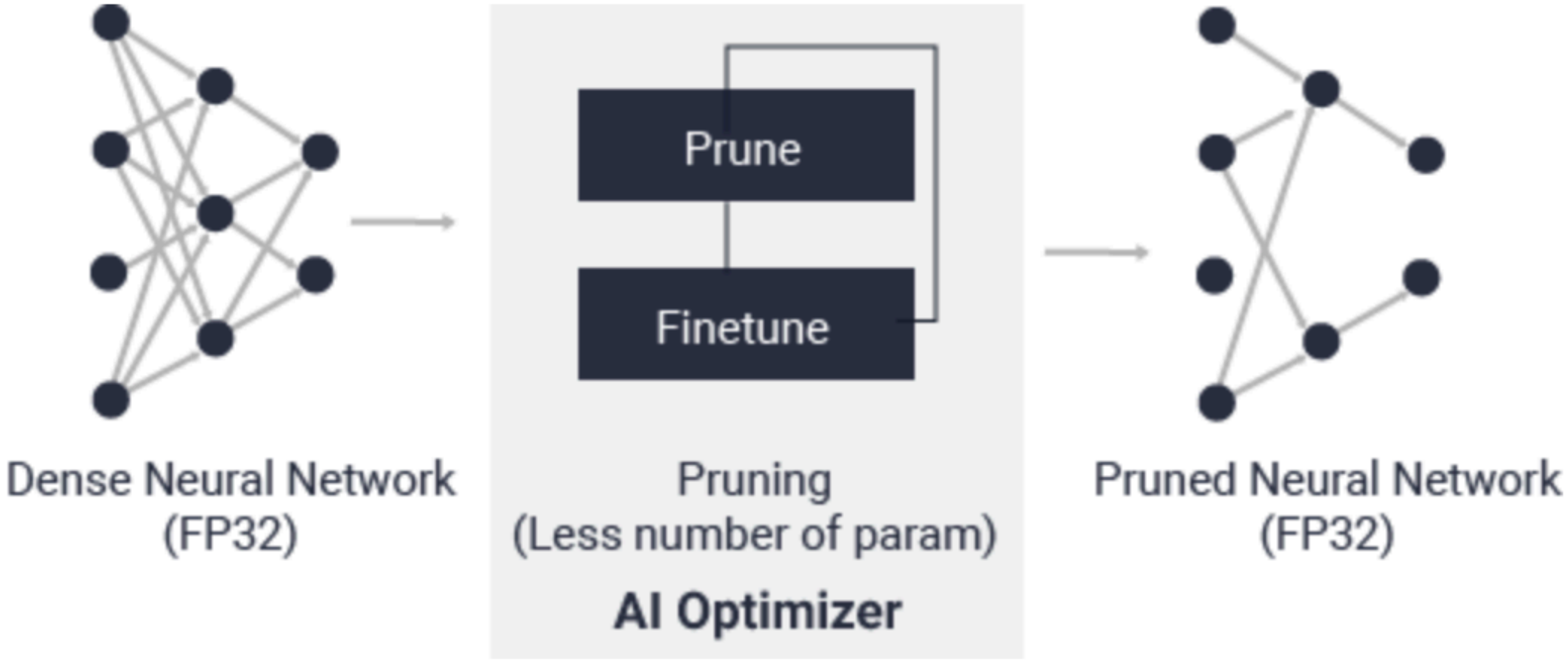

Reduce model complexity by 5x to 50x with minimal accuracy impact

Neural Network Pruning

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  - What is pruning?
  - How should we formulate pruning?
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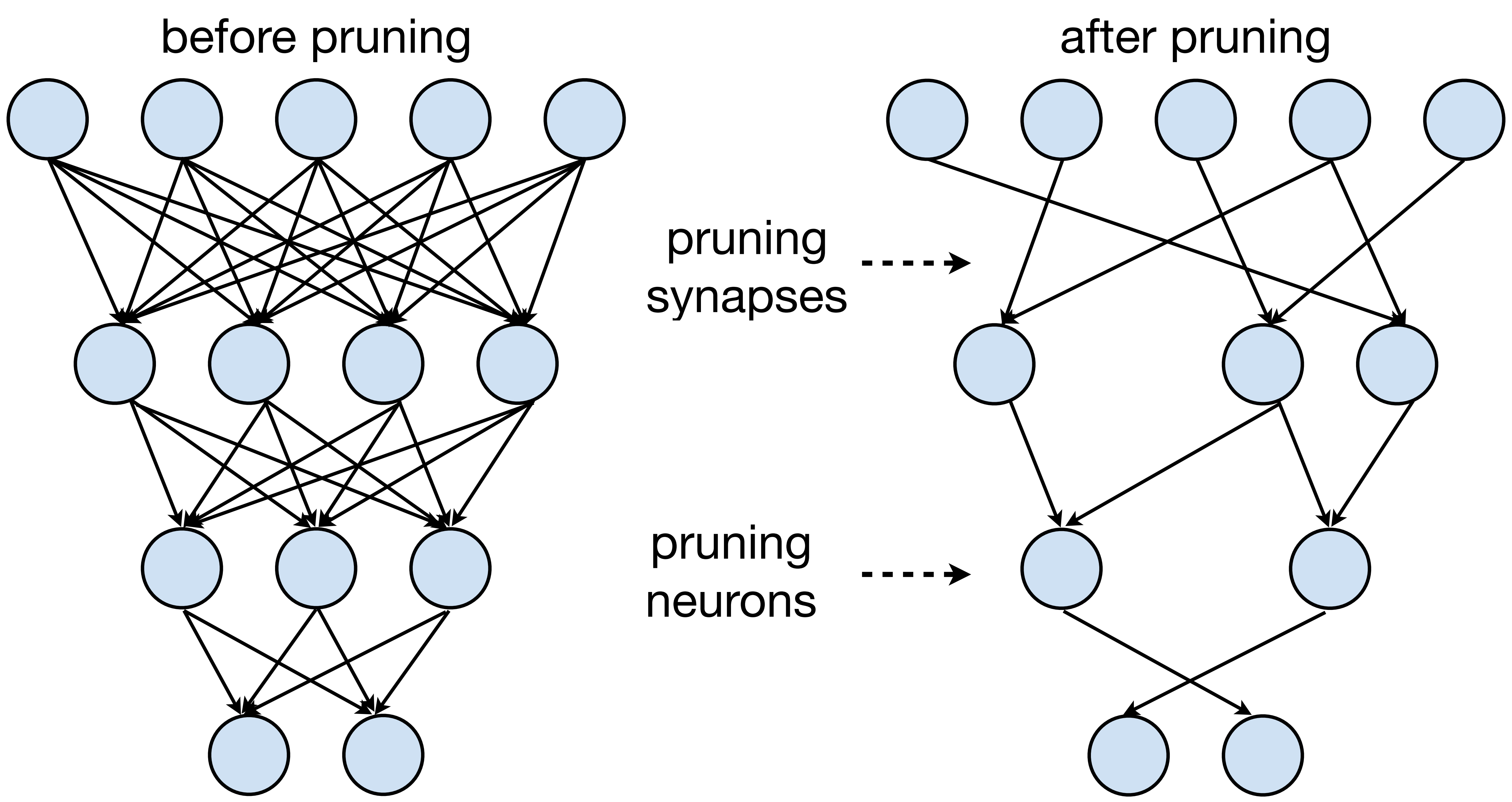


Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Neural Network Pruning

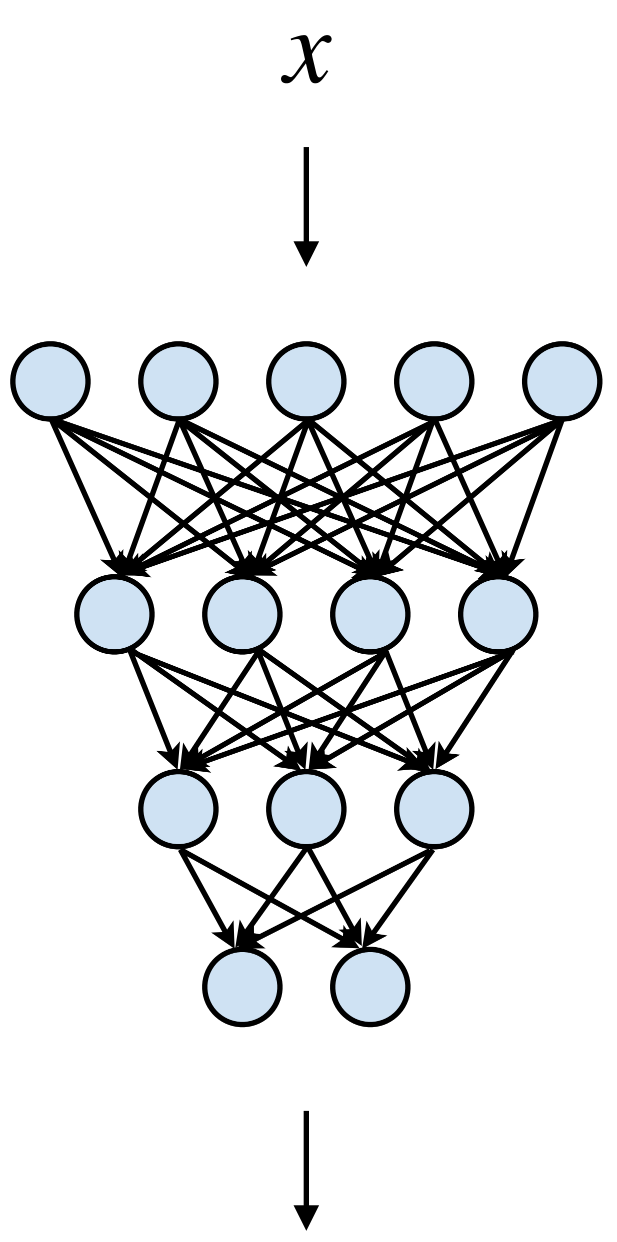
- In general, we could formulate the pruning as follows:

$$\arg\min_{\mathbf{W}_P} L(\mathbf{x}; \mathbf{W}_P)$$

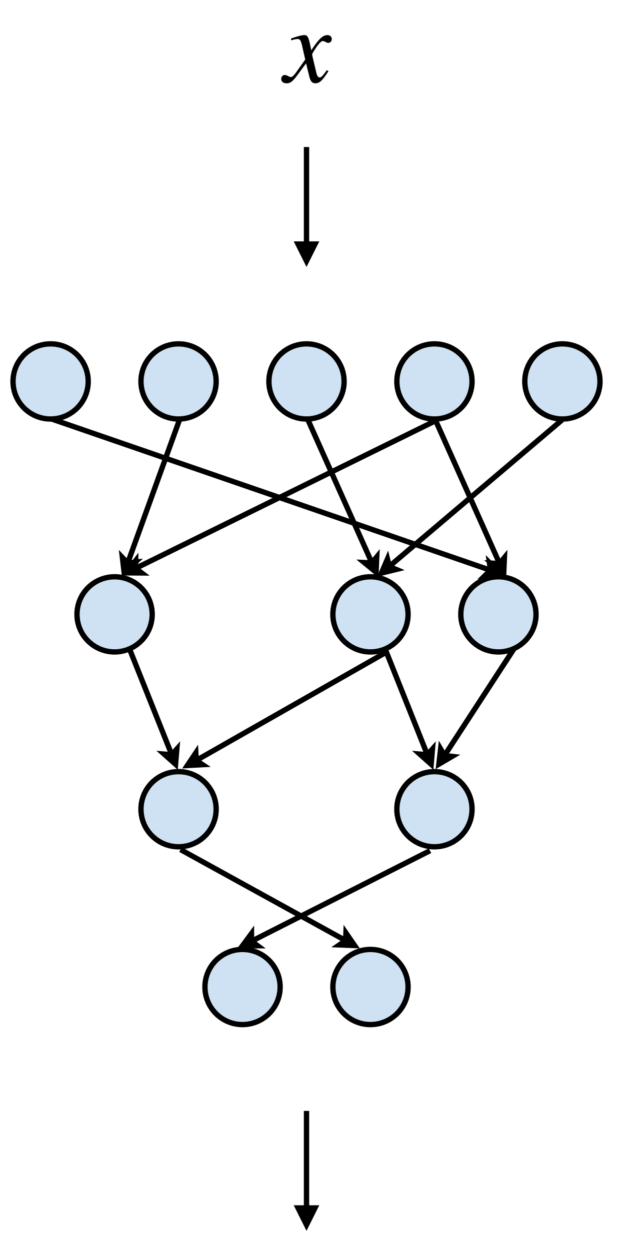
subject to

$$\|\mathbf{W}_p\|_0 < N$$

- L represents the objective function for neural network training;
- $\mathbf{x}$ is input, $\mathbf{W}$ is original weights, $\mathbf{W}_P$ is pruned weights;
- $\|\mathbf{W}_p\|_0$ calculates the #nonzeros in W_P, and N is the target #nonzeros.



$$\arg\min_{\mathbf{W}} L(\mathbf{x}; \mathbf{W})$$



$$\arg\min_{\mathbf{W}_P} L(\mathbf{x}; \mathbf{W}_P)$$

$$s.t. \|\mathbf{W}_P\|_0 \leq N$$

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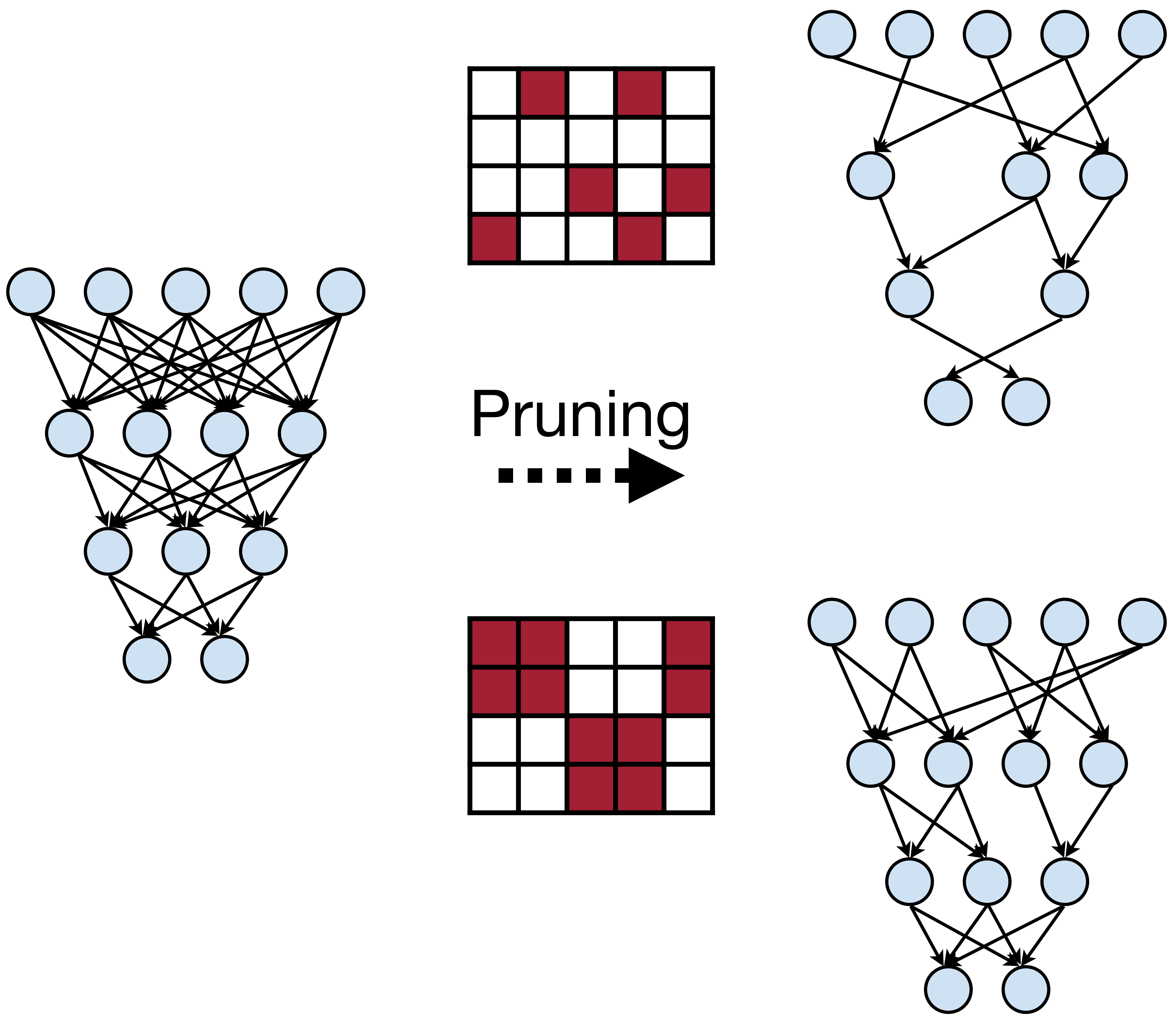

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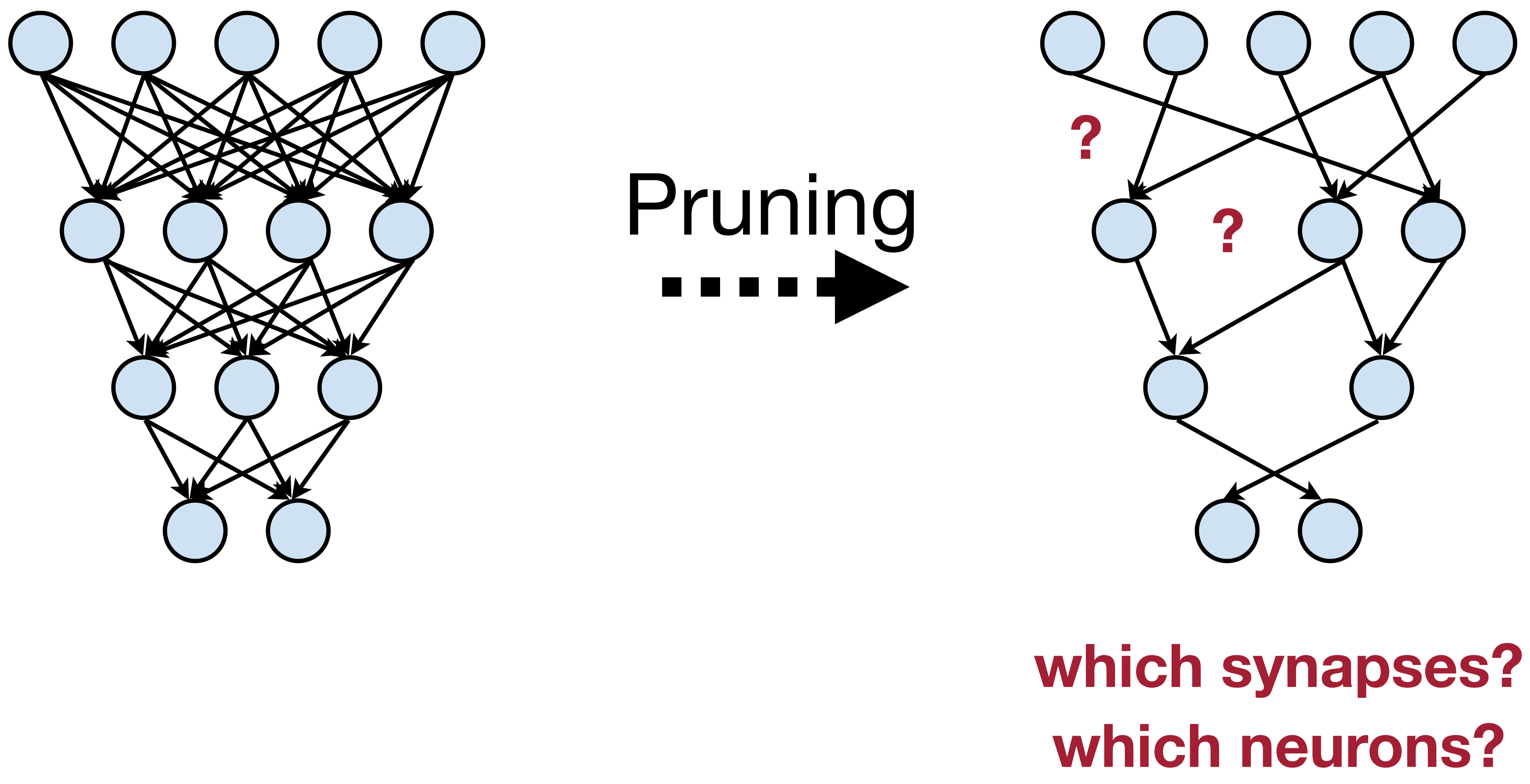

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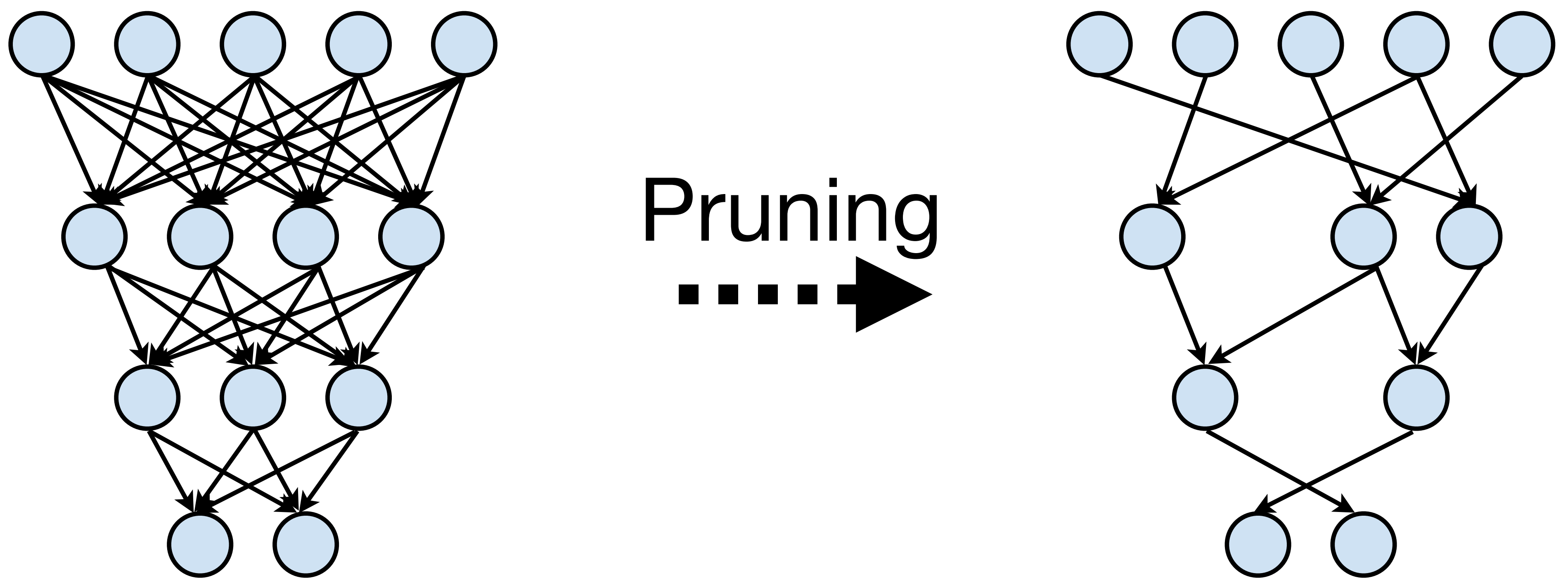

prune 30%?

prune 50%?

prune 70%?

Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

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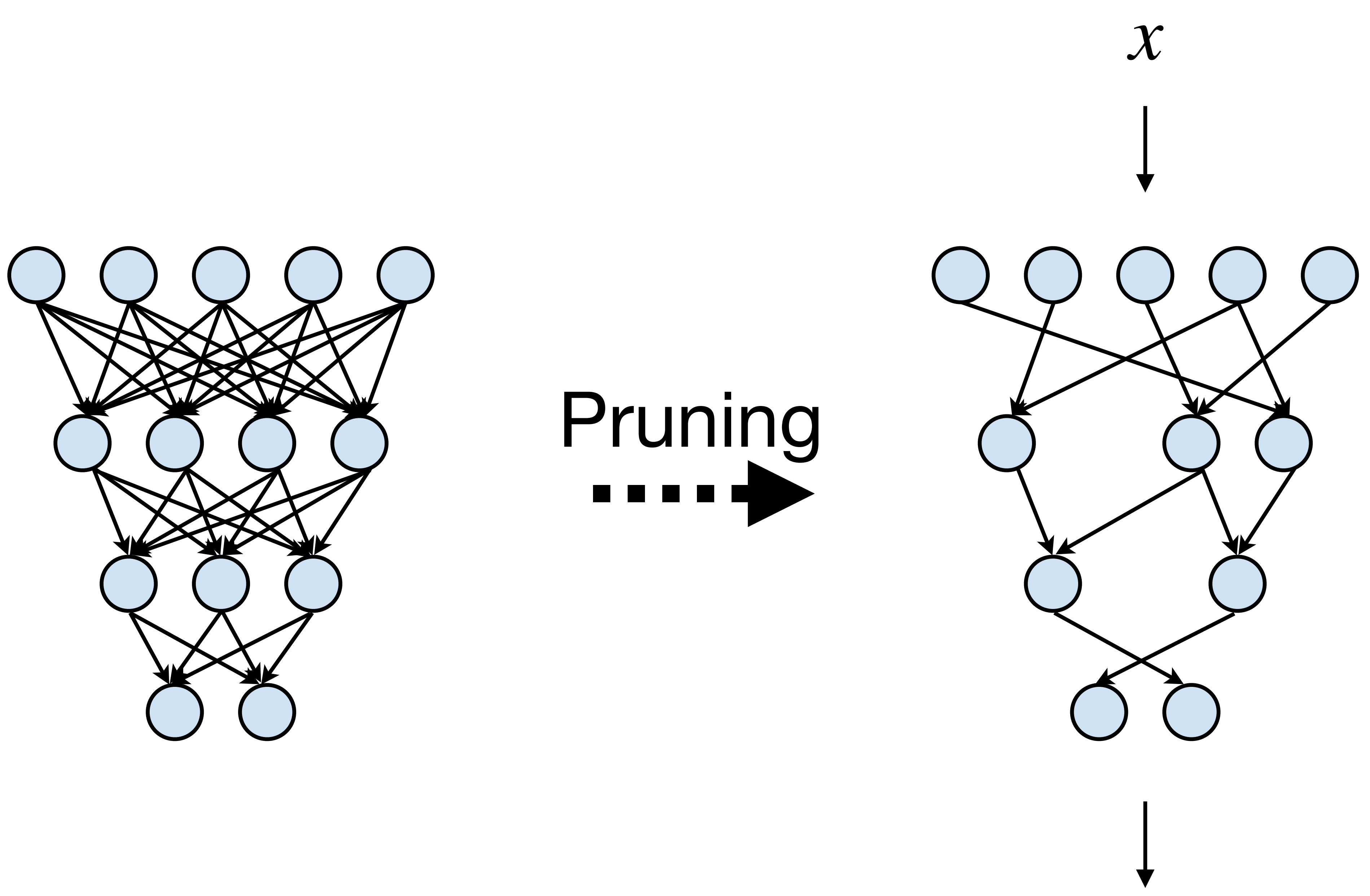

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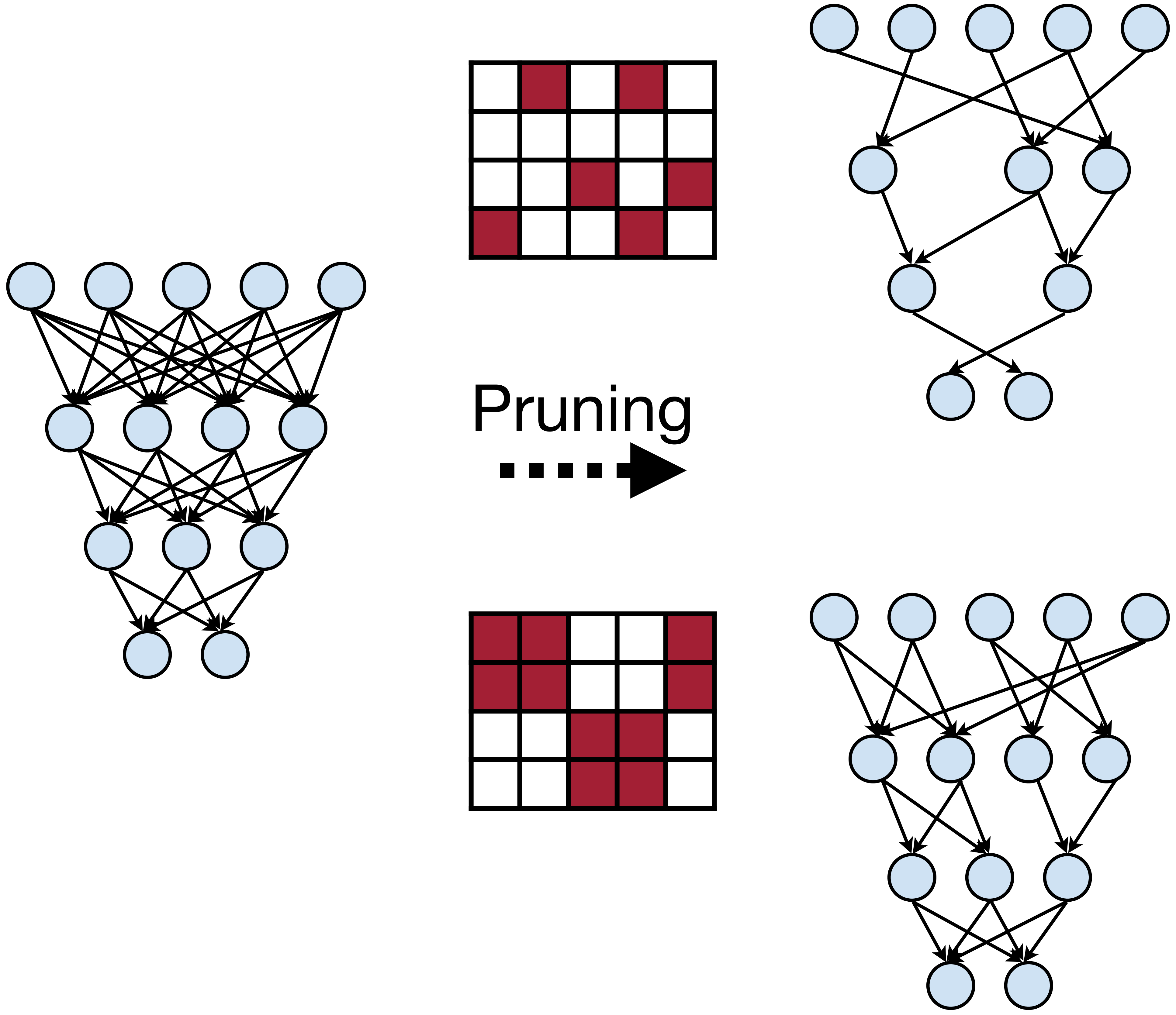

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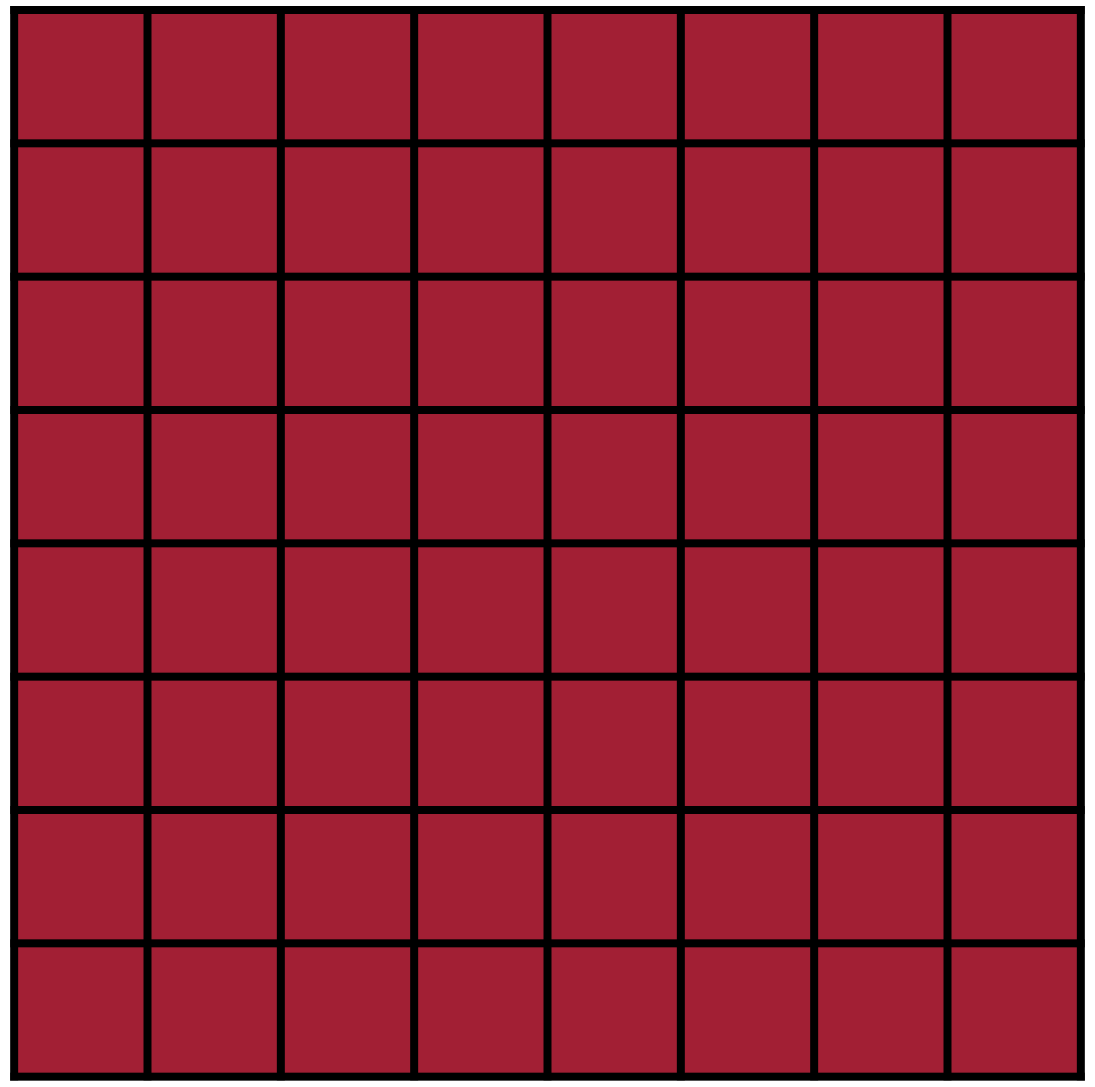
Section 2: Pruning Granularity

Pruning can be performed at different granularities, from structured to non-structured.

Pruning at Different Granularities

A simple example of 2D weight matrix



Pruning at Different Granularities

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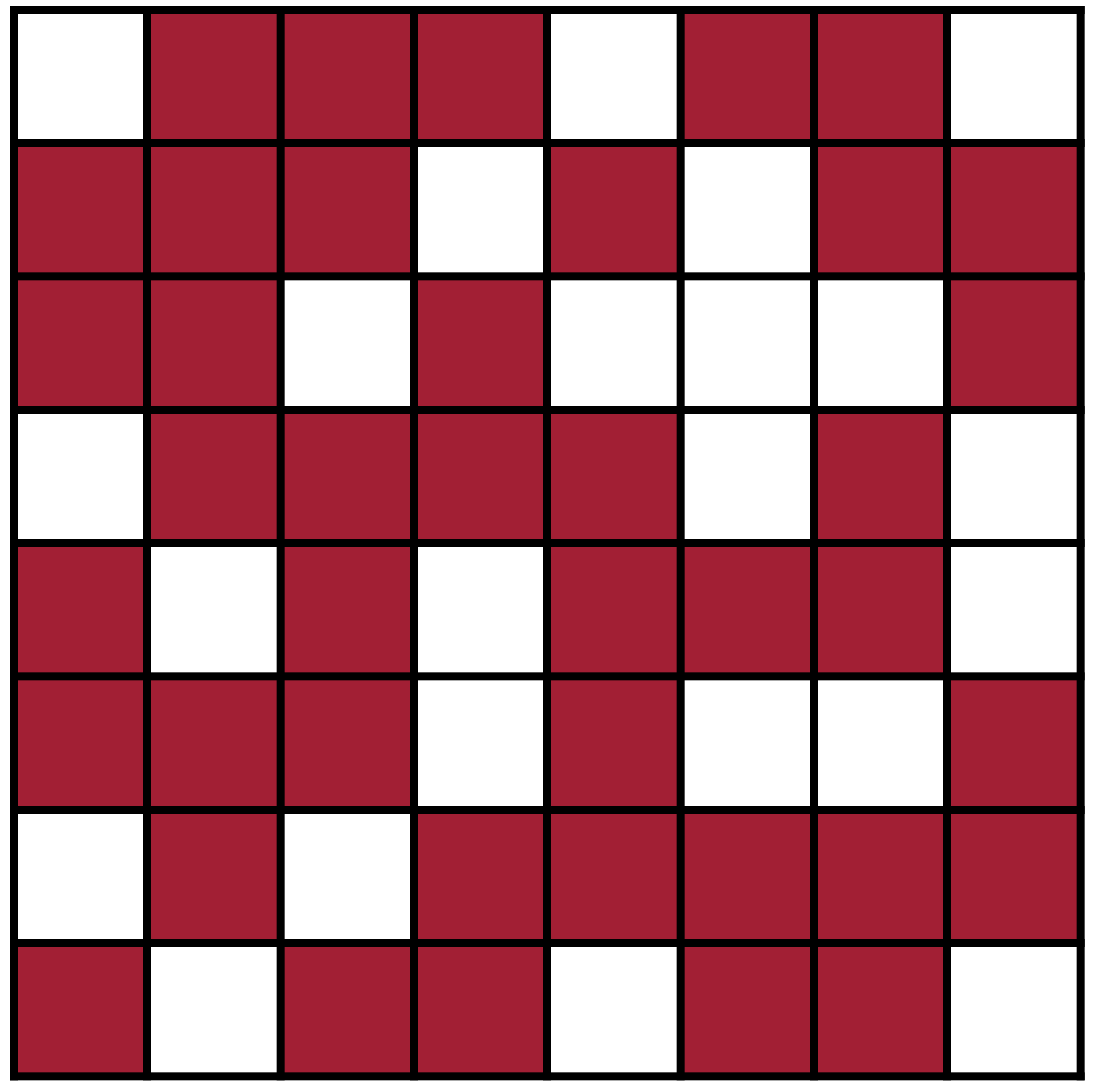
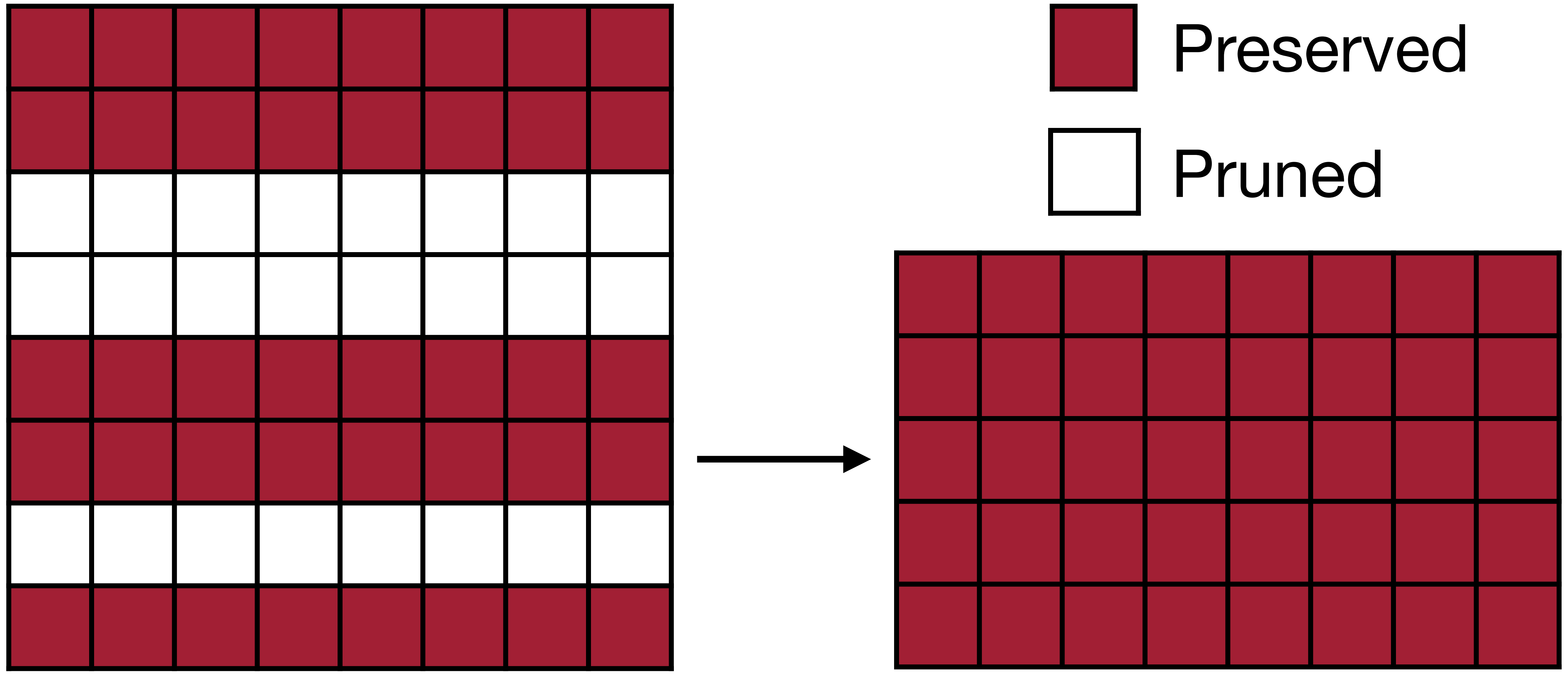




Fine-grained/Unstructured

- More flexible pruning index choice
- Hard to accelerate (irregular data expression)

Bad for 

Coarse-grained/Structured

- Less flexible pruning index choice (a subset of the fine-grained case)
- Easy to accelerate (just a smaller matrix!)

Pruning at Different Granularities

The case of convolutional layers

- The weights of convolutional layers have 4 dimensions $[c_o, c_i, k_h, k_w]$:
    - c_i: input channels (or channels)
    - c_o: output channels (or filters)
    - k_h: kernel size height
    - k_w: kernel size width

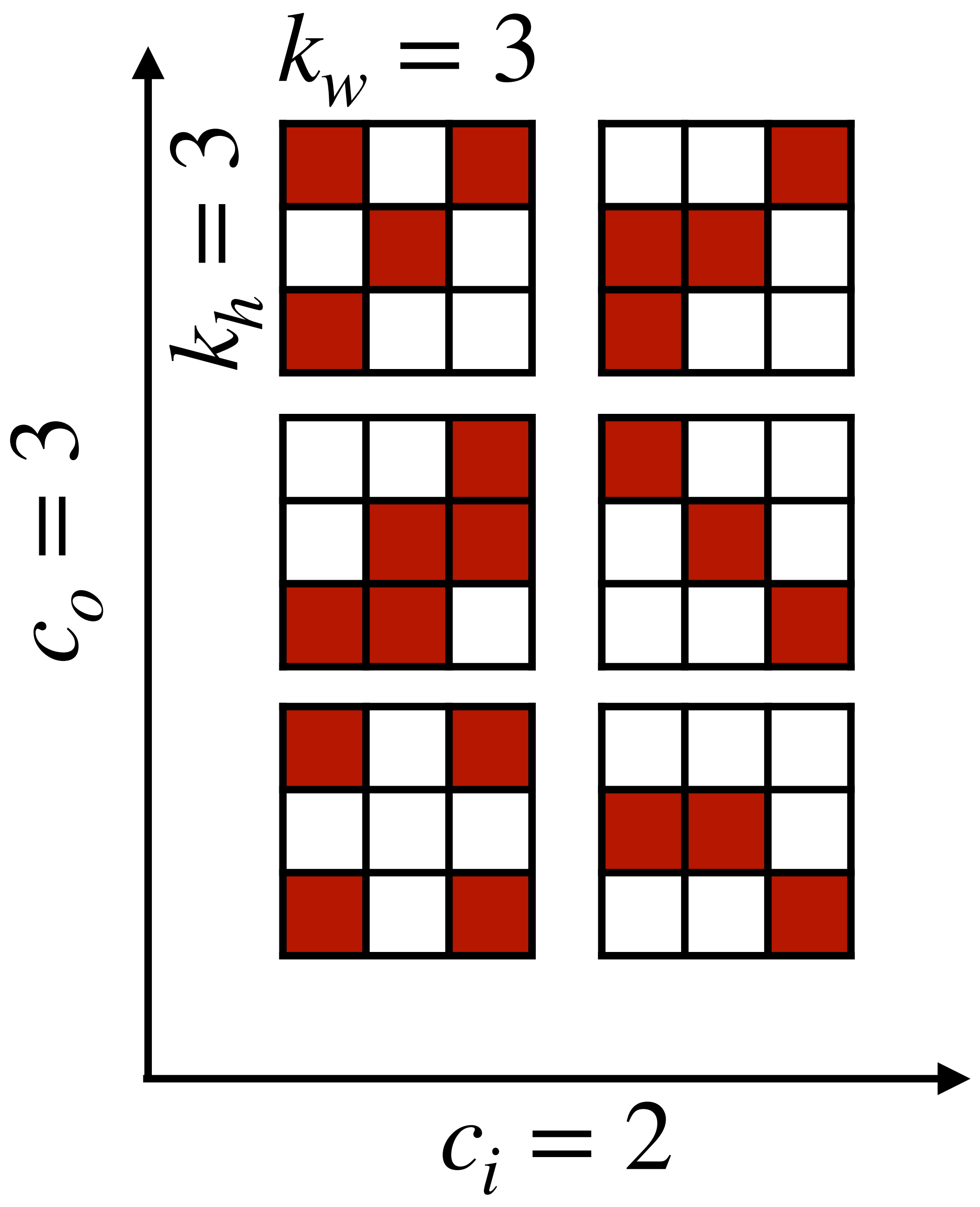
- The 4 dimensions give us more choices to select pruning granularities

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities

Notations

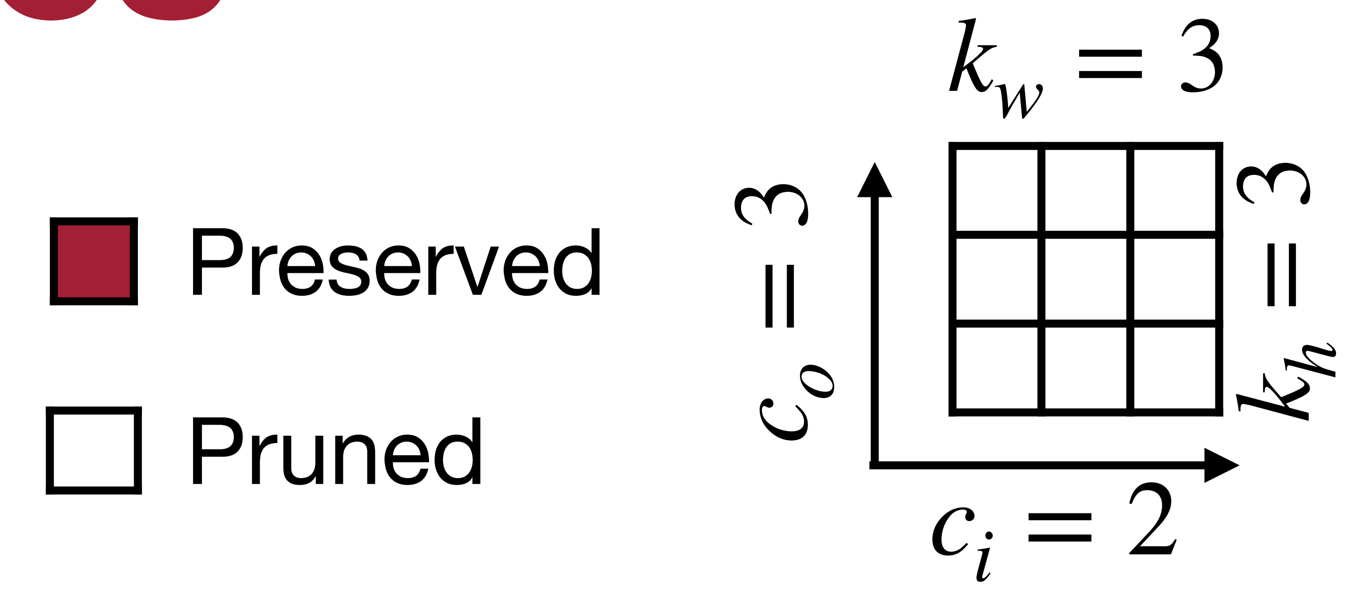


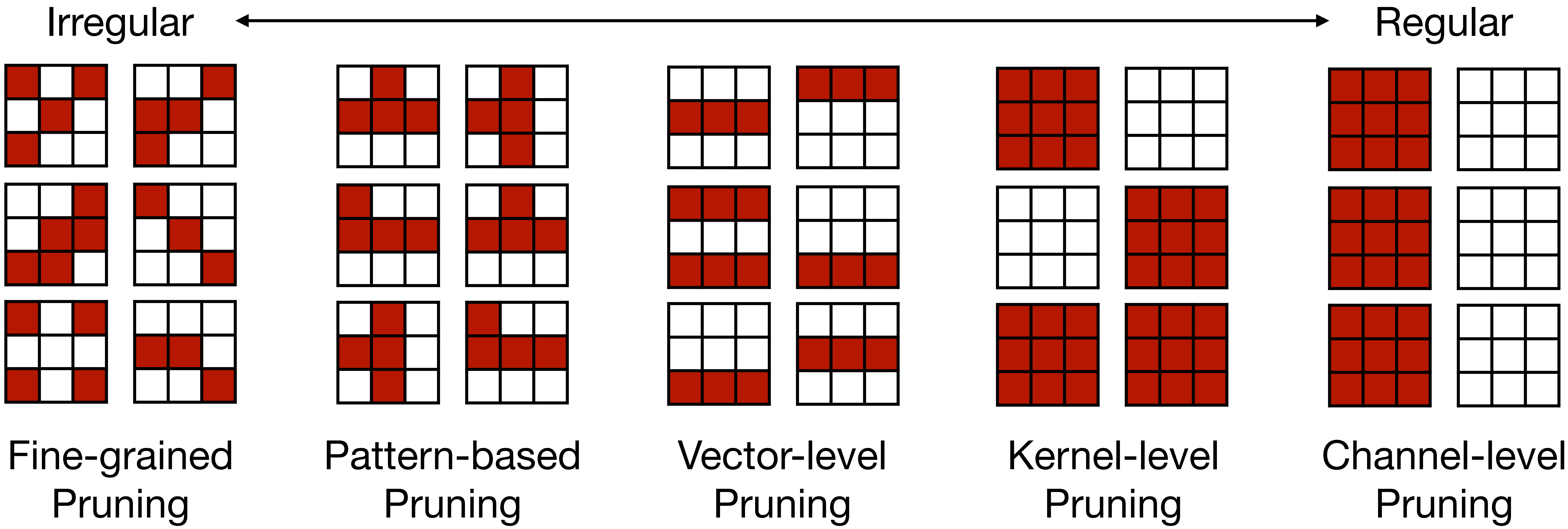
Exploring the granularity of sparsity in convolutional neural networks [Mao *et al.*, CVPR-W]

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities





like Tetris :)

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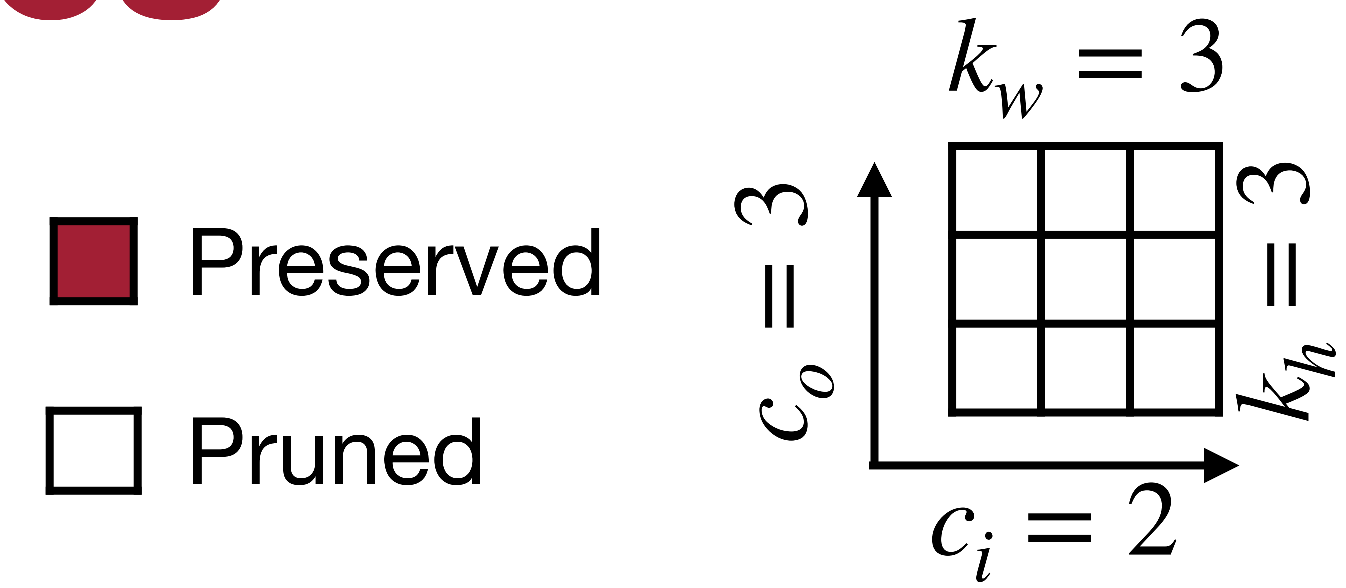
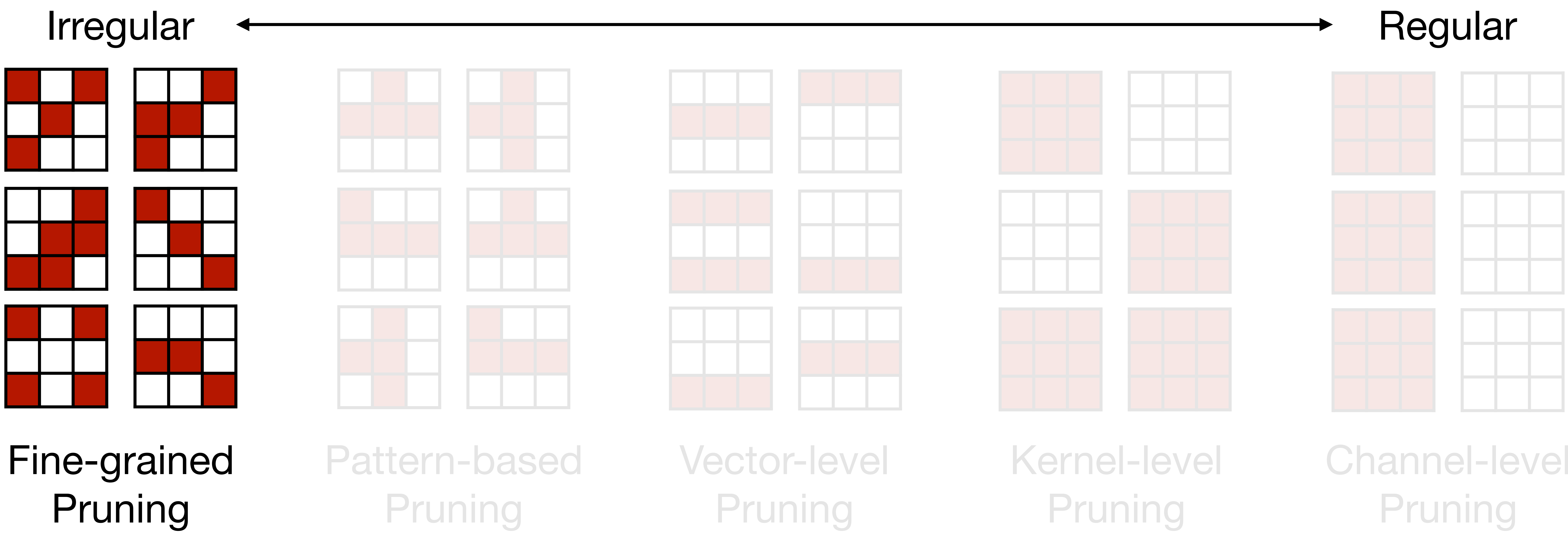
Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities



Pros? **Cons?**

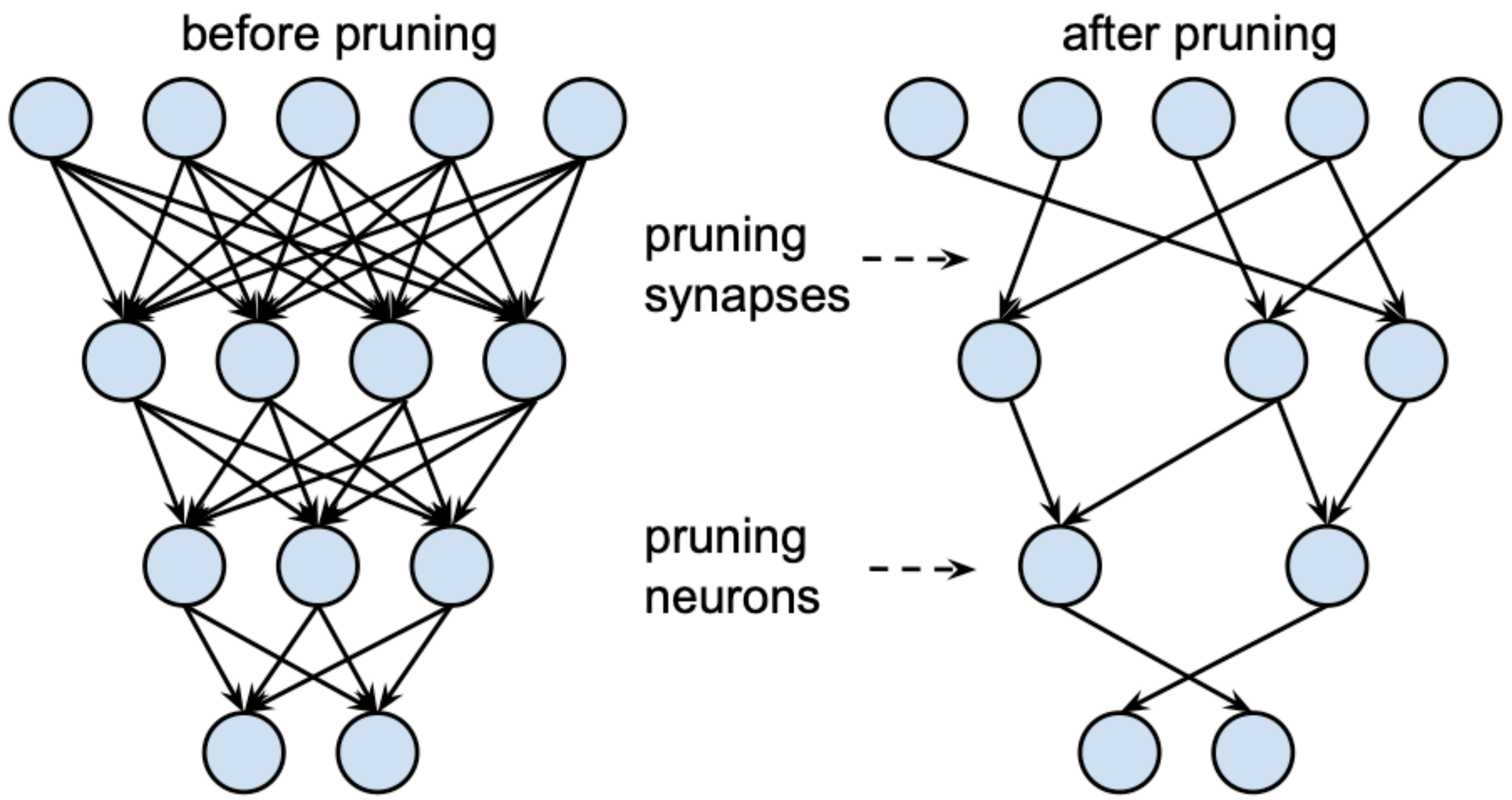


Exploring the granularity of sparsity in convolutional neural networks [Mao *et al.*, CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Fine-grained Pruning** (the case we show before)
  - Flexible pruning indices



Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Pruning at Different Granularities

Let's look into some cases

- **Fine-grained Pruning** (the case we show before)
  - Flexible pruning indices
  - Usually larger compression ratio since we can flexibly find "redundant" weights (we will later discuss how we find them)

Neural Network	#Parameters		
	Before Pruning	After Pruning	Reduction
AlexNet	61 M	6.7 M	**9** ×
VGG-16	138 M	10.3 M	**12** ×
GoogleNet	7 M	2.0 M	**3.5** ×
ResNet50	26 M	7.47 M	**3.4** ×

Efficient Methods and Hardware for Deep Learning [Han S., Stanford University]

Pruning at Different Granularities

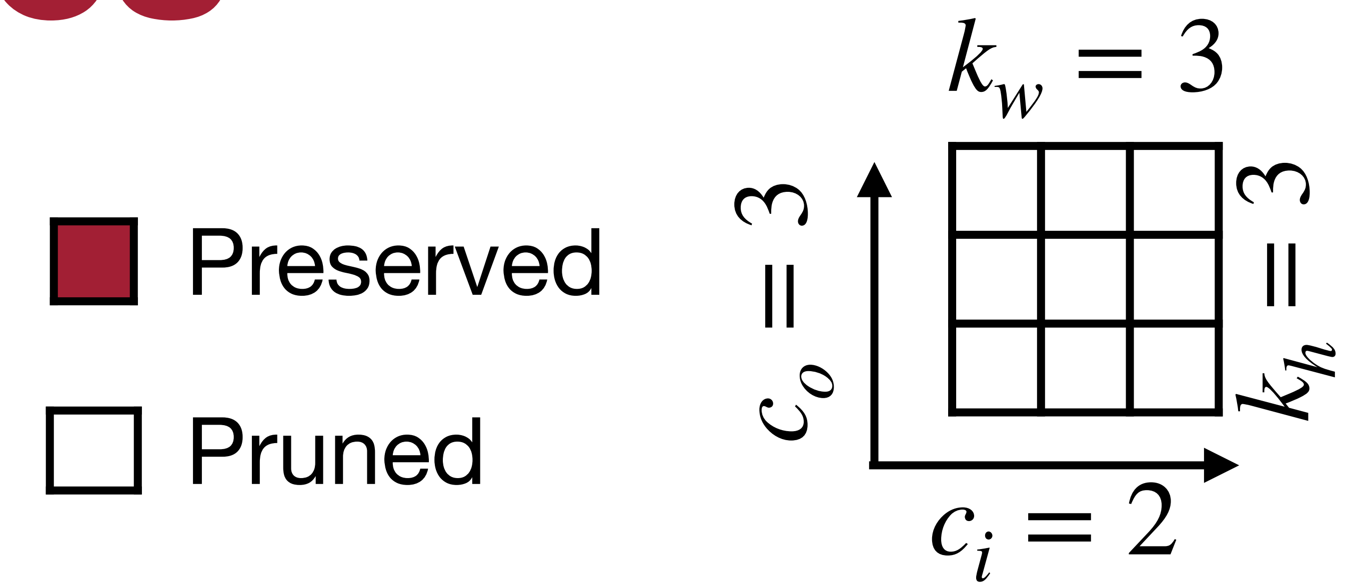
Let's look into some cases

- **Fine-grained Pruning** (the case we show before)
  - Flexible pruning indices
  - Usually larger compression ratio since we can flexibly find "redundant" weights (we will later discuss how we find them)
  - Can deliver speed up on some custom hardware (e.g., EIE) but not GPU (easily)

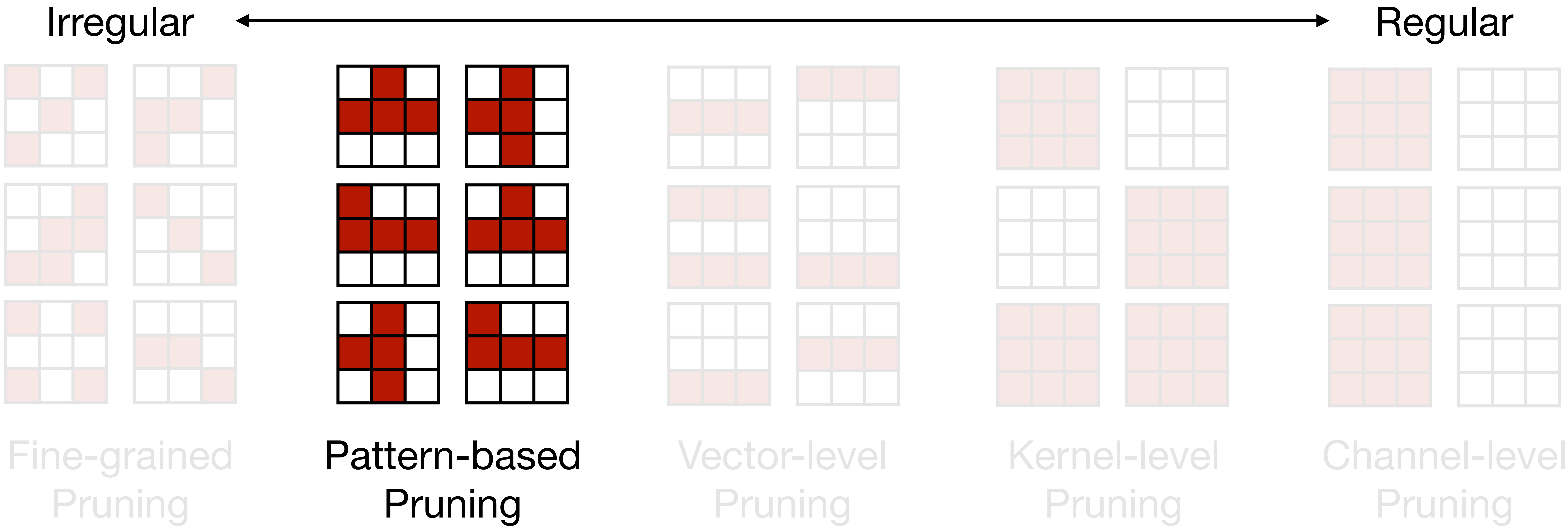
Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities



Pros? Cons?

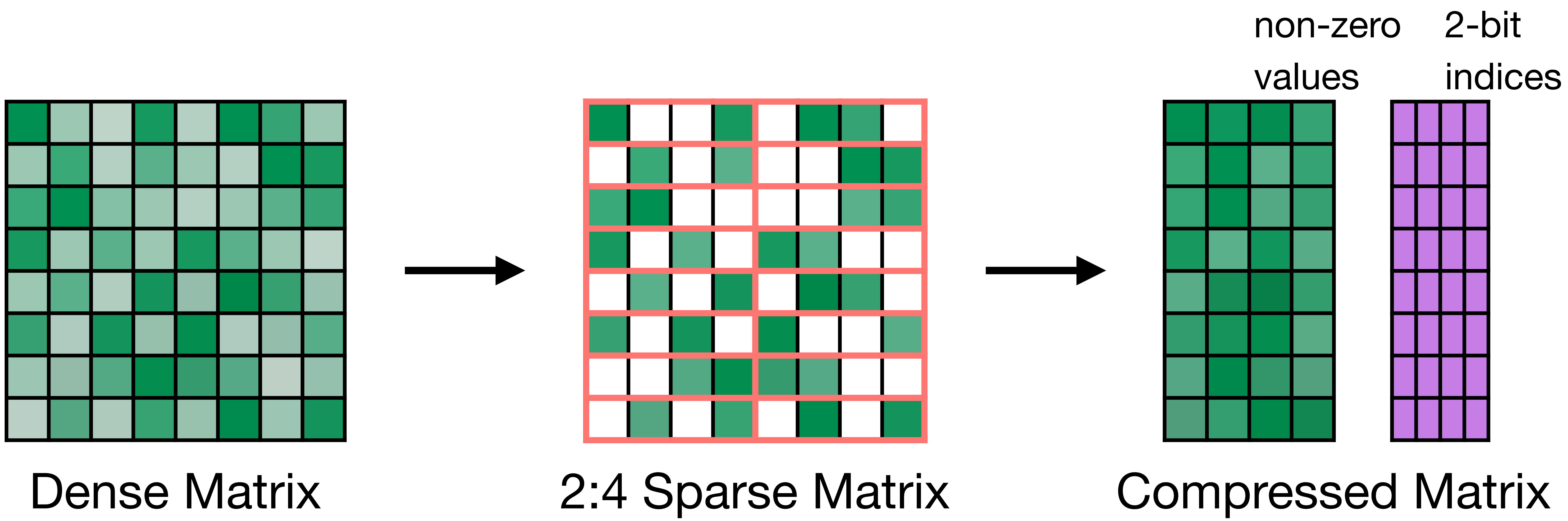


Exploring the granularity of sparsity in convolutional neural networks [Mao *et al.*, CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Pattern-based Pruning: N:M sparsity**
  - N:M sparsity means that in each contiguous M elements, N of them is pruned
  - A classic case is 2:4 sparsity (50% sparsity)
  - It is supported by NVIDIA's Ampere GPU Architecture, which delivers up to 2x speed up



Accelerating Inference with Sparsity Using the NVIDIA Ampere Architecture and NVIDIA TensorRT

Pruning at Different Granularities

Let's look into some cases

- **Pattern-based Pruning: N:M sparsity**
  - N:M sparsity means that in each contiguous M elements, N of them is pruned
  - A classic case is 2:4 sparsity (50% sparsity)
  - It is supported by NVIDIA's Ampere GPU Architecture, which delivers ~2x speed up
  - Usually maintains accuracy (tested on varieties of tasks)

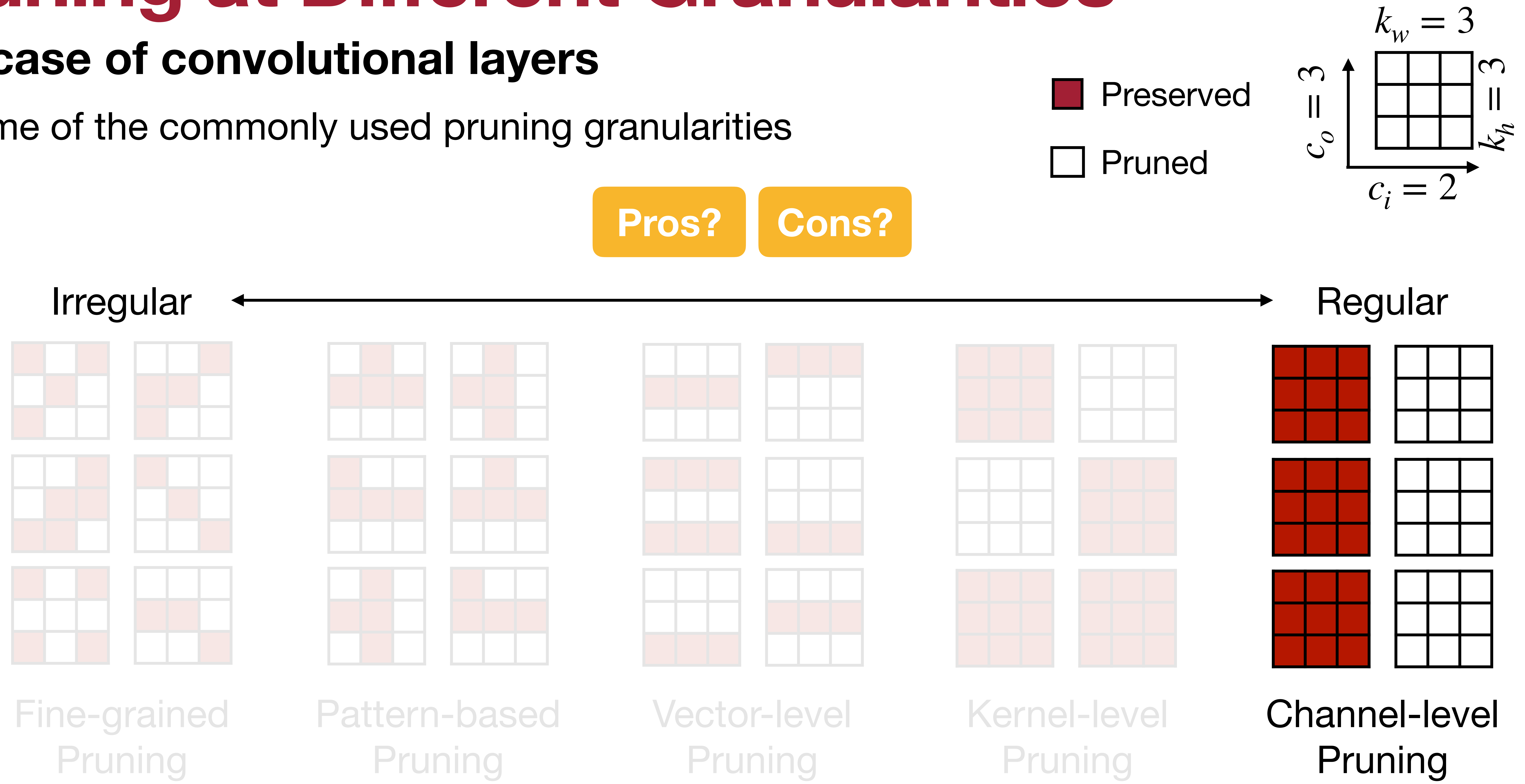
Network	Data Set	Metric	Dense FP16	Sparse FP16
ResNet-50	ImageNet	Top-1	76.1	76.2
ResNeXt-101_32x8d	ImageNet	Top-1	79.3	79.3
Xception	ImageNet	Top-1	79.2	79.2
SSD-RN50	COCO2017	bbAP	24.8	24.8
MaskRCNN-RN50	COCO2017	bbAP	37.9	37.9
FairSeq Transformer	EN-DE WMT'14	BLEU	28.2	28.5
BERT-Large	SQuAD v1.1	F1	91.9	91.9

Accelerating Inference with Sparsity Using the NVIDIA Ampere Architecture and NVIDIA TensorRT

Pruning at Different Granularities

The case of convolutional layers

- Some of the commonly used pruning granularities

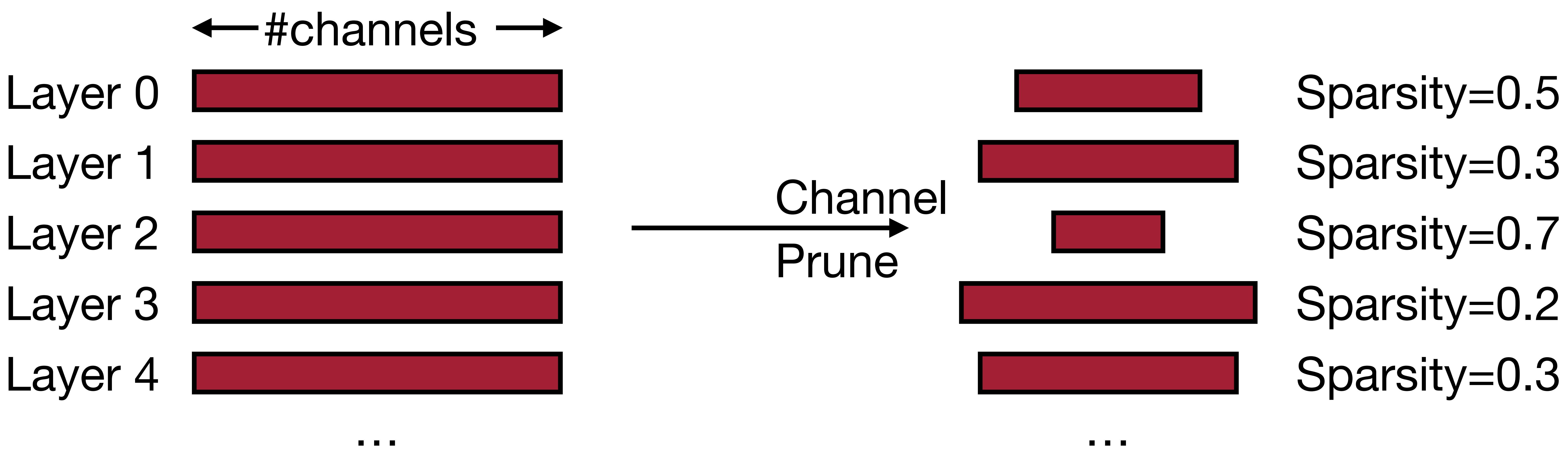


Exploring the granularity of sparsity in convolutional neural networks [Mao *et al.*, CVPR-W]

Pruning at Different Granularities

Let's look into some cases

- **Channel Pruning**
  - Pro: Direct speed up due to reduced channel numbers (leading to an NN with smaller #channels)
  - Con: smaller compression ratio

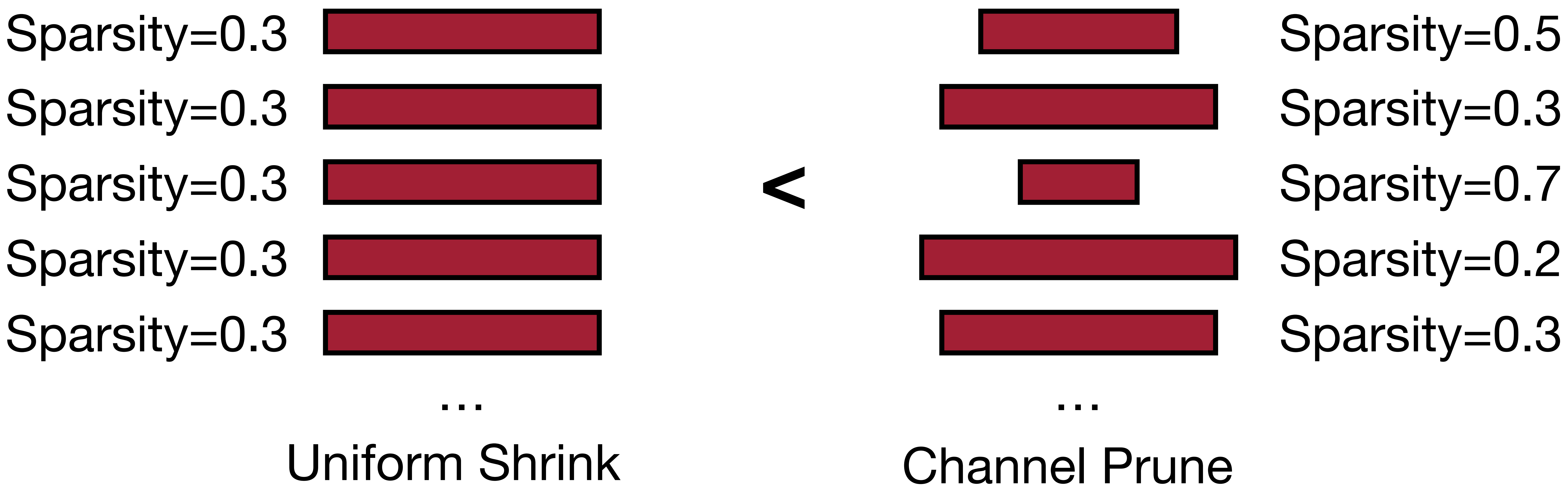


Pruning at Different Granularities

Let's look into some cases

- **Channel Pruning**
  - Pro: Direct speed up due to reduced channel numbers (leading to an NN with smaller #channels)
  - Con: smaller compression ratio

We will later discuss how to find sparsity ratios



Pruning at Different Granularities

Let's look into some cases

- **Channel Pruning**
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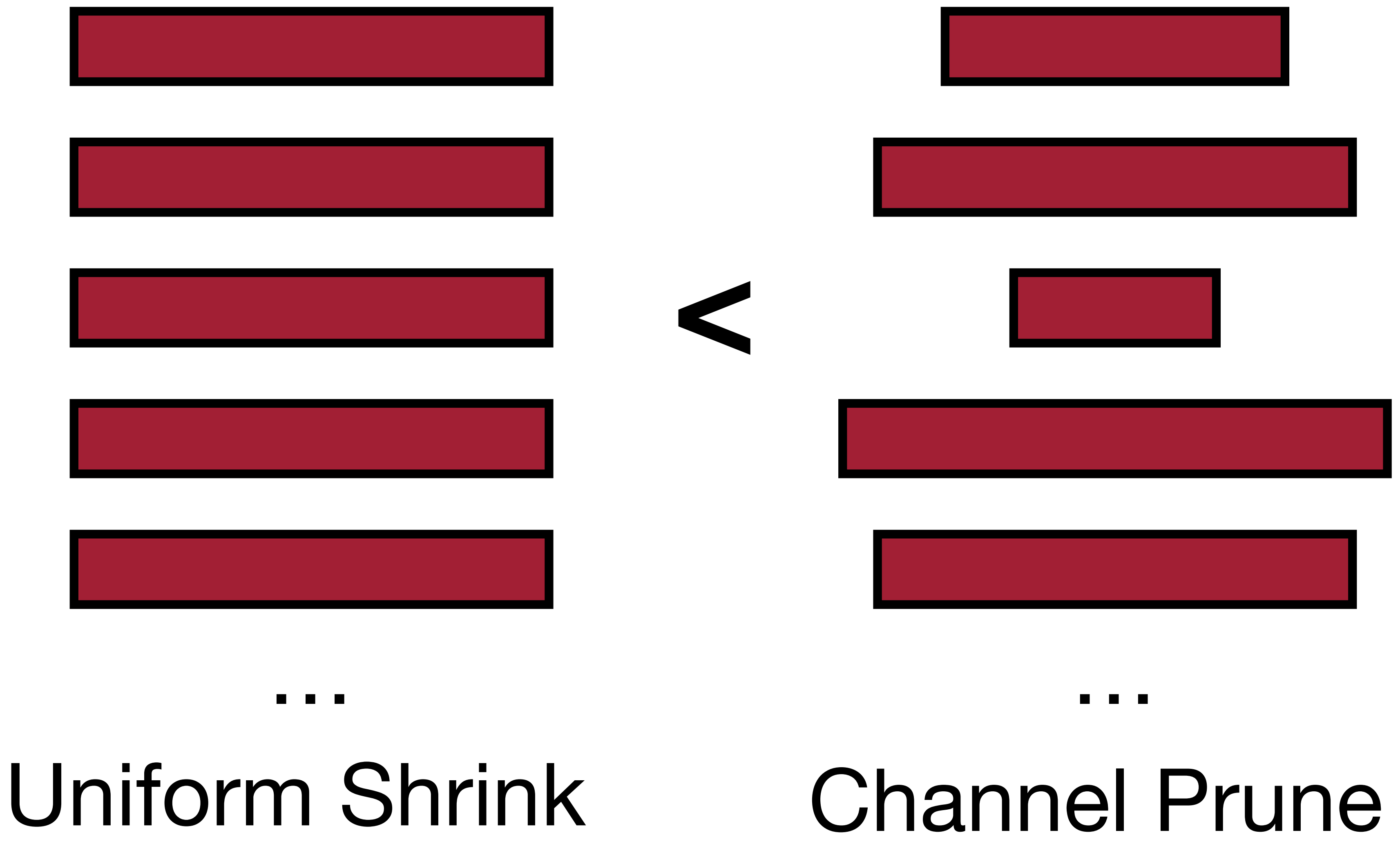
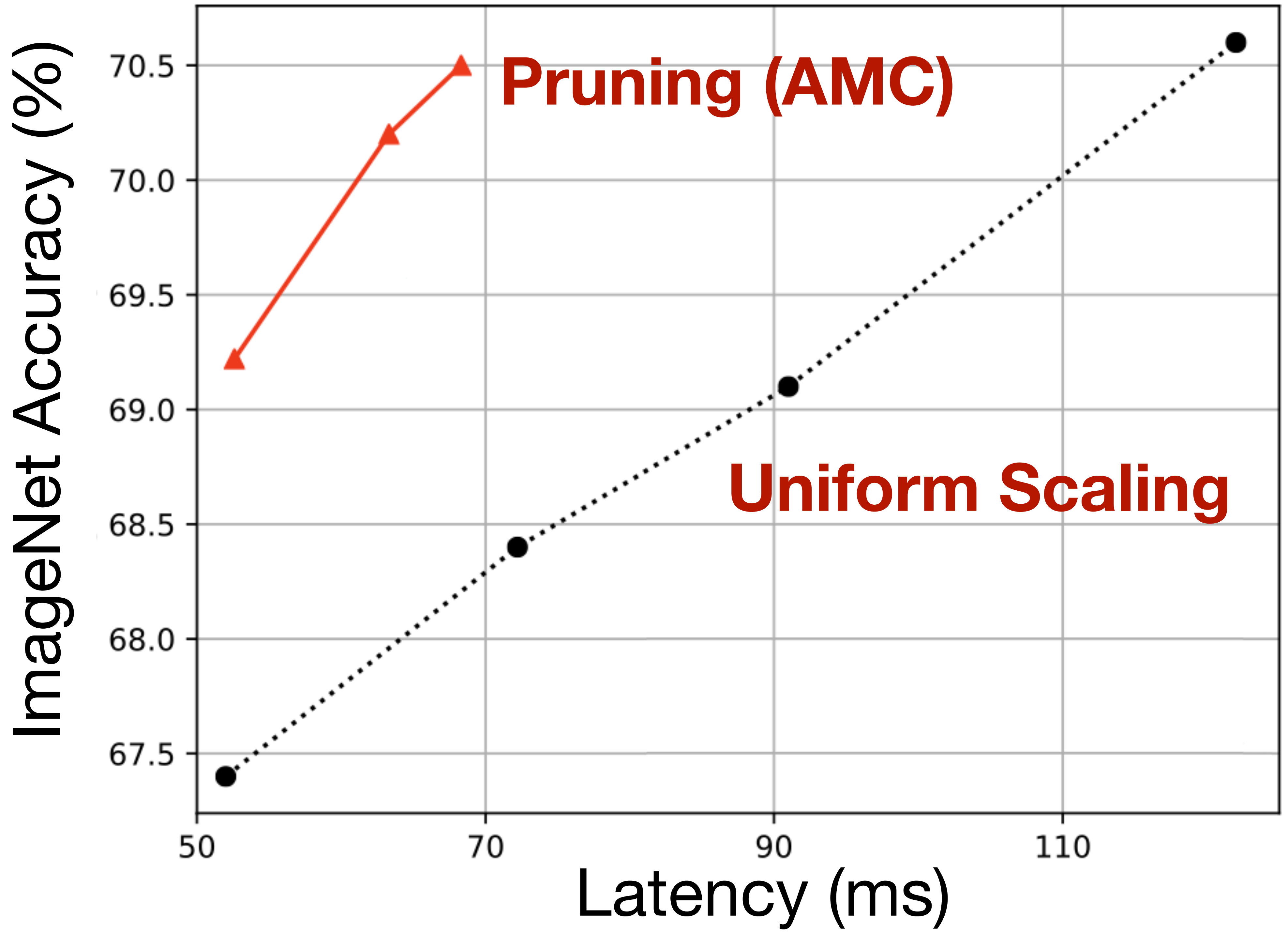
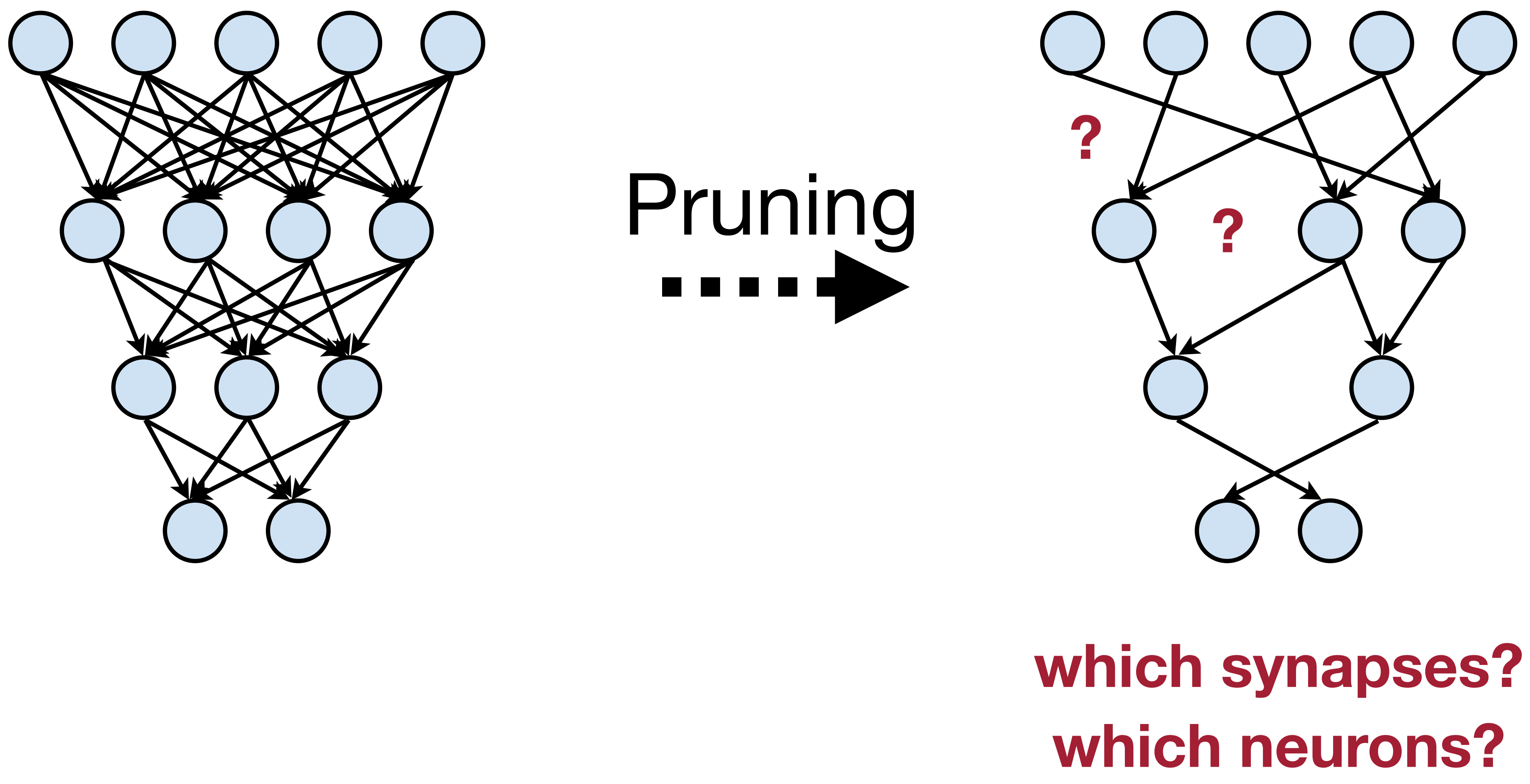




AMC: Automl for Model Compression and Acceleration on Mobile Devices [He *et al.*, ECCV 2018]

Neural Network Pruning

- **Introduction to Pruning**
  - What is pruning?
  - How should we formulate pruning?
- **Determine the Pruning Granularity**
  - In what pattern should we prune the neural network?
- **Determine the Pruning Criterion**
  - What synapses/neurons should we prune?
- **Determine the Pruning Ratio**
  - What should target sparsity be for each layer?
- **Fine-tune/Train Pruned Neural Network**
  - How should we improve performance of pruned models?



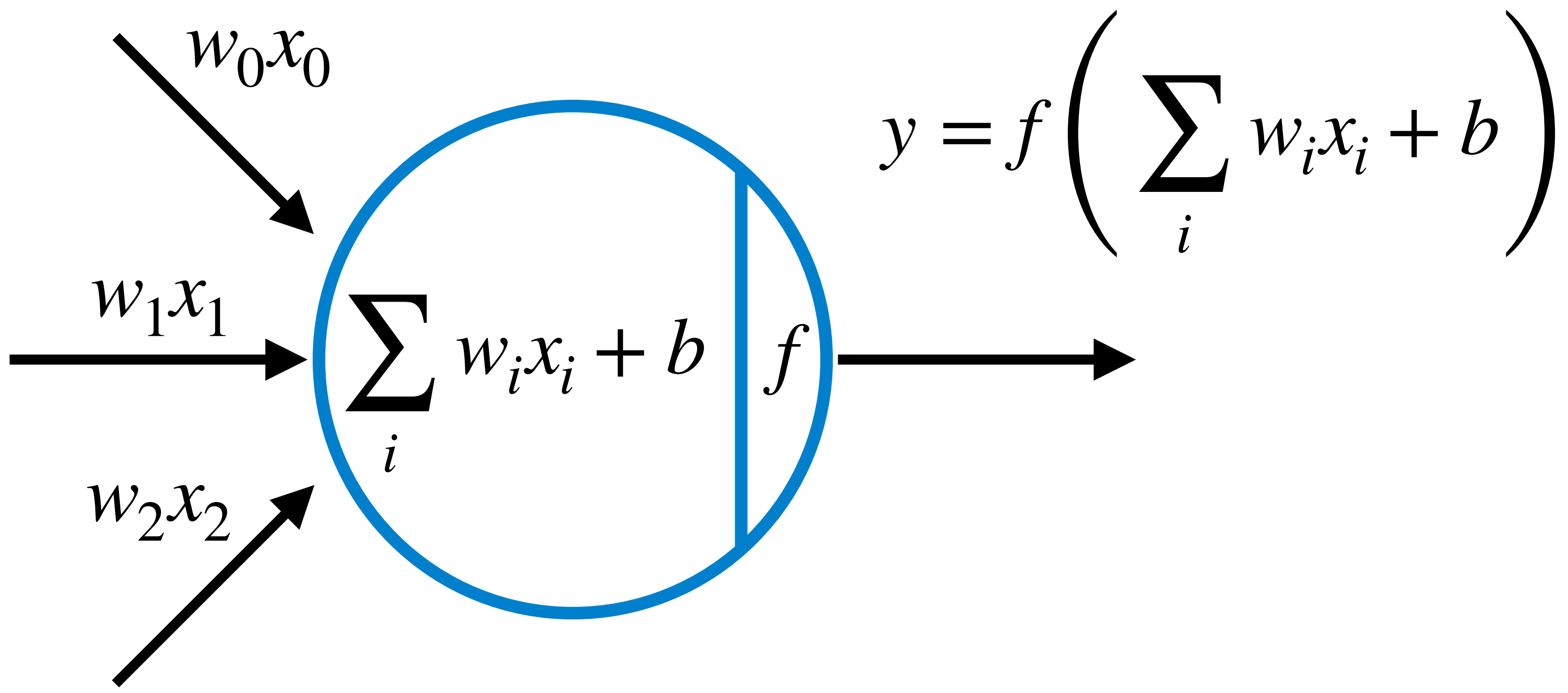
Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Section 3: Pruning Criterion

What synapses and neurons should we prune?

Selection of Synapses to Prune

- When removing parameters from a neural network model,
  - ***the less important*** the parameters being removed are,
  - the better the performance of pruned neural network is.



Example

$f(\cdot) = \text{ReLU}(\cdot), \;\; W = [10, -8, 0.1]$

➡ $y = \text{ReLU}(10x_0 - 8x_1 + 0.1x_2)$

- If only one weight will be removed, which one? Why?
- If only one weight will be kept, which one? Why?

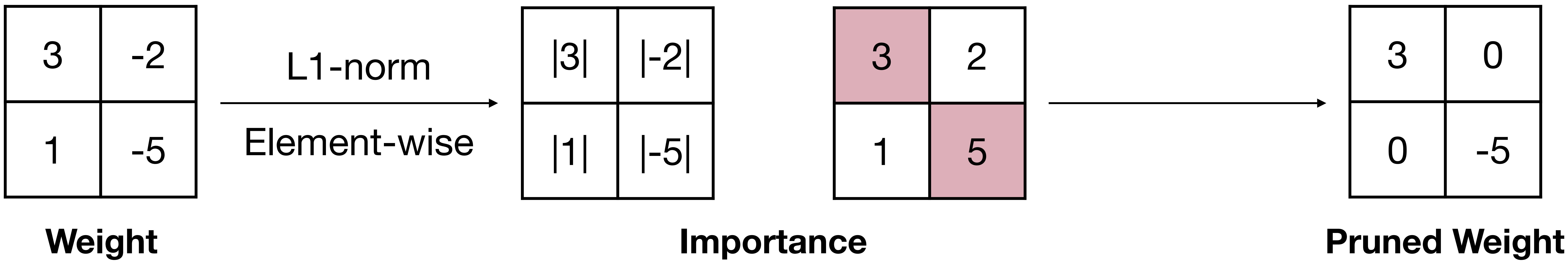
Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
  - For element-wise pruning,

$$Importance = |W|$$

- **Example**



Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
  - For row-wise pruning, the L1-norm magnitude can be defined as,

$$Importance = \sum_{i \in S} |w_i|, \quad \text{where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$

- **Example**

3	-2
1	-5

Weight

$\xrightarrow[\text{Row-wise}]{\text{L1-norm}}$

|3|+|-2|
|1|+|-5|

5
6

Importance

$\longrightarrow$

0	0
1	-5

Pruned Weight

Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
  - For row-wise pruning, the L2-norm magnitude can be defined as,

$$Importance = \sqrt{\sum_{i \in S} |w_i|^2}, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$

- **Example**

3	-2
1	-5

Weight

$\xrightarrow[\text{Row-wise}]{\text{L2-norm}}$

$\sqrt{13} = \sqrt{\|3\|^2 + \|-2\|^2}$
$\sqrt{26} = \sqrt{\|1\|^2 + \|-5\|^2}$

√13
√26

Importance

$\longrightarrow$

0	0
1	-5

Pruned Weight

Learning Both Weights and Connections for Efficient Neural Network [Han *et al.*, NeurIPS 2015]

Magnitude-based Pruning

A heuristic pruning criterion

- Magnitude-based pruning considers weights with ***larger absolute values*** are more important than other weights.
- Magnitude is also known as L_p-norm defined as,

$$\|\mathbf{W}^{(S)}\|_p = \left(\sum_{i \in S} |w_i|^p\right)^{\frac{1}{p}}$$, where $\mathbf{W}^{(S)}$ is a structural set of parameters

- **Example**

3	-2
1	-5

Weight

L2-norm $\longrightarrow$ Row-wise

$\sqrt{13} = \sqrt{\lvert 3 \rvert^2 + \lvert -2 \rvert^2}$
$\sqrt{26} = \sqrt{\lvert 1 \rvert^2 + \lvert -5 \rvert^2}$

√13
√26

Importance

$\longrightarrow$

0	0
1	-5

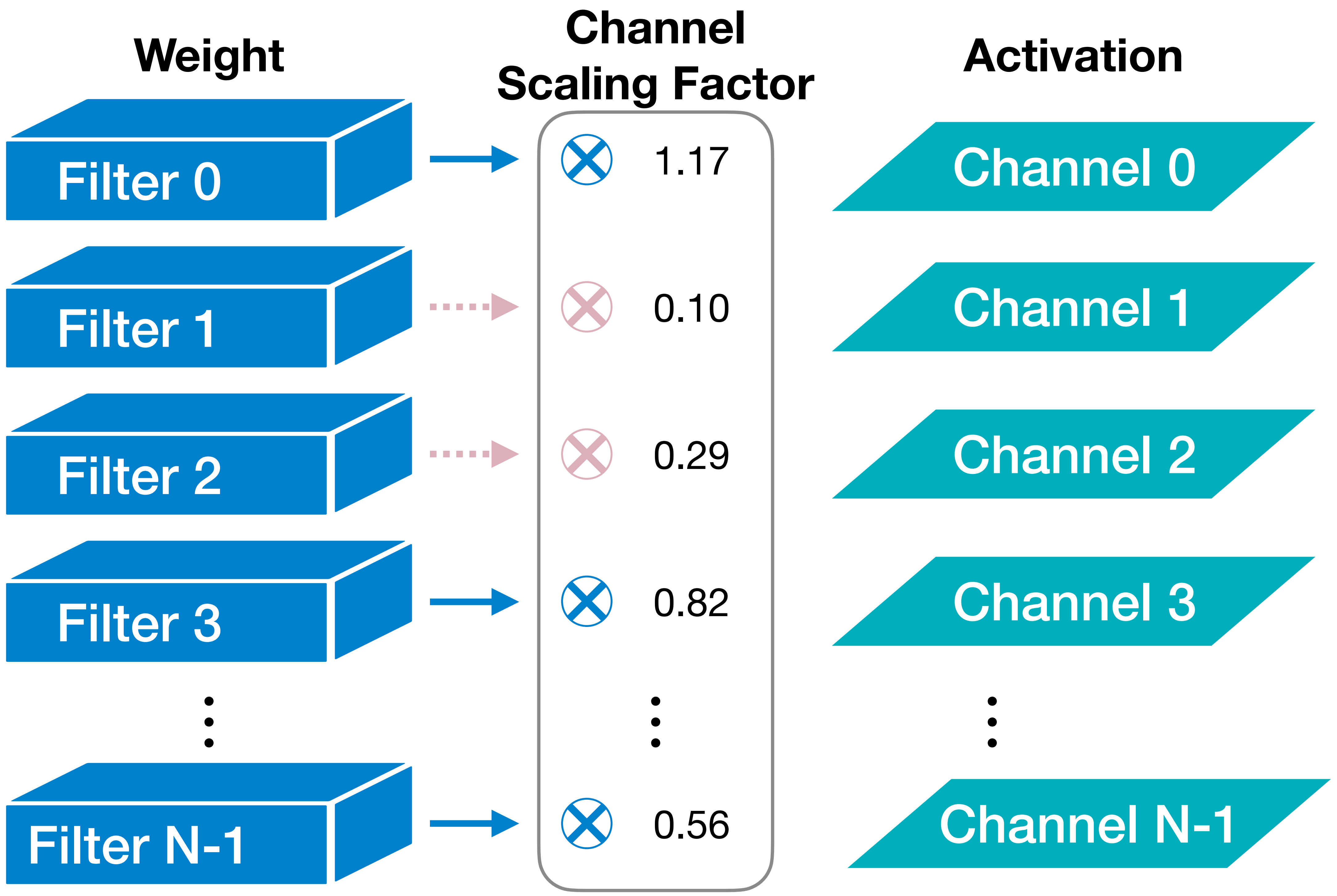
Pruned Weight

Learning Structured Sparsity in Deep Neural Networks [Wen *et al.*, NeurIPS 2016]

Scaling-based Pruning

Pruning criterion for filter pruning

- A scaling factor is associated with each filter (*i.e.*, output channel) in convolutional layers
  - The scaling factor is multiplied to the output of that channel
  - The scaling factors are trainable parameters

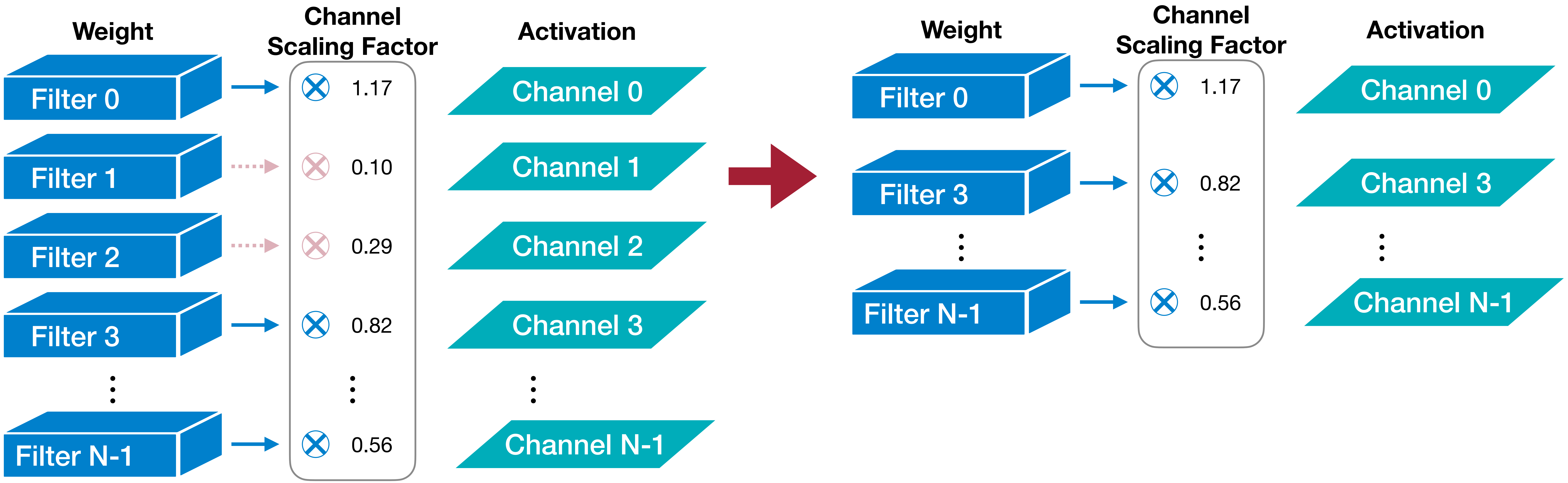


Learning Efficient Convolutional Networks through Network Slimming [Liu *et al.*, ICCV 2017]

Scaling-based Pruning

Pruning criterion for filter pruning

- A scaling factor is associated with each filter (*i.e.*, output channel) in convolutional layers
  - The scaling factor is multiplied to the output of that channel
  - The scaling factors are trainable parameters
- The filters/output channels with small scaling factor magnitude will be pruned



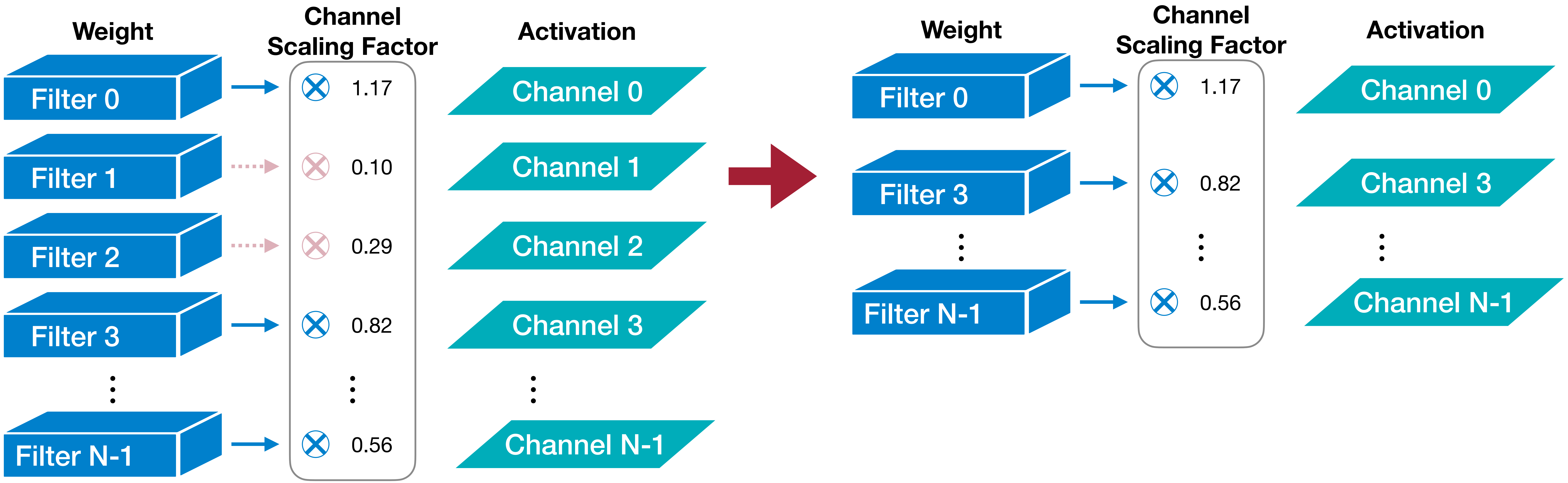
Learning Efficient Convolutional Networks through Network Slimming [Liu *et al.*, ICCV 2017]

Scaling-based Pruning

Pruning criterion for filter pruning

- A scaling factor is associated with each filter (*i.e.*, output channel) in convolutional layers
- The scaling factors can be reused from batch normalization layer

$$\mathbf{z}_o = \gamma \frac{\mathbf{z}_i - \mu_{\mathscr{B}}}{\sqrt{\sigma_{\mathscr{B}}^2 + \epsilon}} + \beta$$



Learning Efficient Convolutional Networks through Network Slimming [Liu *et al.*, ICCV 2017]

Taylor Expansion Analysis on Pruning Error

Evaluate pruning error induced by pruning synapses

- The task of training neural network is to minimize the objective function $L(\mathbf{x}; \mathbf{W})$. The importance of a parameter can be quantified by the error induced by removing it.
- The induced error can be approximated by a Taylor series.

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta \mathbf{W}) = \sum_i g_i \delta w_i + \frac{1}{2} \sum_i h_{ii} \delta w_i^2 + \frac{1}{2} h_{ij} \delta w_i \delta w_j + O(\|\delta \mathbf{W}\|^3)$$

where

$$g_i = \frac{\partial L}{\partial w_i}, \; h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$$

- There are two ways to simplify these approximation.

Second-Order-based Pruning

Minimize the error on loss function introduced by pruning synapses

- The induced error can be approximated by a Taylor series.

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta \mathbf{W}) = \sum_i \cancel{g_i \delta w_i} + \frac{1}{2}\sum_i h_{ii}\delta w_i^2 + \frac{1}{2}\sum_{i \neq j} \cancel{h_{ij}\delta w_i \delta w_j} + \cancel{O(\|\delta \mathbf{W}\|^3)}$$

where

$$g_i = \frac{\partial L}{\partial w_i}, \; h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$$

- Optimal Brain Damage assumes that
  - The objective function L is nearly quadratic: the last term is neglected
  - The neural network training has converged: first-order terms are neglected
  - The error caused by deleting each parameter is independent: cross terms are neglected

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P \,|\, w_i = 0) \approx \frac{1}{2} h_{ii} w_i^2$$

Optimal Brain Damage [LeCun *et al.*, NeurIPS 1989]

Second-Order-based Pruning

Minimize the error on loss function introduced by pruning synapses

- Optimal Brain Damage assumes that
  - The objective function L is nearly quadratic
  - The neural network training has converged
  - The error caused by deleting each parameter is independent

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P \,|\, w_i = 0) \approx \frac{1}{2} h_{ii} w_i^2, \quad \text{where} \quad h_{ii} = \frac{\partial^2 L}{\partial w_i \partial w_j}$$

  - The synapses with smaller induced error $|\delta L_i|$ will be removed; that is to say,

$$importance_{w_i} = |\delta L_i| = \frac{1}{2} h_{ii} w_i^2$$

* h_{ii} is non-negative

Hessian Matrix *H* is difficult to compute.

Optimal Brain Damage [LeCun *et al.*, NeurIPS 1989]

First-Order-based Pruning

Minimize the error on loss function introduced by pruning synapses

- The induced error can be approximated by a Taylor series.

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta \mathbf{W}) = \sum_i g_i \delta w_i + \frac{1}{2} \sum_i \cancel{h_{ii} \delta w_i^2} + \frac{1}{2} \sum_{i \neq j} \cancel{h_{ij} \delta w_i \delta w_j} + \cancel{O(\|\delta \mathbf{W}\|^3)}$$

where

$$g_i = \frac{\partial L}{\partial w_i}, \; h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$$

- If only first-order expansion is considered, under an *i.i.d* assumption,

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P \,|\, w_i = 0) \approx g_i w_i$$

Pruning Convolutional Filters with First Order Taylor Series Ranking [Wang M.]
Importance Estimation for Neural Network Pruning [Molchanov *et al.*, CVPR 2019]

First-Order-based Pruning

Minimize the error on loss function introduced by pruning synapses

- If only first-order expansion is considered, under an *i.i.d* assumption,

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P \mid w_i = 0) \approx g_i w_i, \quad \text{where} \quad g_i = \frac{\partial L}{\partial w_i}$$

- The synapses with smaller induced error $|\delta L_i|$ will be removed; that is to say,

$$importance_{w_i} = |\delta L_i| = |g_i w_i|$$

- Or,

$$importance_{w_i} = |\delta L_i|^2 = \left(g_i w_i\right)^2$$

- For coarse-grained pruning, we have,

$$importance_{\mathbf{W}^{(S)}} = \sum_{i \in S} |\delta L_i|^2 = \sum_{i \in S} (g_i w_i)^2,$$

where $\mathbf{W}^{(S)}$ is the structural set of parameters

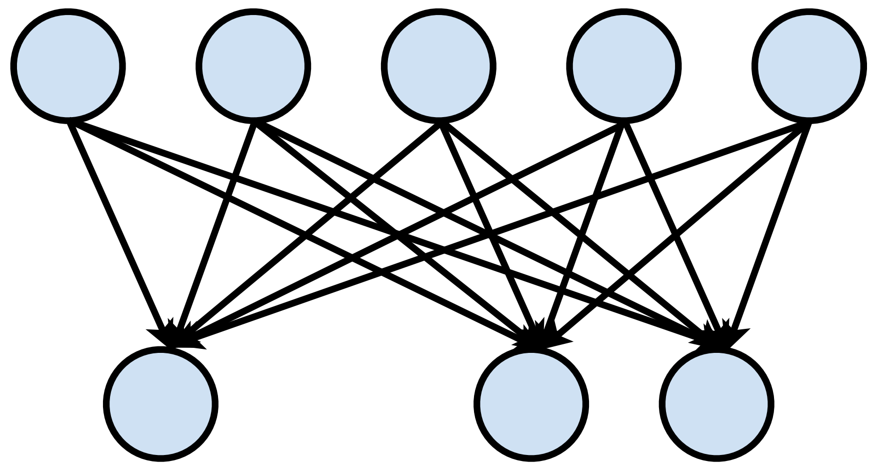
Pruning Convolutional Filters with First Order Taylor Series Ranking [Wang M.]
Importance Estimation for Neural Network Pruning [Molchanov *et al.*, CVPR 2019]

Selection of Neurons to Prune

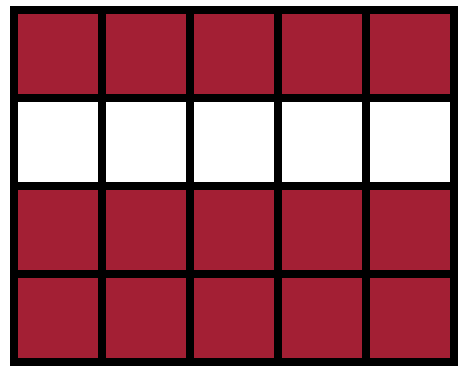
- When removing neurons from a neural network model,
  - ***the less useful*** the neurons being removed are,
  - the better the performance of pruned neural network is.

Recall: Neuron pruning is coarse-grained pruning indeed.

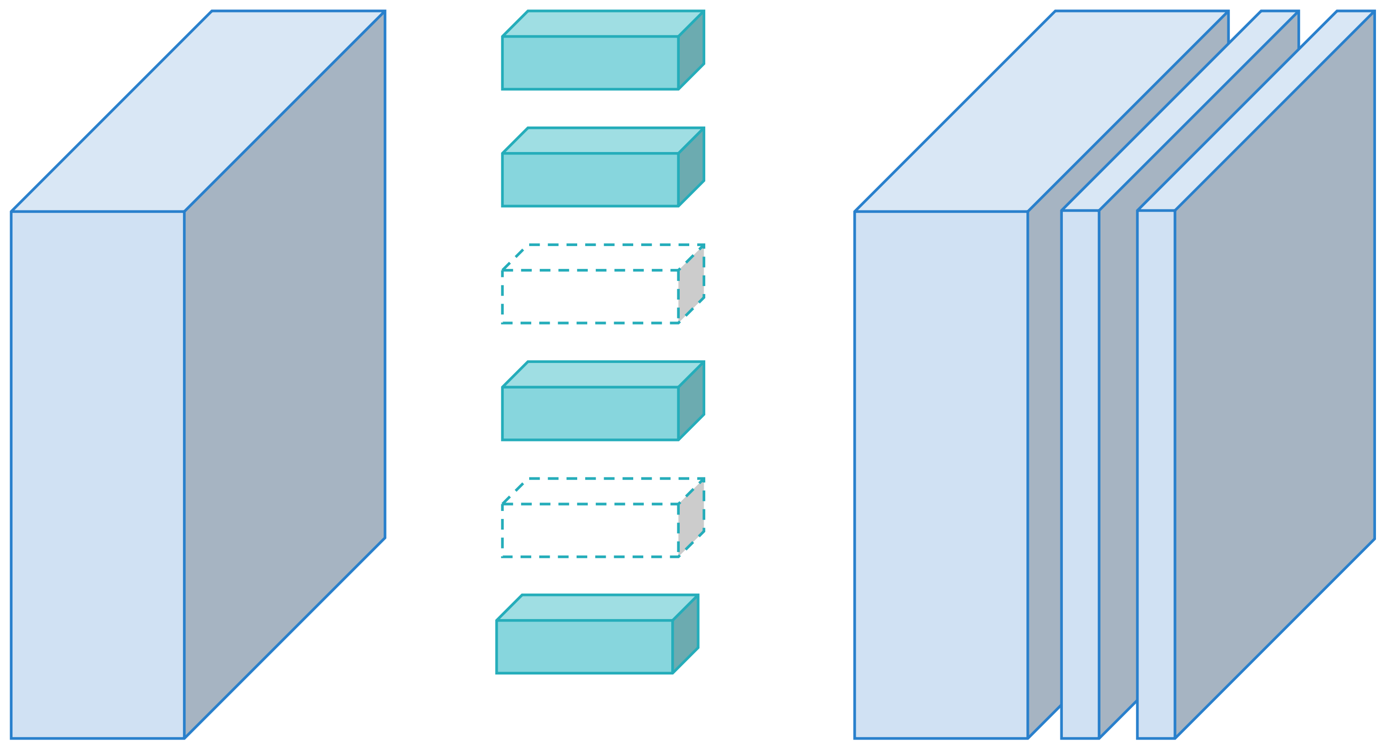
Neuron Pruning in Linear Layer

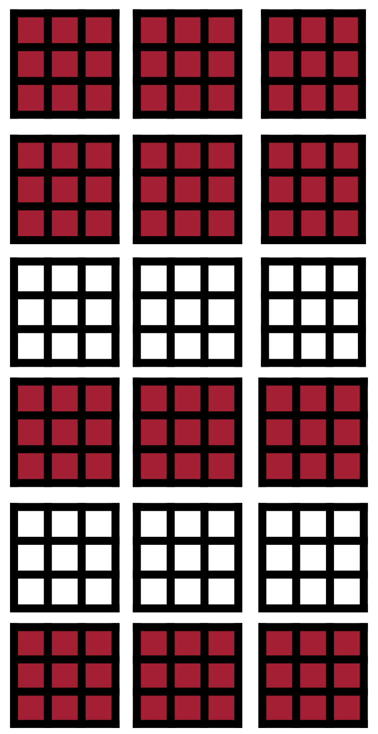


Weight Matrix



Channel Pruning in Convolution Layer





Percentage-of-Zero-Based Pruning

- ReLU activation will generate zeros in the output activation.
- Similar to magnitude of weights, the Average Percentage of Zero activations (APoZ) can be exploited to measure the importance of the neurons.
- The smaller APoZ is, the more importance the neuron has.

Output Activations

Width = 4

Height = 4

0	0.1	0.5	1
1.2	0.6	0.3	0.2
0	0.5	**0**	0.3
0.2	**0**	**0**	0.8

0.1	0.5	**0**	**0**
0.2	0.3	**0**	1
0.1	**0**	**0**	0.5
0.1	0.6	0.7	0.1

0	**0**	0.8	**0**
0.7	**0**	0.6	0.1
1.2	1	**0**	0.2
0.5	**0**	0.3	0.5

Channel = 3

Width = 4

Height = 4

0.5	**0**	0.2	0.1
0	0.2	1.2	**0**
1.2	**0**	0.2	0.3
0.2	0.4	**0**	**0**

0.1	0.5	**0**	**0**
0	0.8	**0**	1
0.1	**0**	0.1	1.0
0.2	**0**	1.0	**0**

0	0.8	0.1	**0**
0.2	**0**	**0**	0.3
0	0.4	**0**	0.5
0.2	**0**	0.3	**0**

Channel = 3

Batch = 2

Average Percentage of Zeros (APoZ)

$$= \frac{5+6}{2 \cdot 4 \cdot 4} = \frac{11}{32}$$

Channel 0

$$= \frac{5+7}{2 \cdot 4 \cdot 4} = \frac{12}{32}$$

Channel 1

$$= \frac{6+8}{2 \cdot 4 \cdot 4} = \frac{14}{32}$$

Channel 2

Network Trimming: A Data-Driven Neuron Pruning Approach towards Efficient Deep Architectures [Hu *et al.*, ArXiv 2017]

First-Order-based Pruning

Minimize the error on loss function introduced by pruning neurons

- Similar to previous Taylor expansion on weights, the induced error of the objective function $L(\mathbf{x}; \mathbf{W})$ can be approximated by a Taylor series expanded on activations.

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x} \,|\, x_i = 0; \mathbf{W}) \approx \frac{\partial L}{\partial x_i} x_i$$

- For a structural set of neurons $\mathbf{x}^{(S)}$ (*e.g.*, a channel plane),

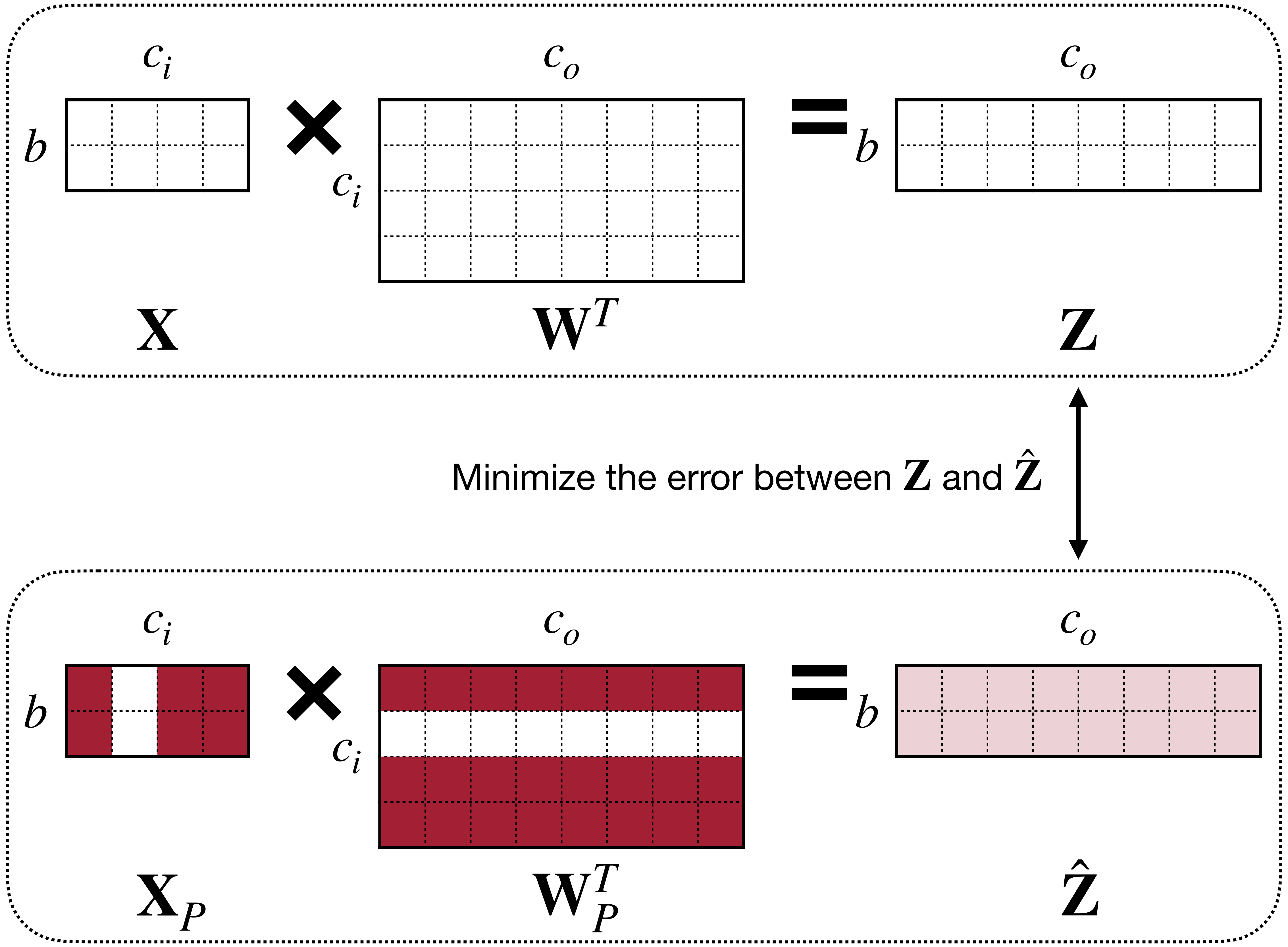
$$|\, \delta L_{\mathbf{x}^{(S)}} \,| = \left| \sum_{i \in S} \frac{\partial L}{\partial x_i} x_i \right|$$

Pruning Convolutional Neural Networks for Resource Efficient Inference [Molchanov *et al.*, ICLR 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- Instead of considering the pruning error of the objective function $L(\mathbf{x}; \mathbf{W})$, regression-based pruning minimizes the reconstruction error of the corresponding layer's outputs.



Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- Let

$$\mathbf{Z} = \mathbf{X}\mathbf{W}^T = \sum_{c=0}^{c_i-1} \mathbf{X}_c \mathbf{W}_c^T$$

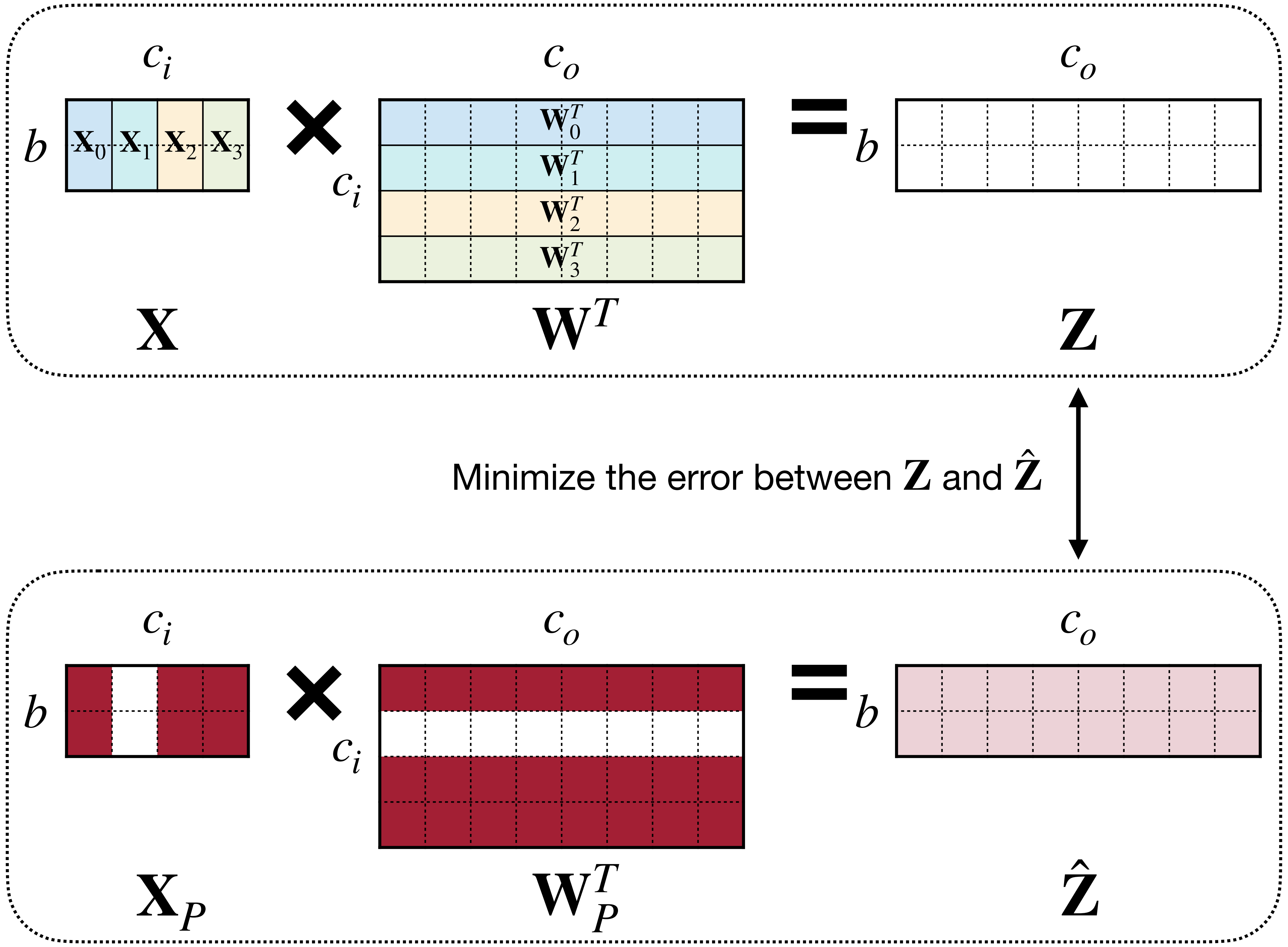
- The problem can be formulate as

$$\arg\min_{\mathbf{W},\,\beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

  subject to

$$\|\beta\|_0 \leq N_c$$

- β is coefficient vector of length c_i for channel selection. $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels.
- Solve the problem in two folds:
  - Fix $\mathbf{W}$, solve β for channel selection **(NP-hard)**
  - Fix β, solve $\mathbf{W}$ to minimize reconstruction error



Channel Pruning for Accelerating Very Deep Neural Networks [He *et al.*, ICCV 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

$$\arg\min_{\beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

subject to

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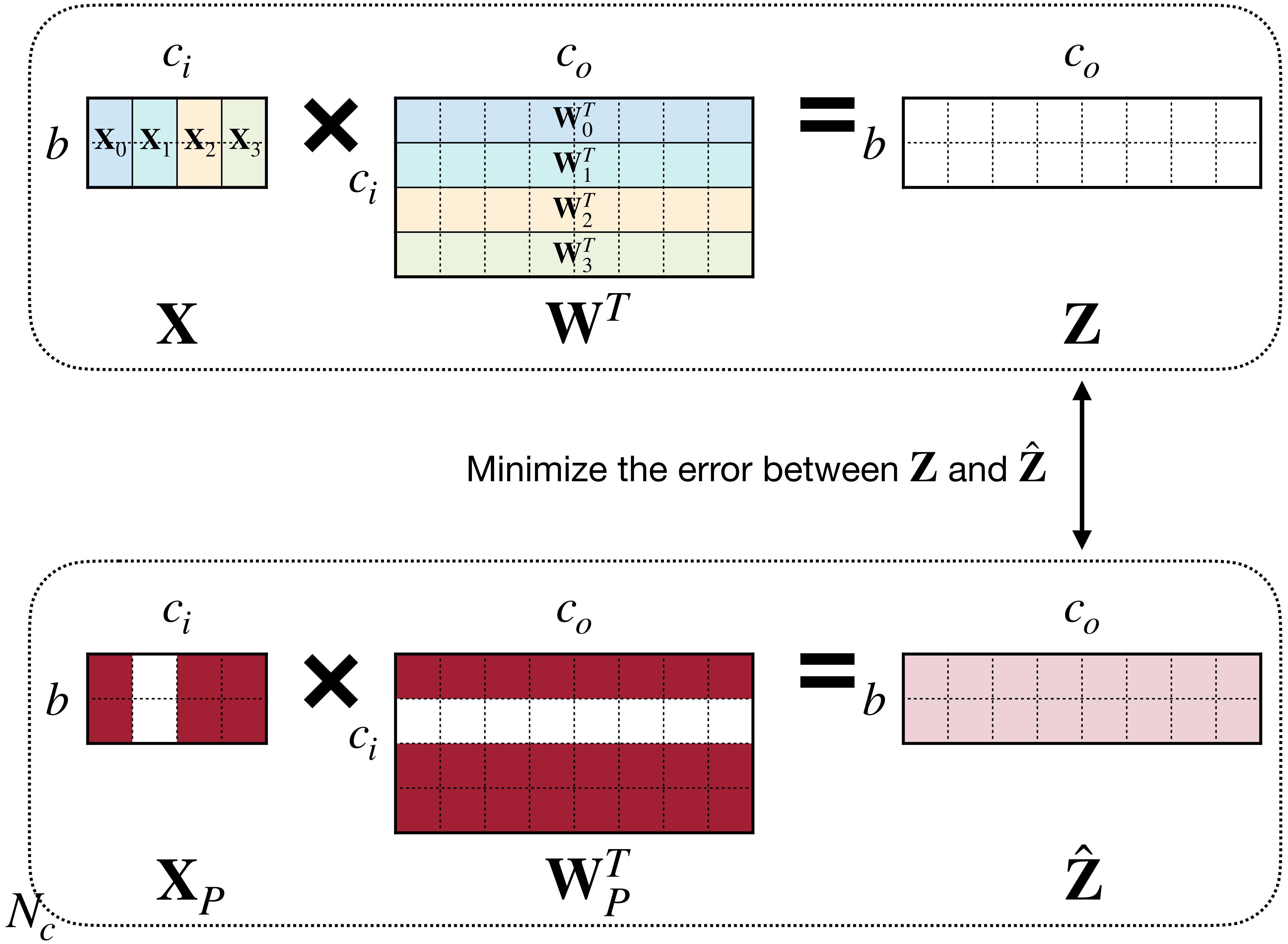
- β is coefficient vector of length c_i for channel selection. $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels.



- The objective can be rewritten as

$$\arg\min_{\beta} \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2 = \|\sum_{c=0}^{c_i-1} \mathbf{X}_c \mathbf{W}_c^T - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

$$= \|\sum_{c=0}^{c_i-1} \left(1 - \beta_c\right) \mathbf{X}_c \mathbf{W}_c^T\|_F^2, \quad s.t. \quad \|\beta\|_0 \leq N_c$$

ThiNet: A Filter Level Pruning Method for Deep Neural Network Compression [Luo *et al.*, ICCV 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

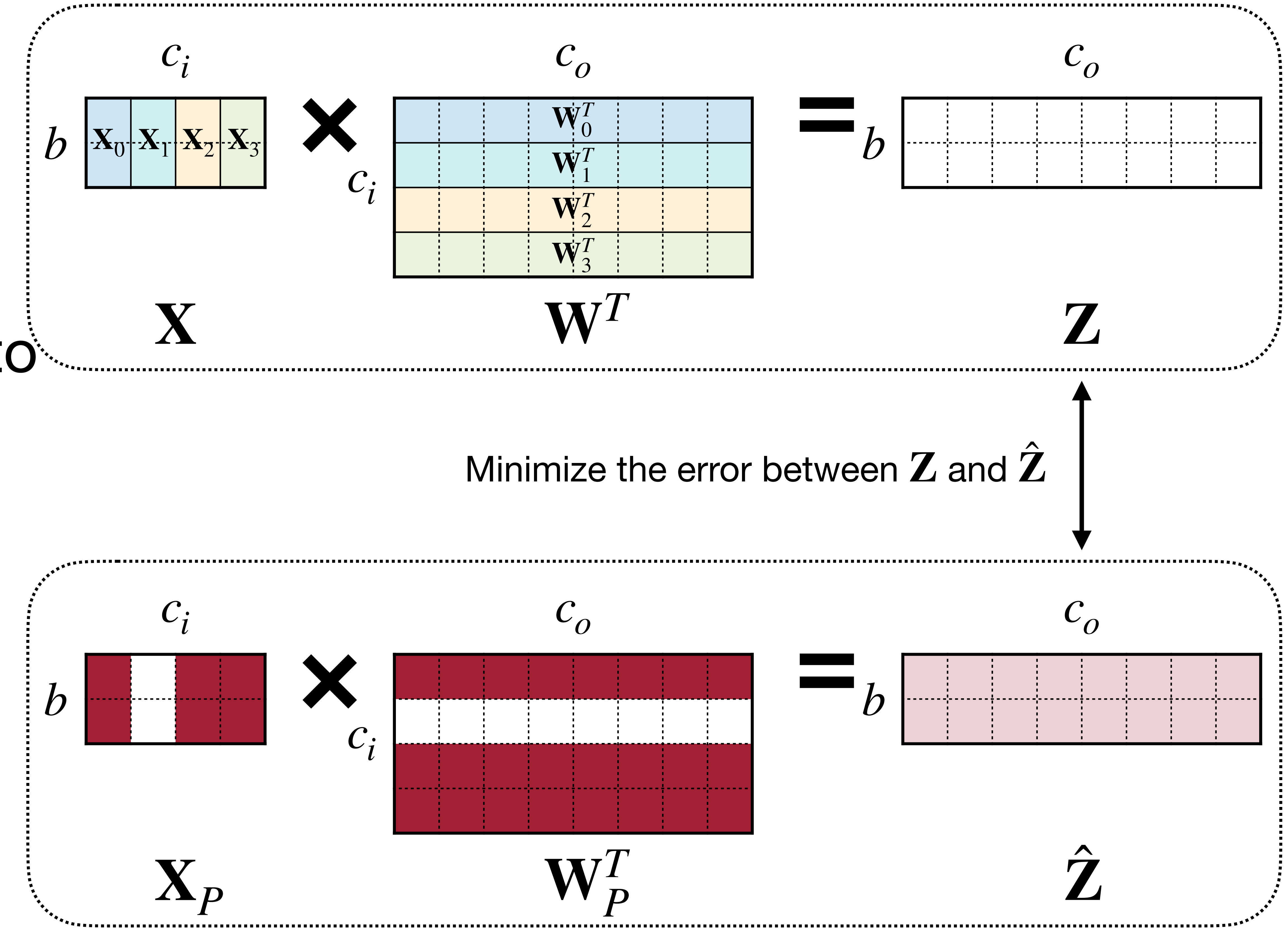
$$\arg\min_{\beta} \| \sum_{c=0}^{c_i-1} \left(1-\beta_c\right) \mathbf{X}_c \mathbf{W}_c^T \|_F^2, \quad s.t. \quad \|\beta\|_0 \leq N_c$$

- ThiNet proposed a greedy solution to optimize

$$\arg\min_{S} \| \sum_{c \in S} \mathbf{X}_c \mathbf{W}_c^T \|_F^2, \quad s.t. \quad \text{card}(S) \leq N_c$$

  - Add only one channel into pruning set S leading to the smallest l2-norm

```
S = []
while len(S) < N:
  min_norm, min_c = +inf, 0
  for c in range(c_i):
    tmpS = S + [c]
    Z = X[:,tmpS] * W[:,tmpS].t()
    norm = Z.norm(2)
    if norm < min_norm:
      min_norm, min_c = norm, c
  S.append(min_c)
```



ThiNet: A Filter Level Pruning Method for Deep Neural Network Compression [Luo *et al.*, ICCV 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

$$\arg\min_{\beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

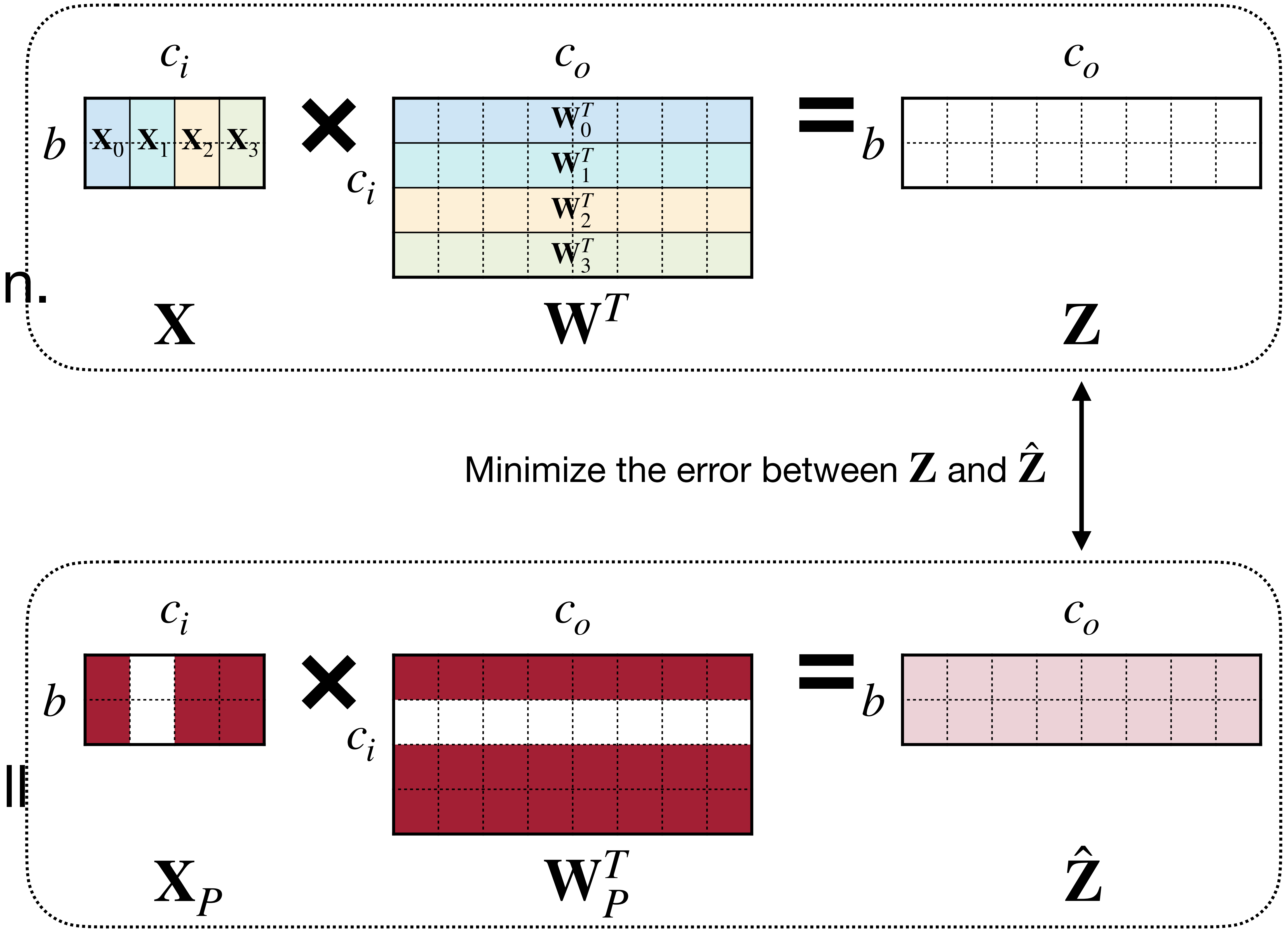
subject to

$$\|\beta\|_0 \le N_c$$

- β is coefficient vector of length c_i for channel selection. $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels.
- Relax the l_0 to l_1 regularization (LASSO):

$$\arg\min_{\beta} \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2 + \lambda\|\beta\|_1$$

  - λ is a penalty coefficient. By increasing λ, there will be more zeros in β.
  - Gradually increase λ and solve the LASSO regression for β, until $\|\beta\|_0 == N_c$ is met.



Channel Pruning for Accelerating Very Deep Neural Networks [He *et al.*, ICCV 2017]

Regression-based Pruning

Minimize reconstruction error of the corresponding layer's outputs

- **Weight Reconstruction**

$$\arg\min_{\mathbf{W}} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

  - β is coefficient vector from previous step

  - This is a classic linear regression problem, which has a unique closed-form solution using the lease squares approach.

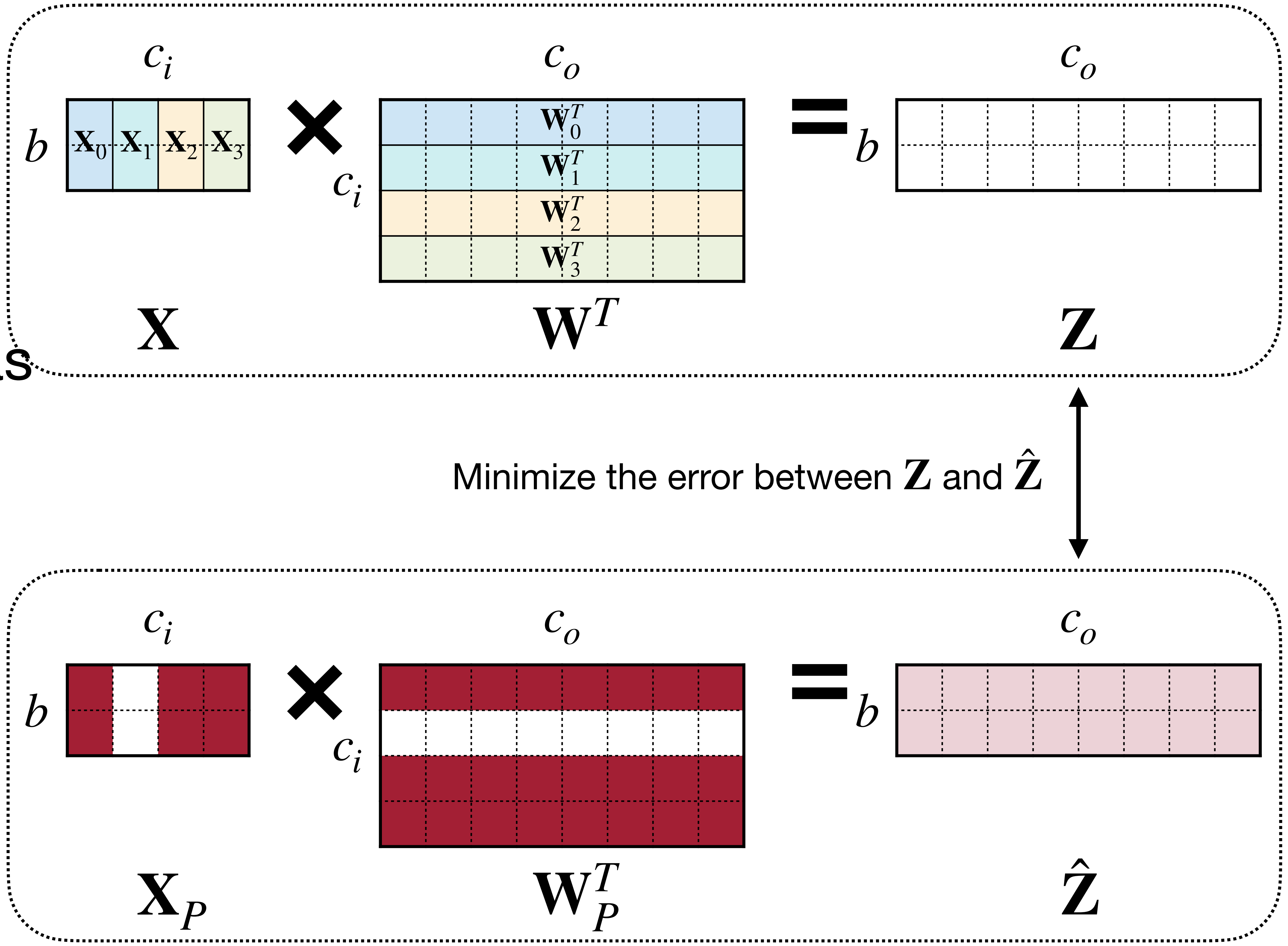
$$\arg\min_{\mathbf{W}} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \mathbf{U}\mathbf{W}^T\|_F^2$$

where

$$\mathbf{U} = \left[\beta_0\mathbf{X}_0 \;\; \beta_1\mathbf{X}_1 \;\; \cdots \;\; \beta_c\mathbf{X}_c \;\; \cdots \;\; \beta_{c_i-1}\mathbf{X}_{c_i-1}\right]$$

and thus,

$$\mathbf{W}^T = \left(\mathbf{U}^T\mathbf{U}\right)^{-1}\mathbf{U}^T\mathbf{Z}$$



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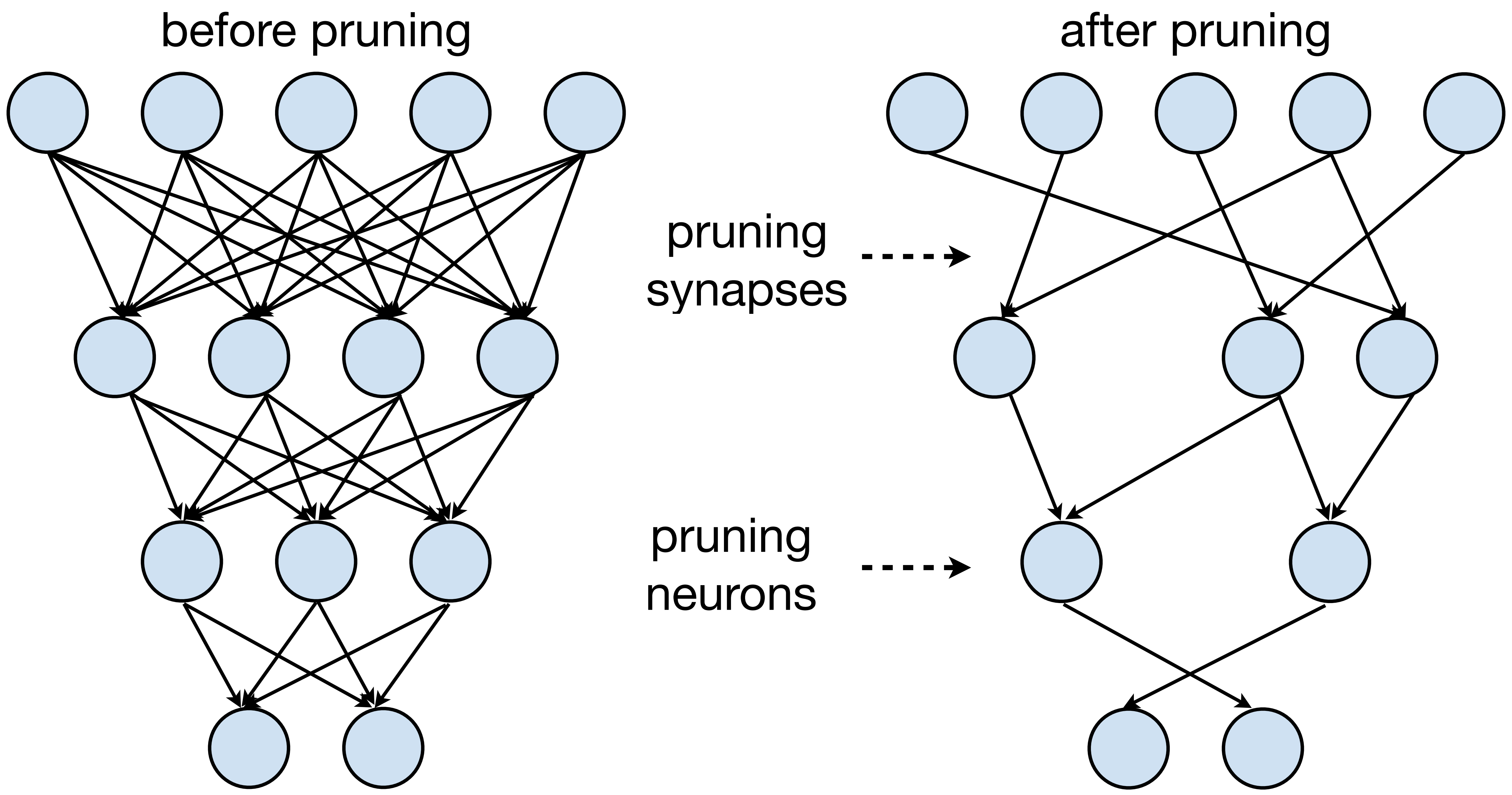
Channel Pruning for Accelerating Very Deep Neural Networks [He *et al.*, ICCV 2017]

Summary of Today's Lecture

In this lecture, we introduced:

- What is pruning
- Granularities of pruning
- Criteria to select weights to prune

- **We will cover in the next lecture:**
  - How to find pruning ratio for each layer
  - How to train/fine-tune the pruned layer
  - Automated ways to find pruning ratios
  - Lottery ticket hypothesis
  - System support for different granularities



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