$$\Phi = -\frac{GM}{\sqrt{R^2+z^2}} \Rightarrow \frac{\partial \Phi}{\partial R} = \frac{GM}{(R^2+z^2)^{3/2}}R \,, \quad \frac{\partial \Phi}{\partial z} = \frac{GM}{(R^2+z^2)^{3/2}} z$$

Consider small displacements in $R \to R_0 + \delta R$ & $z \to 0 + \delta z$, at constant specific angular momentum h, about a circular orbit

z eqn $\frac{\partial \Phi}{\partial z} \simeq \frac{GM}{R^3}\delta z = \Omega^2 \delta z$ so we get $\underline{\underline{\delta \ddot{z} = -\Omega^2 \delta z}}$

R eqn $\frac{\partial \Phi}{\partial R} \simeq \frac{GM}{[(R_0+\delta R)^2 + z^{2\,\to 0}]^{3/2}}(R_0+\delta R) = \frac{GM}{(R_0+\delta R)^2} \simeq \frac{GM}{R_0^2}\left(1 - 2\frac{\delta R}{R_0}\right)$

$R\dot{\phi}^2 = \frac{h^2}{R_0^3} \Rightarrow \frac{h^2}{(R_0+\delta R)^3} \simeq \frac{h^2}{R_0^3}\left(1 - 3\frac{\delta R}{R_0}\right)$, $\ddot{R} \to \delta\ddot{R}$ since $\ddot{R}_0 = 0$.

$$\delta \ddot{R} = \overbrace{\frac{h}{R_0^3} - \frac{GM}{R_0^2}}^{\to 0}\left[-\frac{3h^2}{R_0^4} + \frac{2GM}{R_0^3}\right]\delta R$$

since $h = \sqrt{GMR_0}$ for circular orbit.

$$= \left[-\frac{3GMR_0}{R_0^4} + \frac{2GM}{R_0^3}\right]\delta R = -\frac{GM}{R_0^3}\delta R = -\Omega^2 \delta R$$

$\Rightarrow \delta\ddot{R} = -\Omega^2 \delta R$, $\delta \ddot{z} = -\Omega^2 \delta z$ ← SHM motion with frequency $\Omega \to$ natural oscillation time $\to \underline{\underline{\Omega^{-1}}}$