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Assignment/Lab Title: Design of an Optimisation Model for Industrial Problems

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Design of an Optimisation Model for Industrial Problems

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Contents

List of Figures

List of Tables

1. Introduction

This coursework concerns itself around optimising a biofuel refining supply chain around central and northern France.

1.1. Brief Information

1.1.1. Supply Network Nodes

Details of the key locations for the network are detailed below:

- There are five collections points for corn stover, the raw biomass for this supply chain problem. There are located in Metz, Amiens, Caen, Le Mans and Orleans;
- There are three storages located in Rouen, Troyes and Tours;
- There are two biorefineries located in Reims and Nantes;
- And there are two biofuel customers located in Paris and Le Mans.

All biomass is collected from the collection points and must be stored in the storage points before being refined in the biorefineries. The biomass is then converted into biofuel which is transported to the customers to be sold. A simplified graphical representation of the process can be seen in Figure 1.

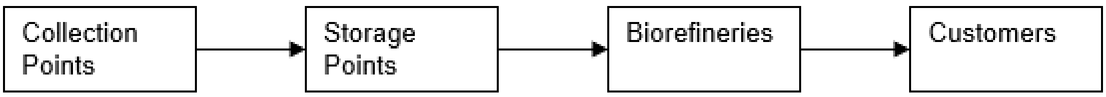

Figure 1 – Simplified Graphical Representation of the Biofuel Supply Chain Process

1.1.2. Resource Availability and Capacities

The availability of corn stover (the biomass) from the collection points are detailed in Table 1. The capacities for the biomass for the storage points and the biorefineries are detailed in Table 2 and Table 3 respectively.

Table 1 – Biomass Availability from the Collection Points

Collection Point	Availability [tonne]
Metz	25,000
Amiens	35,000
Caen	25,000
Le Mans	40,000
Orleans	35,000

Table 2 – Storage Point Capacities

Storage Point	Capacity [tonne]
Rouen	70,000
Troyes	80,000
Tours	60,000

Table 3 – Biorefinery Capacities

Biorefinery	Capacity [tonne]
Reims	90,000
Nantes	75,000

The layout of the supply chain, including the network nodes (locations) can be seen in Figure 2. It is noted that all collected biomass must first go to storages first before being distributed and processed within biorefineries.

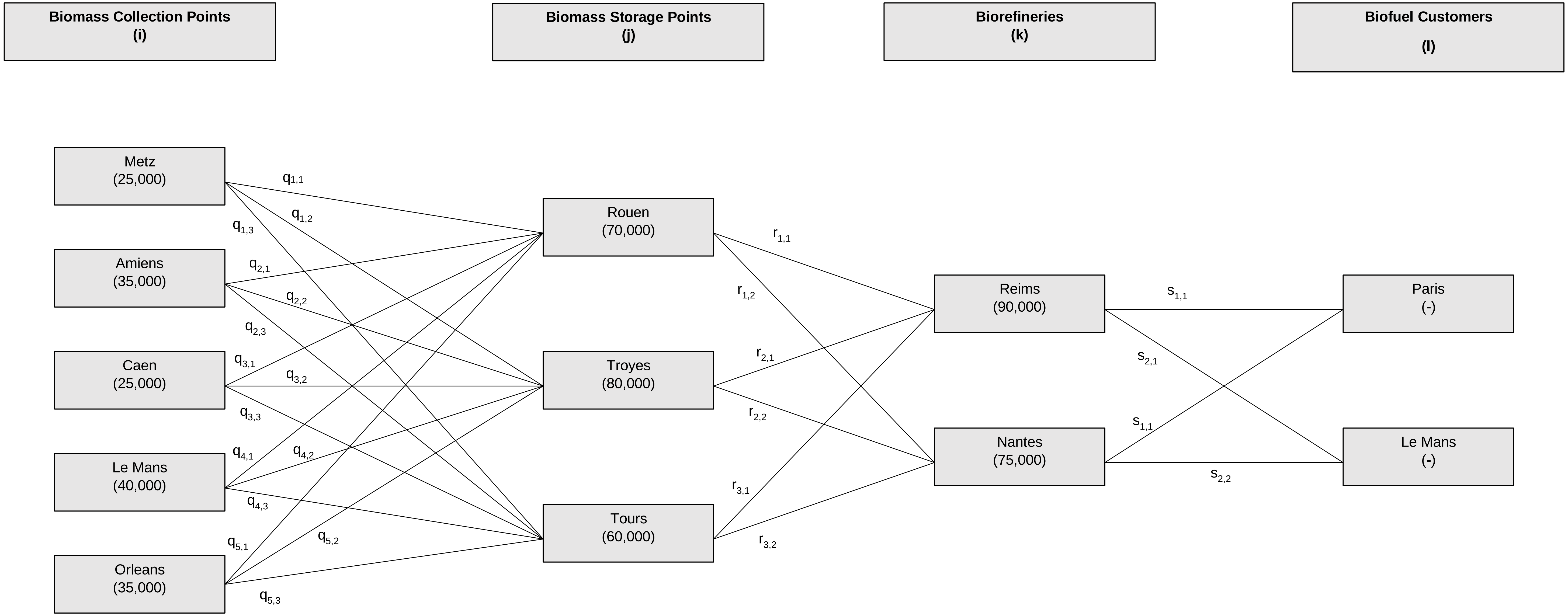

Figure 2 – Graphical Representation of the Biofuel Supply Chain Process

Numbers in brackets represent availability of biomass of the biomass collection points and capacity for the storage points and biorefineries.

i,j,k,l represent the biomass collection points, storage points, biorefineries and customers respectively.

q,r,s represent the quantities being delivered from the biomass collection points to the storage points, storage points to the biorefineries, and the biorefineries to the customers respectively.

1.1.3. Storage Information

The cost of storing biomass per tonne within all the storage can be calculated by

$$Cost_{storage} = x_1^2 + x_2^2 + x_3^2 - x_1 - x_2 - x_3 + 6.0$$

Equation 1

Where x_1 is the cost of the energy requirements, x_2 is the operational cost associated with labour and equipment and x_3 is the cost of repayment of capital repayment.

The optimum solution of the storage cost can be calculated by minimising Equation 1.

1.1.4. Biorefineries & Conversation Technology Information

The conversion rates for converting biomass into biofuel can be found in Table 4. It is noted that the Nantes' refinery can use either technology 1 or 2 whilst the Reims' refinery can only use technology 1. Each scenario will test using just technology 1 for both biorefineries and then using technology 1 in the Reims refinery and technology 2 in the Nantes refinery.

Table 4 – Biorefinery & Technology Conversion Rates

Biorefinery	Technology	Conversion Rates
Reims	Technology 1	30%
Nantes	Technology 2	32%

1.1.5. Transportation Information

The cost of transporting material between sites is shown in Table 5.

Table 5 – Storage Point Capacities

Travel Points	Cost of travel [£ per mile per tonne]
Collection Points – Storage Points	£0.70
Storage Points – Biorefineries	£0.50
Biorefineries – Customers	£0.50

The distances between the locations have been found by using Google Maps (Alphabet, 2020). These are shown in Table 6.

Table 6 – Transportation Distances between Locations

Origin	Destination	Distance [miles]
Metz	Rouen	297
	Troyes	156
	Tours	360
Amiens	Rouen	74.4
	Troyes	185
	Tours	236
Caen	Rouen	81.4
	Troyes	256
	Tours	166
Le Mans	Rouen	131
	Troyes	212
	Tours	60.9
Orleans	Rouen	149
	Troyes	131
	Tours	72.9
Rouen	Reims	181
	Nantes	240
Troyes	Reims	78.7
	Nantes	322
Tours	Reims	233
	Nantes	135
Reims	Paris	89.6
	Le Mans	213
Nantes	Paris	239
	Le Mans	114

1.1.6. Other Key Economic Considerations

Table 7 shows the costs of purchasing and refining the biomass (per tonne of biomass) along with the sale price for the biofuel.

Table 7 – Key Economic Considerations

Travel Points	Value [per tonne]
Cost of biomass	£15
Biomass Refining	£45
Price of Biofuel	£975

1.1.7. Scenario Conditions

Additional constraints are added for the two scenarios given. The two scenarios focus on producing maximum profit

1.1.7.1. Scenario 1 Conditions

This scenario focuses on maximising the supply to the customers given:

1. Paris demands 25,000 tonnes of biofuel
2. Le Mans demands 20,000 tonnes of biofuel

1.1.7.2. Scenario 2 Conditions

This scenario focuses on maximising the utilisation of the biorefineries capacities given:

1. The Reims' biorefinery has a capacity of 60,000 tonnes
2. The Nantes' biorefinery has a capacity of 40,000 tonnes
3. All biofuel produced is split equally between the customers

2. GAMS – Optimisation Modelling

This section focuses on the different forms of data entry and handling into the GAMS environment. There are to be four cases to optimise. Cases 1 and 2 will model scenario 1, where Case 1 uses just technology 1 in the biorefineries and Case 2 uses technology 1 in the Reims' biorefinery and technology 2 in the Nantes' biorefinery. Cases 3 and 4 will model scenario 2, where Case 3 uses just technology 1 in the biorefineries and Case 2 uses technology 4 in the Reims' biorefinery and technology 2 in the Nantes' biorefinery.

2.1. Scalars

Scalars are values which are constant throughout the GAMS simulation. The scalars used within this optimisation problem can be found in Table 8. It should be noted that the minimised optimal solution of the has been included as a scalar for practicality (this will have been calculated through a previous GAMS optimisation).

Table 8 – Scalars within the GAMS Simulation

Scalar	Description	Value	Units
TruckCostSmall	Cost of transporting biomass using small trucks	0.7	£/tonne/mile
TruckCostBig	Cost of transporting biomass using large trucks	0.5	£/tonne/mile
BiofuelPrice	Price of biofuel	975	£/tonne
BiomassCost	Cost of biomass	15	£/tonne
RefiningCost	The cost of refining biomass into biofuel	45	£/tonne (of biomass)
StorageOptimiumCost	The cost of storing biomass	5.25	£5.25

2.2. Sets

"Sets are fundamental building blocks in any GAMS model." (GAMS Development Corporation, 2020). The sets have been classified into their types of locations i.e. collection points, storage points, biorefineries and customers. The sets used in the optimisation problem can be found in Table 9.

Table 9 – Sets within the GAMS Simulation

Parameter	Description	Contents of Set
i	Collection Points	CP_Metz, CP_Amines, CP_Caen, CP_LeMans, CP_Orleans
j	Storage Points	SP_Rouen, SP_Troyes, SP_Tours
k	Bio-Refineries	BR_Reims, BR_Nantes
l	Customers	C_Paris, C_LeMans

Note: The following prefixes used within the sets are defined as: CP – Collection Points, SP – Storage Points, BR – Biorefineries, C – Customers

2.3. Parameters

Parameters are essentially unordered lists with corresponding values. The availabilities, capacities, conversion rates and specified customer demand have been set as parameters. The parameters used within the optimisation problem can be found in Table 10.

Table 10 – Parameters within the GAMS Simulation

Set	Description	Contents of Parameter
a(i)	Availability of biomass at the collection points in tonnes	CP_Metz 25000 CP_Amines 35000 CP_Caen 25000 CP_LeMans 40000 CP_Orleans 35000
b(j)	Capacity of biomass at the storage points in tonnes	SP_Rouen 70000 SP_Troyes 80000 SP_Tours 60000
c(k)	Capacity at the biorefineries in tonnes for biomass (applicable to scenario 1 only)	BR_Reims 90000 BR_Nantes 75000
e(k)	The conversion rates using technology 1 and technology 2 in the biorefineries (technology 1 for Reims, technology 2 for Nantes)	BR_Reims 0.3 BR_Nantes 0.32
f(k)	The conversion rates using technology 1 in the biorefineries	BR_Reims 0.3 BR_Nantes 0.30
g(l)	The demand of the customers in tonnes for biofuel (applicable to scenario 1 only).	C_Paris 25000 C_LeMans 20000
h(k)	Capacity at the biorefineries in tonnes for biomass (applicable to scenario 2 only)	BR_Reims 60000 BR_Nantes 40000

2.4. Tables

All distances were obtained through the use of the route finder provided by Google Maps (Alphabet, 2020). The distances are the quickest routes at the time of search and are provided in miles. It is assumed that the trucks can physically take the routes suggested by Google Maps.

The distances between the collection points to the storage points, the storage points to the biorefineries and the biorefineries to the customers are represented by Table 11, Table 12 and Table 13 respectively.

The matrices $d1_{i,j}$, $d2_{i,j}$ and $d3_{i,j}$ represent the tabulated distances in matrix form.

Table 11 – Distances from Biomass Collection Points to Storage Points

	SP_Rouen	SP_Troyes	SP_Tours
CP_Metz	297	156	360
CP_Amines	74.4	185	236
CP_Caen	81.4	256	166
CP_LeMans	131	212	60.9
CP_Orleans	149	131	72.9

Table 11 can be represented in matrix form below.

$$d1_{i,j} = \begin{pmatrix} 297 & 156 & 360 \\ 74.4 & 185 & 236 \\ 81.4 & 256 & 166 \\ 131 & 212 & 60.9 \\ 149 & 131 & 72.9 \end{pmatrix}$$

Table 12 – Distances from Storage Points to Biorefineries

	BR_Reims	BR_Nantes
SP_Rouen	181	240

CP_Amines	78.7	322
SP_Tours	233	135

Table 12 can be represented in matrix form below.

$$d2_{j,k} = \begin{pmatrix} 181 & 240 \\ 78.7 & 322 \\ 233 & 135 \end{pmatrix}$$

Table 13 – Distances from Biorefineries to Customers

	C_Paris	C_LeMans
BR_Reims	89.6	213
BR_Nantes	239	114

Table 13 can be represented in matrix form below.

$$d3_{k,l} = \begin{pmatrix} 89.6 & 213 \\ 239 & 114 \end{pmatrix}$$

2.5. Variables

The variables for the storage cost optimisation can be found in Table 14. Variables y, x_1, x_2, and x_3 must be non-negative values as they cannot feasibly be negative in the physical world. The cost of storage (y), energy requirements (x_1) and costs of operation (x_2) and capital repayment (x_3) can never be below 0.

Whereas, the variables for the main optimisation problem can be found in Table 15. The variables q, r and s must be non-negative as they cannot feasibly be negative in the physical world. q, r and s represent the quantities of material being transported between their respective sites and henceforth must be positive. The profits have not been constrained as it is possible (but unexpected) to generate negative values.

Table 14 – Variables within the GAMS Simulation for Determining the Optimum Storage Cost

Variable	Description
y	Cost of storing the biomass per tonne of biomass
x_1	Energy requirements
x_2	Operational cost associated with labour and equipment
x_3	Repayment of capital investment;

Table 15 – Variables within the GAMS Simulation for the Logistical Optimisation Problem

Variable	Description
q(i,j)	Quantity of biomass transported between the collection points and storage points in tonnes
r(j,k)	Quantity of biomass transported between the storage points and biorefineries in tonnes
s(k,l)	Quantity of biomass transported between the biorefineries and customers in tonnes
P111	Case 1: Profit for scenario 1 with technology 1 in both biorefineries
P112	Case 2: Profit for scenario 1 with technology 1 in Reims and technology 2 in Nantes
P211	Case 3: Profit for scenario 2 with technology 1 in both biorefineries
P212	Case 4: Profit for scenario 2 with technology 1 in Reims and technology 2 in Nantes;

2.6. Constraints

The constraints help by reducing the degrees of freedom for the problem. This allows the formation of the feasible region where an optimum solution can be determined. Both scenarios share several constraints, but there are scenario-specific constraints for each scenario.

2.6.1. Constraints for both Scenarios

The first constraint is the supply availability constraint. The total amount of biomass taken from the collection points cannot exceed that available at the collection points henceforth:

$$\sum_{j} q(i,j) < a(i)$$

The second constraint is the storage capacity constraint. It follows a similar principle to the previous constraint. The total amount of biomass going towards the storages cannot be greater than the capacity available at the storage points therefore:

$$\sum_{i} q(i,j) < b(j)$$

The third constraint is the storage mass balance constraint. The product arriving at the site must equal the product leaving so that no accumulation can occur. This is a basic mass balance simulating a steady state process.

$$\sum_{i} q(i,j) = \sum_{j} r(j,k)$$

The fourth constraint is the refinery mass balance. This follows the same principle as the third constraint. The product arriving at the site must equal the product leaving so that no accumulation can occur. This is a basic mass balance simulating a steady state process. This constraint is for when both refineries utilise technology 1. The conversions for the refineries are included in the constraint. This constraint will be used for Case 1 and Case 3.

$$\sum_{j} r(j,k) \times f(k) = \sum_{l} s(k,l)$$

The fifth constraint is the same as the fourth but the Nantes' biorefinery will use technology 2. Therefore, the conversion parameter has changed. This constraint will be used for Case 2 and Case 4.

$$\sum_{j} r(j,k) \times e(k) = \sum_{l} s(k,l)$$

2.6.2. Scenario 1 Specific Constraints

This is the first scenario 1 specific constraint and it refers to the biorefinery capacity. The total amount of biomass going towards the storages cannot exceed the capacity available at the biorefineries henceforth:

$$\sum_{j} r(j,k) < c(k)$$

This is the second scenario 1 specific constraint and it refers to the customer demand. The total amount of biofuel leaving the biorefineries must be equal to the specified customer demand henceforth:

$$\sum_{k} s(k,l) = g(l)$$

2.6.3. Scenario 2 Specific Constraints

This is the first scenario 2 specific constraint and it refers to the biorefinery capacity. The total amount of biomass going towards the storages cannot exceed the capacity available at the biorefineries. As the biorefinery utilisation is to be maximised, the amount of product going to the biorefineries must be set to the capacity, henceforth:

$$\sum_{j} r(j,k) = h(k)$$

This is the second scenario 2 specific constraint and it refers to the customer demand. The total amount of biofuel leaving the biorefineries must be equally split between the customers, therefore the customer in Paris will receive the same volume of product as the one in Le Mans henceforth:

$$\sum_{k} s(k,\text{"}C\_Paris\text{"}) = \sum_{k} s(k,\text{"}C\_LeMans\text{"})$$

2.7. Profit Equation

The profit can be determined from the income of the process minus the expenses of the process.

$$Profit = Income - Expenses$$

The income comes from the sale of biofuel to the customers, the expenses are from the cost of biomass, storing the biomass, refining the biomass and transporting the biomass/biofuel.

$$Profit = Sale\ of\ Biofuel - Cost\ of\ Biomass - Cost\ of\ Biomass\ Refining - Cost\ of\ Storage - Cost\ of\ Transport$$

Even though there are four cases covering the two scenarios, only one profit equation is required. The differences between the four cases are affected by the constraints only.

The components of the profit equation are be expressed in the following way:

Sale of Biofuel

$$BioFuelPrice \times \sum_{k,l} s(k,l)$$

Cost of Biomass

$$BiomassCost \times \sum_{i,j} q(i,j)$$

Cost of Biomass Refining

$$RefiningCost \times \sum_{j,k} r(j,k)$$

Cost of Storage

$$StorageOptimiumCost \times \sum_{i,j} q(i,j)$$

Cost of Transport – Collection Points to Storage Points

$$TruckCostSmall \times \sum_{i,j} d1(i,j) \times q(i,j)$$

Cost of Transport – Storage Points to Biorefineries

$$TruckCostBig \times \sum_{j,k} d2(j,k) \times r(j,k)$$

Cost of Transport – Biorefineries to Customers

$$TruckCostBig \times \sum_{k.l} d3(k,l) \times s(k,l)$$

Therefore the overall equation can be represented by:

$$\begin{aligned} Profit = {} & BioFuelPrice \times \sum_{k,l} s(k,l) - BiomassCost \times \sum_{i,j} q(i,j) - RefiningCost \times \sum_{j,k} r(j,k) \\ & - StorageOptimiumCost \times \sum_{i,j} q(i,j) - TruckCostSmall \\ & \times \sum_{i,j} d1(i,j) \times q(i,j) - TruckCostBig \times \sum_{j,k} d2(j,k) \times r(j,k) - TruckCostBig \\ & \times \sum_{k.l} d3(k,l) \times s(k,l) \end{aligned}$$

2.8. Solver Selection

There were two optimisation problems to resolve:

1. The cost of storage optimisation and;
2. The overall logistical network optimisation problem

2.8.1. Storage Cost Optimisation Solver

The storage cost optimisation problem is an unconstrained, non-linear problem. A solver was required that could provide a global optimum (in this case, a global minimum) that can solve unconstrained, non-linear problems. The BARON solver is a "Branch-And-Reduce Optimization Navigator for proven global solutions" (GAMS Development Corporation, 2020) which can solve non-linear problems. Therefore, it meets the criteria required.

2.8.2. Overall Logistical Network Optimisation Solver

This optimisation problem is a constrained linear problem. There are several solvers than can be used for this problem: BDMLP which is a "LP and MIP solver that comes with any GAMS system"; CPLEX which is a "High-performance LP/MIP solver"; SOPLEX which is a "High-performance LP solver"; XA which is a "Large scale LP/MIP solver" (GAMS Development Corporation, 2020) etc. All of these solvers have been tested and work successfully for the optimisation problem. The SOPLEX solver has been selected as it seems to have a key focus on high-performance for solely linear problems. There are other successful solvers like CONOPT which would also successfully solve the problem, but that has a focus on non-linear problems, so its use may be questionable when compared to alternatives mentioned.

3. Results

As seen in Table 16, the case with the highest profit is Case 2 with a profit of £13,000,000. This is the expected result as the physical volume of biofuel product that could be sold is greater than that in Case 3 and Case 4, whom could process significantly less product. Case 2 generates more profit than Case 1 as the Nantes biorefinery has a higher conversion rate by using technology 2, than technology 1. This allows a lower expenditure for the purchase of raw material and transporting it.

Case 3 and Case 4 couldn't generate as much profit as their biorefinery capacity was significantly produced, resulting in less product being produced and a condition was placed on the distribution of biofuel to the customers, resulting in a increase in transportation costs for the cases. Case 3 generates less profit than Case 4 due to its lower conversation rate as explained previously.

Thus, the order of profitable cases is reasonable.

Table 16 – GAMS-Calculated Profits for the Different Cases

Case	GAMS-Calculated Profit
Case 1	£12,100,000
Case 2	£13,000,000
Case 3	£8,490,000
Case 4	£9,250,000

3.1. Storage Cost Calculation

The optimised GAMS-calculated storage cost is £5.25.

The cost of storage is a non-linear unconstrained optimisation problem. The problem equation is found below. All terms for the equation are defined in Section 1.1.3.

$$Cost_{storage} = y = x_1^2 + x_2^2 + x_3^2 - x_1 - x_2 - x_3 + 6.0$$

3.1.1. First Order Derivation (Finding the Gradient)

The solution of x_1, x_2 and x_3 can be found by finding the gradient through the partial differentiation with respect to each variable.

$$\nabla y = \begin{pmatrix} \frac{\partial y}{\partial x_1} \\ \frac{\partial y}{\partial x_2} \\ \frac{\partial y}{\partial x_3} \end{pmatrix}$$

$$\frac{\partial y}{\partial x_1} = 2x_1 - 1$$

$$\frac{\partial y}{\partial x_2} = 2x_2 - 1$$

$$\frac{\partial y}{\partial x_3} = 2x_3 - 1$$

$$\nabla f(x) = \begin{pmatrix} 2x_1 - 1 \\ 2x_2 - 1 \\ 2x_3 - 1 \end{pmatrix}$$

When ∇y is equal to zero, the gradient is found providing the solution to x_1, x_2 and x_3,

$$\nabla f(x) = \begin{pmatrix} 2x_1 - 1 \\ 2x_2 - 1 \\ 2x_3 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}$$

The values of x_1, x_2 and x_3 are have been found for when the gradient is equal to zero. It should now be determined if these are a local or global minimum, maximum or stationary point solution.

3.1.2. Second Order Derivation (Finding the Hessian Derivative)

Creation of the Hessian matrix, yields

$$\nabla y = \begin{pmatrix} \frac{\partial^2 y}{\partial x_1} & \frac{\partial^2 y}{\partial x_1 x_2} & \frac{\partial^2 y}{\partial x_{1x_3}} \\ \frac{\partial^2 y}{\partial x_2 x_1} & \frac{\partial^2 y}{\partial x_2} & \frac{\partial^2 y}{\partial x_2 x_3} \\ \frac{\partial^2 y}{\partial x_3 x_1} & \frac{\partial^2 y}{\partial x_3 x_2} & \frac{\partial^2 y}{\partial x_3} \end{pmatrix}$$

The second partial derivatives are:

$$\frac{\partial^2 y}{\partial x_1} = 2$$

$$\frac{\partial^2 y}{\partial x_2} = 2$$

$$\frac{\partial^2 y}{\partial x_3} = 2$$

All other derivatives equal zero. Therefore

$$\nabla^2 y = \begin{pmatrix} \frac{\partial^2 y}{\partial x_1} & \frac{\partial^2 y}{\partial x_1 x_2} & \frac{\partial^2 y}{\partial x_{1x_3}} \\ \frac{\partial^2 y}{\partial x_2 x_1} & \frac{\partial^2 y}{\partial x_2} & \frac{\partial^2 y}{\partial x_2 x_3} \\ \frac{\partial^2 y}{\partial x_3 x_1} & \frac{\partial^2 y}{\partial x_3 x_2} & \frac{\partial^2 y}{\partial x_3} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

The determinate is:

$$det\ (\nabla^2 y) = 2 \times (-1)^{1+1} \times \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + 0 \times (-1)^{1+2} \times \det\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} + 2 \times (-1)^{1+2} \times \det\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$= 2 \times (2 \times 2 - 0 \times 0) = 8$$

$$\therefore det\ (\nabla^2 y) > 0$$

Since the $det\ (\nabla^2 y)$ is greater than zero, it is a positive definite.

3.1.3. Calculating the Eigenvalues:

$$\det(\lambda I - \nabla^2 y) = 0$$

$$= \det \begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix}$$

$$= \det(\nabla^2 y) = 2 \times (-1)^{1+1} \times \det \begin{pmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{pmatrix}$$

$$+ 0 \times (-1)^{1+2} \times \det \begin{pmatrix} 0 & 0 \\ 0 & 2-\lambda \end{pmatrix} + 2 \times (-1)^{1+2} \times \det \begin{pmatrix} 0 & 2-\lambda \\ 0 & 0 \end{pmatrix}$$

$$= 2 \times (\lambda - 2)^2 = 0$$

$$\therefore \lambda = 2$$

$$\therefore \lambda > 0$$

3.1.4. Storage Cost Calculation Results

Since the eigenvalues are greater than zero, the second derivative classified as a positive definite for the values found from the first derivative when equal to zero. The solution is a global minimum. The storage cost global minimisation has been determined for when $x_1 = 0.5$, $x_2 = 0.5$ and $x_3 = 0.5$.

Thus, the cost of storage is

$$Cost_{storage} = x_1^2 + x_2^2 + x_3^2 - x_1 - x_2 - x_3 + 6.0$$

$$Cost_{storage} = 0.5^2 + 0.5^2 + 0.5^2 - 0.5 - 0.5 - 0.5 + 6.0 = 5.25$$

From the following calculation procedure, the optimal minimised cost of storage per tonne is £5.25. This is consistent with the GAMS-calculated value.

3.2. Scenario 1

The graphical representations of the optimised solutions for Case 1 and Case 2 are represented in Figure 3 and Figure 4 respectively.

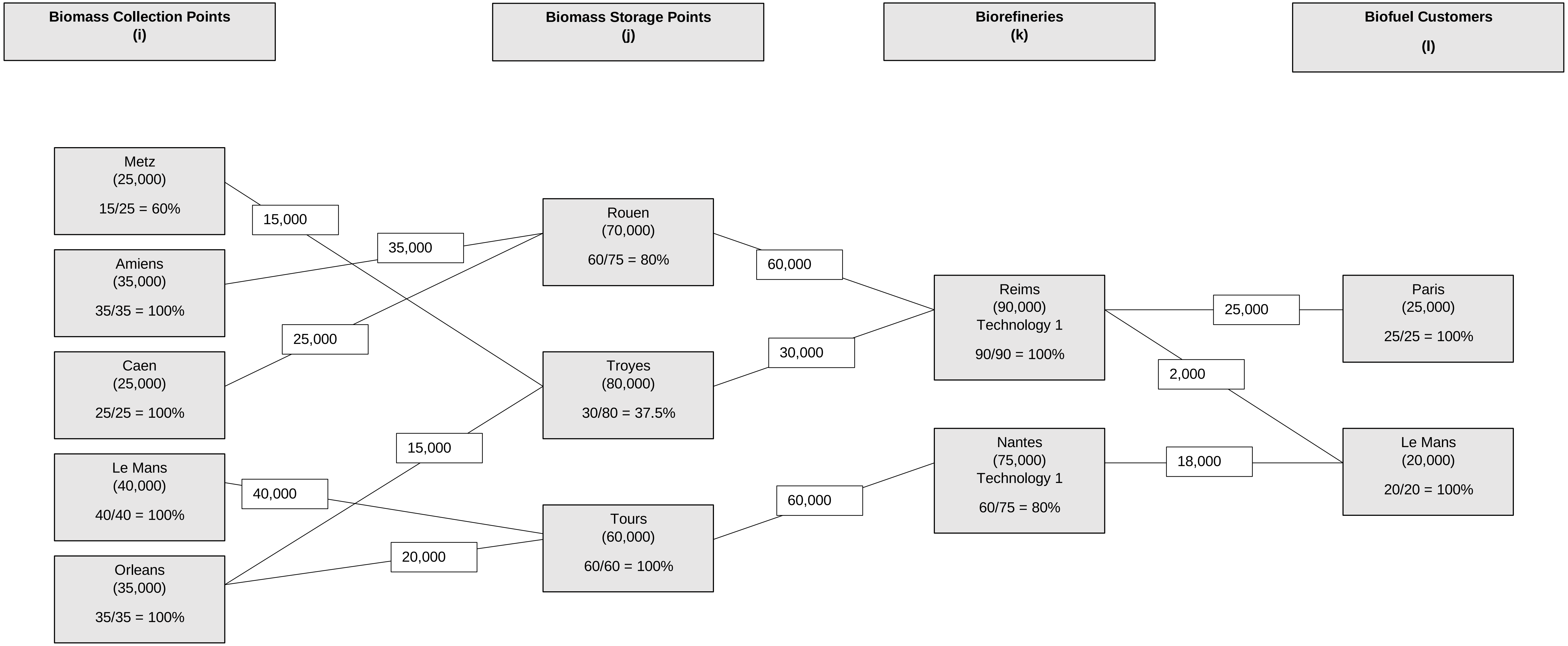

Figure 3 - Graphical Representation of Case 1: Scenario 1 where Both Biorefineries Use Technology 1

Numbers in brackets represent availability of biomass of the biomass collection points and capacity for the storage points and biorefineries and demand for the customers.

Utilisation rates are calculated within the grey boxes.

Volume of biomass/biofuel in tonnes being transported between locations are presented in the white boxes.

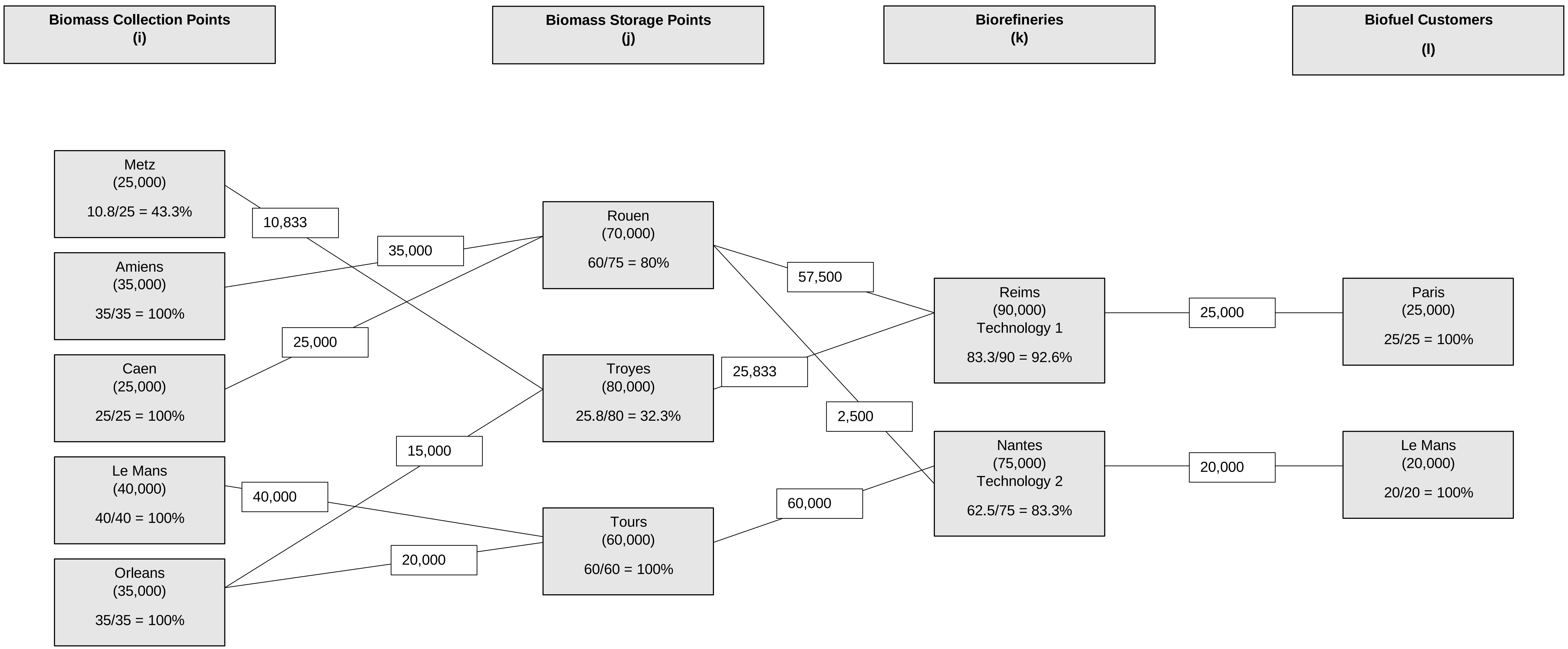

Figure 4 - Graphical Representation of Case 2: Scenario 1 where the Reims Biorefinery Uses Technology 1 and the Nantes Biorefinery Uses Technology 2

Numbers in brackets represent availability of biomass of the biomass collection points and capacity for the storage points and biorefineries and demand for the customers.

Utilisation rates are calculated within the grey boxes.

Volume of biomass/biofuel in tonnes being transported between locations are presented in the white boxes.

3.2.1. Scenario 1 Discussion and Analysis

In Cases 1 & 2, the only collection point to not be fully utilised is the collection point in Metz with a utilisation rate of 60% and 43.3% respectively. This is reasonable as Metz is significantly further from the storage points than the other collection points as shown in Table 6 (297, 156 and 360 miles from Rouen, Troyes and Tours respectively), hence it is a more expensive collection point to use in comparison, when considering the transportation costs. The demand of biofuel is not sufficient to require that all biomass from all the collection points is required. Given these reasons the Metz is not fully utilised. It is noted that less biomass is required from Metz in Case 2, this is due to the use of technology 2 in the Nantes refinery which has a higher conversion.

The Tours storage point is fully utilised in both cases which is to be expected given that it's so close to the Le Mans and Orleans collection points (60.9 and 72.9 miles respectively). The Rouen has a utilisation rate of 80% which is reasonable as it takes all the biomass from Caen and Amiens whom are extremely close collection points (81.4 miles and 74.4 miles respectively). It takes the full biomass availability from Amines and Caen. Troyes has the lowest utilisation rate of 37.5% for Case 1 and 32.3% for Case 2. This is reasonable as it's the storage furthest from the collection points. It currently takes the spare availability from Orleans and makes up the remainder from Metz to meet the customer demand. The utilisation is reduced in Case 2 as technology 2 provides a higher conversion, so less biomass is required for the given biofuel.

The Nantes biorefinery has a utilisation rate of 80% in Case 1 and 83.3% in Case 2. In both cases, all the stored biomass in Tours goes to Nantes, due to Tours being 98 miles closer to Nantes than Reims. In Case 2, 2,500 tonnes of biomass goes from Rouen to Nantes, this is more economical due to the higher conversion rate (2% higher per tonne biomass), reducing the refining cost per tonne of biofuel produced, with the saving being greater than the transportation cost of changing the logistical network. The Reims biorefinery has utilisation rate of 100% in Case 1 and 92.5% in Case 2. All biomass from Troyes goes to Reims due to its proximity compared to Nantes. All the biomass from Rouen goes to Reims again due to it being the closest biorefinery. In Case 2, 2,500 tonnes go to Nantes for reasons already explained.

In Case 1, all biofuel product from Nantes (80% utilisation rate) goes to Le Mans with the majority of biofuel from Reims (which has a 100% utilisation rate) going to Paris. A small portion from Reims goes towards Paris to meet the total customer demand. It is clear that the distance between Reims and Le Mans is financially cheaper than the distance between Rouen & Troyes to the Nantes refinery. In Case 2, all biofuel from Nantes (83.3% utilisation rate) goes to Le Mans and all biofuel from Reims (92.6% utilisation rate) goes to Paris. This is because of the higher conversion in the Nantes refinery providing cheaper transportation costs earlier in the network and the refining savings are greater than that post refinery due to the higher conversion.

Case 2 yields a profit of £13 million compared to Case 1 which yields a profit of £12.1 million. Therefore the saving produced by using technology 2 in Nantes, increases the profit by £900,000. So technology 2 should be used.

It is important to ensure the profits are validated for the optimisation cases so hand-calculated verifications have been conducted.

3.2.1.1. Case 1 Profit Validation

Table 17 – Income for Case 1

Product [tonnes]	Price of product/tonne	Income, £
45,000	975	43,900,000

Table 18 – Cost of Transport for Case 1

Locations	Quantity, tonnes	Miles	Cost/mile/tonne	Cost, £
Collection Points - Storage Points				
Metz - Troyes	15,000	156	0.7	1,640,000
Amiens - Rouen	35,000	74.4	0.7	1,820,000
Caen - Rouen	25,000	81.4	0.7	1,420,000
Le Mans - Tours	40,000	60.9	0.7	1,700,000
Orleans - Troyes	15,000	131	0.7	1,380,000
Orleans - Tours	20,000	72.9	0.7	1,020,000
Storage Points - Biorefineries				
Rouen - Reims	60,000	181	0.5	5,430,000
Troyes - Reims	30,000	78.7	0.5	1,180,000
Tours - Nantes	60,000	135	0.5	4,050,000
Biorefineries - Customers				
Reims - Paris	25,000	89.6	0.5	1,120,000
Reims - Le Mans	2,000	213	0.5	213,000
Nantes - Le Mans	18,000	114	0.5	1,030,000
Total				22,000,000

Table 19 – Other Costs for Case 1

Costs	Quantity, tonnes	Cost/tonne	Cost, £
Refining Cost	150,000	45	6,750,000
Biomass Cost	150,000	15	2,250,000
Storage Cost	150,000	5.25	787,500
Total			9,790,000

Taking the total income and costs from Table 17, Table 18 and Table 19, the profit is £43,900,000 - £22,000,000 - £9,790,000 = £12,100,000 which is consistent with GAMS-generated solution as shown in Table 16.

3.2.1.2. Case 2 Profit Validation

Table 20 – Income for Case 2

Product [tonnes]	Price of product/tonne	Income, £
45,000	975	43,900,000

Table 21 – Cost of Transport for Case 2

Locations	Quantity, tonnes	Miles	Cost/mile/tonne	Cost, £
Collection Points - Storage Points				
Metz - Troyes	10,800	156	0.7	1,180,000
Amiens - Rouen	35,000	74.4	0.7	1,820,000
Caen - Rouen	25,000	81.4	0.7	1,420,000
Le Mans - Tours	40,000	60.9	0.7	1,710,000
Orleans - Troyes	15,000	131	0.7	1,380,000
Orleans - Tours	20,000	72.9	0.7	1,020,000
Storage Points – Biorefineries				
Rouen - Reims	57,500	181	0.5	5,200,000
Rouen - Nantes	2,500	240	0.5	300,000
Troyes - Reims	25,800	78.7	0.5	1,020,000
Tours - Nantes	60,000	135	0.5	4,050,000
Biorefineries – Customers				
Reims - Paris	25,000	89.6	0.5	1,120,000
Nantes - Le Mans	20,000	114	0.5	1,140,000
Total				21,400,000

Table 22 – Other Costs for Case 2

Costs	Quantity, tonnes	Cost/tonne	Cost, £
Refining Cost	146,000	45	6,560,000
Biomass Cost	146,000	15	2,190,000
Storage Cost	146,000	5.25	765,000
Total			9,520,000

Taking the total income and costs from Table 20,

Table 21 and Table 22, the profit is £43,900,000 - £21,400,000- £9,520,000 = £13,000,000 which is consistent with GAMS-generated solution as shown in Table 16.

3.3. Scenario 2

The graphical representations of the optimised solutions for Case 3 and Case 4 are represented in Figure 5 and Figure 6 respectively.

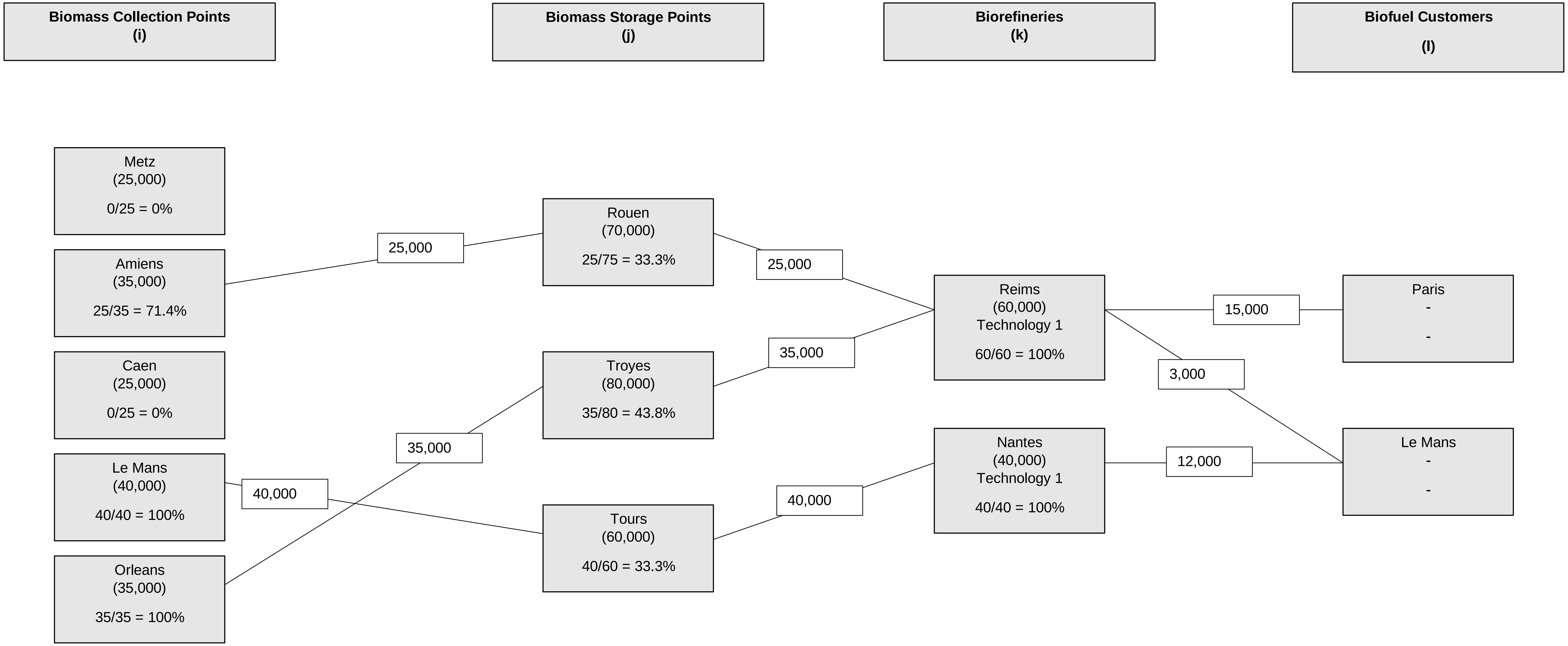

Figure 5 - Graphical Representation of Case 3: Scenario 2 where Both Biorefineries Use Technology 1

Numbers in brackets represent availability of biomass of the biomass collection points and capacity for the storage points and biorefineries.

Utilisation rates are calculated within the grey boxes.

Volume of biomass/biofuel in tonnes being transported between locations are presented in the white boxes.

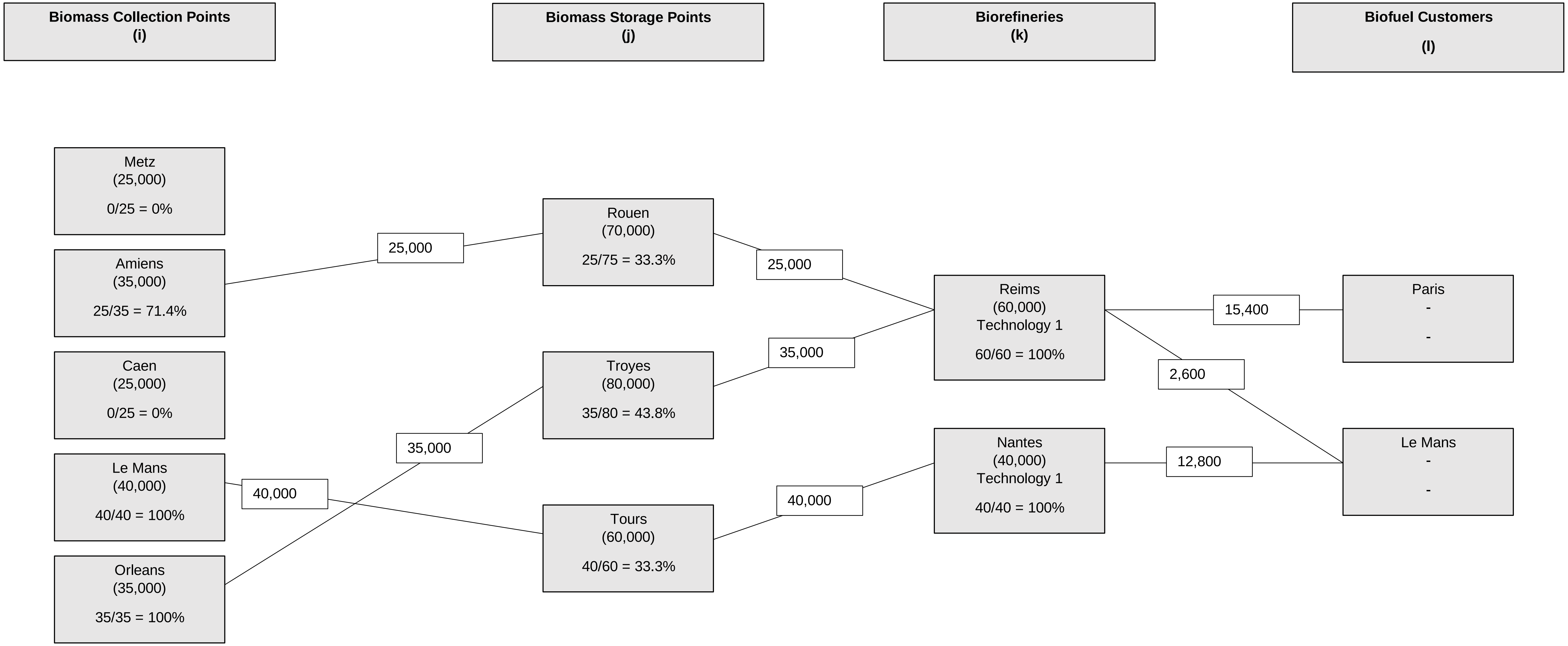

Figure 6 - Graphical Representation of Case 4: Scenario 2 where the Reims Biorefinery Uses Technology 1 and the Nantes Biorefinery Uses Technology 2

Numbers in brackets represent availability of biomass of the biomass collection points and capacity for the storage points and biorefineries.

Utilisation rates are calculated within the grey boxes.

Volume of biomass/biofuel in tonnes being transported between locations are presented in the white boxes.

3.3.1. Scenario 2 Discussion and Analysis

In Cases 3 & 4, the only collection points to not be utilised (i.e. utilisation rate of 0%) are the collection points in Metz and Caen. This is reasonable as Metz is significantly further from the storage points than the other collection points as shown in Table 6 (297, 156 and 360 miles from Rouen, Troyes and Tours respectively) and Caen closest storage point is further than Orleans, Le Mans and Amiens' closest storage points. Each collection point that is being used is delivering biomass to only their closest storage point because the biorefineries have a much smaller capacity than that in scenario 1. The demand of biofuel is not sufficient to require that all biomass from all the collection points is required. Given these reasons the Metz and Caen are not utilised. Amiens has a utilisation rate of 71.4% as its closest storage point is further than Le Mans and Orleans' (whom have 100% utilisation rates) respective storage points.

The storage points of Rouen, Troyes and Tours have utilisation rates of 33.3%, 43.8% and 33.3%. This is expected as they're taking biomass from their closest collection points. This is for both cases.

All biomass from Rouen and Troyes goes to the Reims biorefinery as it's the closest biorefinery and all biomass from Tours goes to the Nantes biorefinery. This applies to both cases. The utilisation rate of the Reims and Nantes' biorefineries are 100% as their capacities are being maximised. Their capacities provide the limiting factor on for the amount of biomass being processed within the network. In Case 4, technology 2 provides a higher conversion rate, producing more biofuel per tonne of biomass.

All biofuel produced in the Nantes refinery goes towards Le Mans and the majority of the biofuel from the Reims refinery goes to Paris with a portion going to Le Mans to ensure that both customers receive the same amount of product as specified by a Scenario 2 condition. The refineries focuses their supplies on their nearest customers to minimise transportation costs.

Case 4 yields a profit of £9.25 million compared to Case 1 which yields a profit of £8.49 million. Therefore the saving produced by using technology 2 in Nantes, increases the profit by £760,000. So technology 2 should be used.

It is important to ensure the profits are validated for the optimisation cases so hand-calculated verifications have been conducted.

3.3.1.1. Case 3 Profit Validation

Table 23 – Income for Case 3

Product [tonnes]	Price of product/tonne	Income, £
30,000	975	29,300,000

Table 24 – Cost of Transport for Case 3

Locations	Quantity, tonnes	Miles	Cost/mile/tonne	Cost, £
Collection Points - Storage Points				
Amiens - Rouen	25,000	74.4	0.7	1,300,000
Le Mans - Tours	40,000	60.9	0.7	1,710,000
Orleans - Troyes	35,000	131	0.7	3,210,000
Storage Points – Biorefineries				
Rouen - Reims	25,000	181	0.5	2,260,000

Troyes - Reims	35,000	78.7	0.5	1,380,000
Tours - Nantes	40,000	135	0.5	2,700,000
Biorefineries – Customers				
Reims - Paris	15,000	89.6	0.5	672,000
Reims - Le Mans	3,000	213	0.5	320,000
Nantes - Le Mans	12,000	114	0.5	684,000
Total				14,200,000

Table 25 – Other Costs for Case 3

Costs	Quantity, tonnes	Cost/tonne	Cost, £
Refining Cost	100,000	45	4,500,000
Biomass Cost	100,000	15	1,500,000
Storage Cost	100,000	5.25	525,000
Total			6,525,000

Taking the total income and costs from Table 23, Table 24 and Table 25, the profit is £29,250,000 - £14,200,000 - £6,525,000 = £8,580,000 which is consistent with GAMS-generated solution as shown in Table 16. (Note: minor discrepancy with rounding in hand-calculated answer but solution has been fully validated within Excel.)

3.3.1.2. Case 4 Profit Validation

Table 26 – Income for Case 4

Product [tonnes]	Price of product/tonne	Income, £
30,800	975	30,000,000

Table 27 – Cost of Transport for Case 4

Locations	Quantity, tonnes	Miles	Cost/mile/tonne	Cost, £
Collection Points - Storage Points				
Amiens - Rouen	25,000	74.4	0.7	1,300,000
Le Mans - Tours	40,000	60.9	0.7	1,710,000
Orleans - Troyes	35,000	131	0.7	3,210,000
Storage Points – Biorefineries				
Rouen - Reims	25,000	181	0.5	2,260,000
Troyes - Reims	35,000	78.7	0.5	1,380,000
Tours - Nantes	40,000	135	0.5	2,700,000
Biorefineries – Customers				
Reims - Paris	15,400	89.6	0.5	690,000
Reims - Le Mans	2,600	213	0.5	277,000
Nantes - Le Mans	12,800	114	0.5	730,000
Total				14,300,000

Table 28 – Other Costs for Case 4

Costs	Quantity, tonnes	Cost/tonne	Cost, £
Refining Cost	100,000	45	4,500,000
Biomass Cost	100,000	15	1,500,000
Storage Cost	100,000	5.25	525,000
Total			6,525,000

Taking the total income and costs from Table 26, Table 27 and Table 28, the profit is £30,000,000 - £14,300,000 - £6,525,000 = £9,180,000 which is consistent with GAMS-generated solution as shown in Table 16. (Note: minor discrepancy with rounding in hand-calculated answer but solution has been fully validated within Excel.)

3.4. Final Discussion and Analysis

It is noted from the profit hand calculations that the transportation costs greatly exceed all other costs combined for all four cases. Therefore factors that affect the transport costs may provide a significant effect to the model via utilisation rates.

The model uses distances obtained from Google maps, but a few assumptions were made than may affect the costing of the model. The distances used were from city to city, but not to the exact locations of collection points, storage points, biorefineries and customers, which may produce a small discrepancy. The distances are taken from the quickest routes, but some roads may not be suitable for large trucks, due to low bridge or small and tight turns, as Google Maps provides a focus of cars, this may require detours that could change the distances and affect the optimisation problem. France is infamous for private toll roads, it is uncertain if toll costs will provide a significant effect to the optimisation and whether drivers may attempt to take routes that could try and avoid them hence changing the distances.

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