Linear Control Systems
Master 1

Academic year of 2019-2020

LCS - HW3 - Control of blood sugar concentration

Work of Bareel Pierre-Yves, Gevaert Lou and Paquay Joachim

(s162827, s161603 and s154432)

1 Summary of project

In the previous homework, we established the state-space representation of our open-loop system. First of all, the input of our system are U(t) and m(t), the output is G(t) and our state variables are G(t), X(t) and I(t). We will analyse the evolution of the variables from 0 to 1440 minutes, all variables have thus the same domain.

		Notation	Image	Unit
Inputs	u_1	m	[0 125]	mg
	u_2	U	[0 35]	mU/min
State	x_1	G	[70 120]	mg/dl
	x_2	X	[0 0.0018]	mg/dl
	x_3	I	[0 110]	mU/ml
Output	y	G	[70 120]	mg/dl

As our system was not linear, we had to linearize it at an equilibrium point and the ABCD matrices we obtained are below :

$$A = \begin{bmatrix} -p_1 - Xe & -Ge & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -n \end{bmatrix} B = \begin{bmatrix} 0 & \frac{1}{V_G} \\ 0 & 0 \\ \frac{1}{V_I} & 0 \end{bmatrix} \tag{1}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{2}$$

We now have to go a step further in the design of our controller. First, we will add a state feedback to our system. To do so, it is considered that all the states are measurable. The aim of the state feedback is to stabilize our system. We can write the inputs as $u = -Kx + k_r r$. The A matrix will therefore become A-BK and we will have the ability to fix the values of K to have the stability that we want for our system.

2 State feedback controller = computation of K and kr

The state feedback we added imply that $u = -Kx + k_r r = -k_1 G - k_2 X - k_3 I + k_r r$ and $\dot{x} = (A - BK)x + Bk_r r$. The values of K are chosen by the dynamic performances that we want to obtain. In our case, only the entrance u that concerns the injection of insulin is controllable, we will thus avoid the other entrance m(t) by taking this matrix K :

$$K = \begin{bmatrix} k_1 & k_2 & k_3 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

To compute the **dynamic performance**, i.e. K, one has to compute

$$\begin{aligned} det(sI - A + BK) &= det \begin{bmatrix} s + p_1 + X_e & G_e & 0 \\ 0 & s + p2 & -p_3 \\ \frac{k_1}{V_G} & \frac{k_2}{V_G} & s + n + \frac{k_3}{V_G} \end{bmatrix} \\ &= s^3 + s^2(n + \frac{k_3}{V_I} + p_1 + p_2 + X_e) \\ &+ s(p_2 n + \frac{p_2 k_3}{V_I} + \frac{p_3 k_2}{V_I} + p_1 n + \frac{p_1 k_3}{V_I} + p_1 p_2 + X_e n + \frac{X_e k_3}{V_I} + X_e p_2) \\ &+ p_1 p_2 n + X_e p_2 n + \frac{1}{V_I}(p_1 p_2 k_3 + p_1 p_3 k_2 + X_e p_2 k_3 + X_e k_2 p_3 - G_e p_3 k_1) \end{aligned} \tag{4}$$

According to the theory, the values of matrix K for a second order system can be computed by equalizing the coefficients of det(sI - A + BK) with the coefficients of the polynomial here under.

$$p(s) = s^2 + 2\zeta_c\omega_c s + \omega_c^2 \tag{5}$$

However, we have a third order system so we have to find a way to "get rid" of one eigenvalue. One eigenvalue of matrix A (s = -n) is way bigger than the two others so the corresponding exponential will rapidly converge to 0 and this is the one that is not dominant for the dynamics of our system. Therefore, equation (4) is written in the form :

$$(s+n)(Is^2 + Js + K) = Is^3 + s^2(J + In) + s(K + Jn) + Kn \tag{6}$$

By resolving this transformation,isolating k_1, k_2 and k_3 and comparing the terms of the same order, one can find that

$$\begin{cases} k_1 = \dfrac{1}{G_e p_3}(k_3(p_1p_2 + X_e p_2 - np_1 - np_2 - nX_e + n^2) + k_2 p_3(p_1 + X_e - n)) \\ k_2 = \dfrac{V_I}{p_3}(\omega_c^2 - \dfrac{k_3}{V_I}(p_1 + p_2 + X_e - n) - p_1p_2 - X_e p_2) \\ k_3 = V_I(2\zeta_c\omega_c - p_1 - p_2 - X_e) \end{cases} \tag{7}$$

In order to find k_r, corresponding to the **static performance**, one simply has to resolve :

$$k_r = \frac{-1}{C(A - BK)^{-1}B} \tag{8}$$

3 Observer = computation of L

As we did for the state feedback controller, we will start by computing the matrix L defined as :

$$L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \tag{9}$$

And by computing det(sI - A + LC), we will find a 3^{rd} order polynomial which depends on the variables l_1, l_2 and l_3.

$$\begin{aligned} det(sI - A + LC) = & s^3 + s^2(p_1 + p_2 + X_e + l_1 + n) \\ & + s(p_1p_2 + X_e p_2 + l_1 p_2 + p_1 n + X_e n + l_1 n + p_2 n - G_e l_2) \\ & + np_1p_2 + nX_e p_2 + nl_1 p_2 - G_e l_2 n - G_e p_3 l_3 \end{aligned} \tag{10}$$

The characteristically equation for the observer is given by :

$$p(s) = s^2 + 2\zeta_o\omega_o s + \omega_o^2 \tag{11}$$

In the same way as the state-feedback, one could use the equation (6), (11) and the result of the equation (10) in order to obtain the terms l_1, l_2 and l_3.
It gives :

$$\begin{cases} l_1 = 2\zeta_o\omega_o - p_1 - p_2 - X_e \\ l_2 = \dfrac{\omega_o^2 - p_1p_2 - X_e p_2 - l_1 p_2}{G_e} \\ l_3 = 0 \end{cases} \tag{12}$$

4 Constraints and simulations specifications

There are several parameters that can be chosen but they have to follow a few constraints.

•The reference can vary but in a certain range of value, we considered that the optimal target was 90mg/dl but theoretically the level of glucose could vary from 70 to 120mg/dl, since we will converge to r=0, we can make r vary from -20 to +30mg/dl.

•The input signal u1 : Limitation of insulin injection : First of all the pump is not able to deliver insuline at an infinite speed, according to a manufacturer website[1], the maximum speed is u1=1500 mU/min but the patient can not receive an infinite amount of insulin. If the pump is constantly active, u1 is limited to 35 mU/min to not exceed the maximum dose.

•The input signal u2 : Meal consumption : We took realistic values for the food intake. We considered that the person would take three meals of 125 mg of glucose and one snack of 90 mg of glucose in one day. We therefore created a train of pulses having the corresponding periods and that the width of the pulses was about 30 min, the time of a lunch approximately.

•The rise time and the settling time are not very important since the patient can wait a few minutes for the insulin to kick in without risking anything

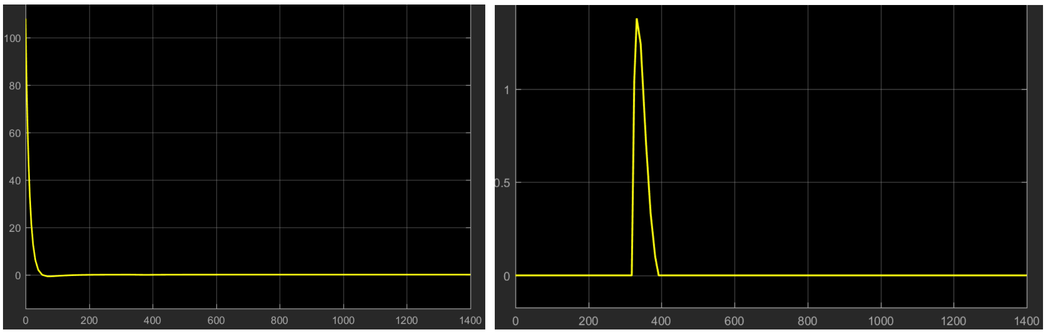
•Overshoot : We want to have a small (or nonexistent) overshoot. That is why we chose the parameters xci$_c andxci_0 equalto$0.975

5 Simulations and discussion

To do the simulations on Simulink, we implemented the *Openloop* system with the linearized equations from time 0 to 1440 minutes :

$$\begin{cases} \dot{G}(t) = -(p_1 + Xe)G(t) + X(t)G_e + \dfrac{m(t)}{V_G} \\ \dot{X}(t) = -p_2 X(t) + p_3 I(t) \\ \dot{I}(t) = -nI(t) + \dfrac{U(t)}{V_I} \end{cases} \tag{13}$$

With the initial conditions Xinitial = $[1.2Gb \quad 0 \quad 1.2Ib \quad 1.2Gb \quad 0 \quad 1.2Ib]$



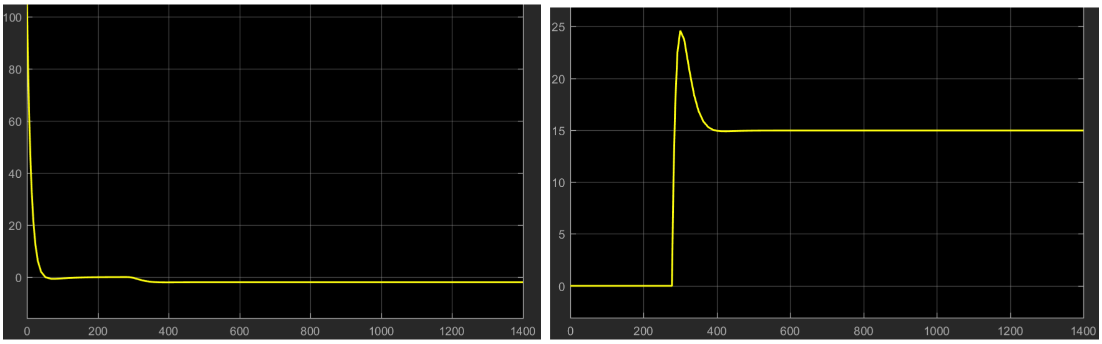
(a) Evolution of G(t) in mg/dl

(b) Evolution of U(t) in mU/min

We noticed that there is a delay and that the peak of insulin should be simultaneous with the decrease of glucose in the blood. We will see on the plot with the meals that this delay is constant and comes from the observer since it was not present before we added it.

5.1 Response to a reference variation

The reference should be put at 0 since the system is linearized in which case we have this result : If we take a lower reference e.g. r = -2, we have :



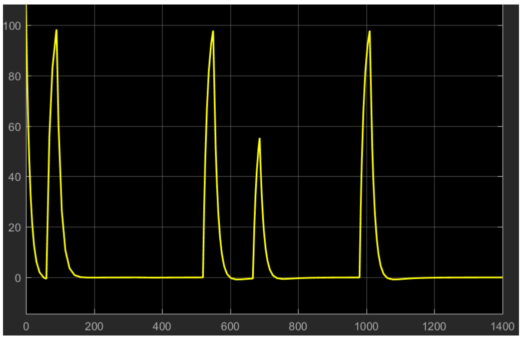
(a) Evolution of G(t) in mG/dl

(b) Evolution of U(t) in mU/min

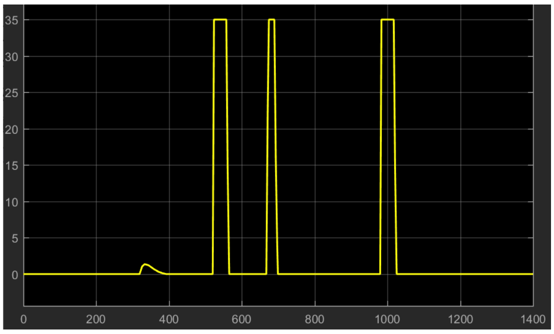
At the equilibrium, the pump is injecting 15mU/min. Indeed, if the pump was not injection insulin, the level of glucose would stabilized at 0 and not r=-2. The curve of glucose's evolution is stabilizing at -2 which is expected but the value of equilibrium of U(t) is unpredictable.

5.2 Response to a perturbation/disturbance

We added perturbations so that it looks like the person is taking meals and snacks. The entrance u_2 seen by the observer will be delayed (by 20s) compared to the entrance of the Openloop.



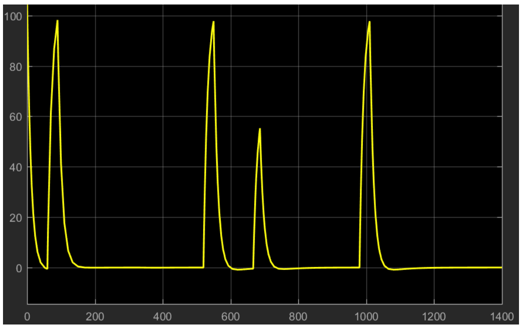
(a) Evolution of G(t) in mg/dl



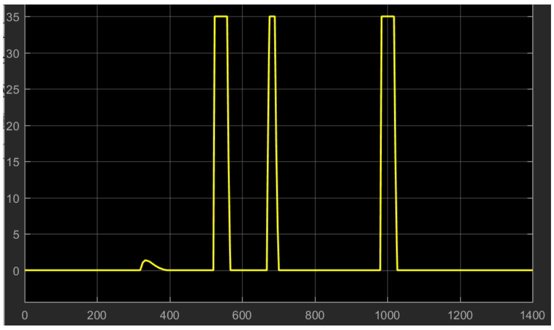
(b) Evolution of U(t) in mU/min

5.3 Response to noise

First of all we changed the entrance of the observer to put the same disturbance as the *Openloop* system but with a delay of 20 sec.

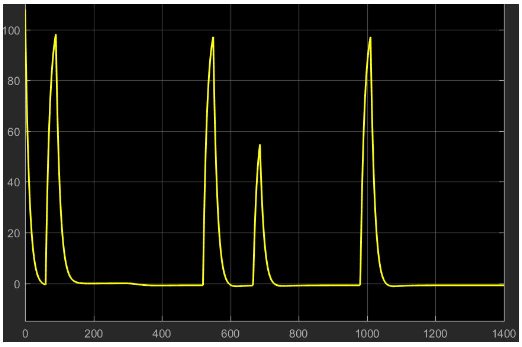


(a) Evolution of G(t) in mg/dl

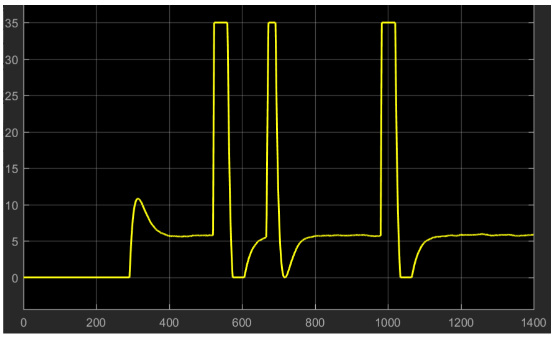


(b) Evolution of U(t) in mU/min

Then we added noise (with a mean of 5 and a variance of 1) between the output of the *Openloop* and the entrance of the *Observer* :



(a) Evolution of G(t) in mg/dl



(b) Evolution of U(t) in mU/min

Références

[1] Retrived from *https://www.medtronic-diabetes.be/*

[2] Bergman, R. N., Ider, Y. Z., Bowden, C. R., and Cobelli, C. (1979). *Quantitative estimation of insulin sensitivity. American Journal of Physiology - Endocrinology and Metabolism, 236(6) :E667–677*

[3] R N Bergman, Y Z Ider, C R Bowden, and C Cobelli (1979 Jun 01). *Quantitative estimation of insulin sensitivity. American Journal of Physiology-Endocrinology and MetabolismVolume 236, Issue 6*