EEE3090F (2023)
Practical Assignment 1: Filter Design

This practical assignment, along with Practical 1 and your tutorials, aims to cement your understanding and familiarity with the theory presented on filters in your lectures.

This assignment consists of two sections: Question 1 involves theoretical analysis of a Sallen-Key filter while Question 2 involves the development and simulation of your own filter using LTSpice.

Instructions

Show all calculations with an appropriate level of detail. You are encouraged to type these out (good practice using LaTeX) but high quality scans/photos of hand calculations are acceptable.

In Question 2 you will be asked to simulate three separate circuits. Please include all three circuits in your submitted .asc file. Make use of LTSpice "Text" tool to annotate which circuit is which.

Please use the model of the LM358 supplied to you to simulate your op-amp circuits. The model and instructions on how to import it are available on the assignment page.

Submission Instructions

Submit the following:

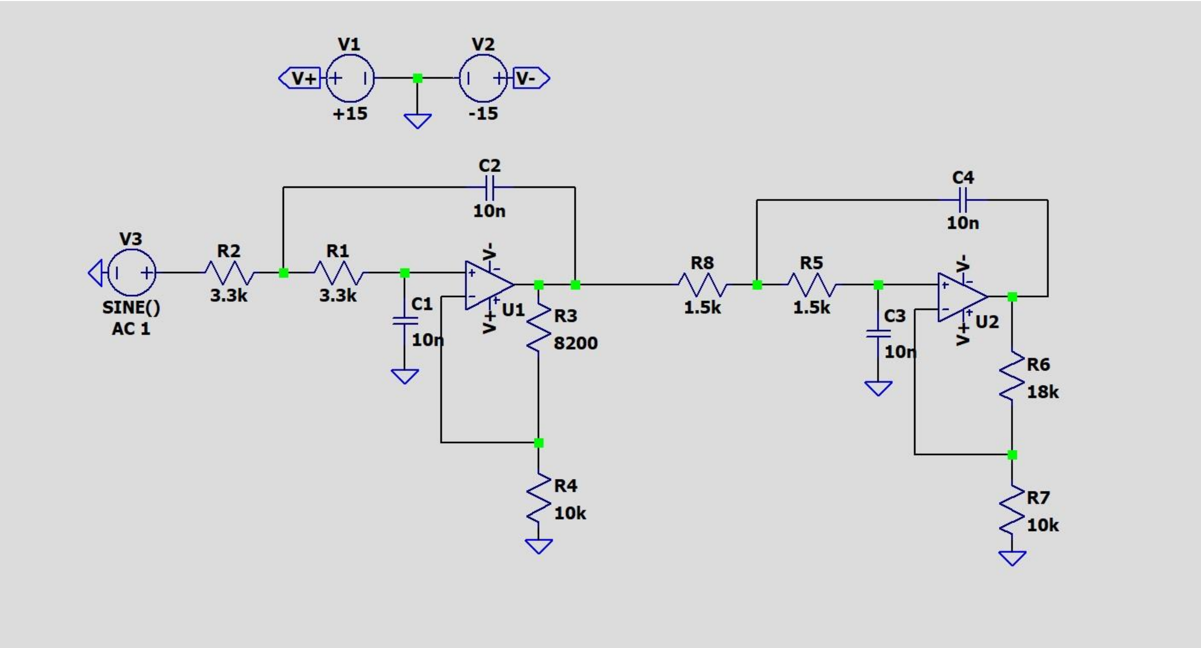
1. One PDF report containing all written and illustrated answers. Use the file naming scheme exactly as: "EEE3090F_2023_Practical_Assignment_1_<STDNUMXXX>.pdf"
2. One LTSpice simulation file (.asc) containing all the circuits you have simulated and used to generate your answers. Use the same naming scheme as your pdf

Compress all your files into a single .zip folder with the same naming scheme described above.

Ensure your report is well formatted, containing a title page, page numbers, and any other structural elements you feel necessary. Your report must at the very least display:

- Your full name and student number,
- The title of this assignment,
- The date submitted.

Question 1: Theory [14]



1.1 Describe what is apparent about this filter:

a. Is this a passive or active filter **[1]**

b. Is this a high-pass, low-pass or band-pass filter **[1]**

a) Active Filter

b) Low-pass filter

1.2 Calculate the gain (K) for each stage of the filter. Complete the following table and show your calculations below the table **[2]**

	Stage 1	Stage 2
Gain	1,82	2,8

$K_1 = 1 + \frac{R_3}{R_4} = 1 + \frac{8200}{10000}$

$= 1,82$

$K_2 = 1 + \frac{R_6}{R_7}$

$= 1 + \frac{18000}{10000}$

$= 2,8$

1.3 Calculate the passband gain (in dB) for this filter and the expected gain at the -3 dB cut-off frequency f_c **[2]**

Passband Gain $= 20\log(1,82 \times 8,2) = 14,14$

Thus the -3dB is $11,14$dB.

1.4 Calculate f_p for each filter stage. **[3]**

Stage	Calculation	Result

1	$f_p = \frac{1}{2\pi RC}$ $= \frac{1}{2\pi \cdot 3300 \cdot 10\times10^{-9}}$	4822,88 Hz
2	$f_p = \frac{1}{2\pi RC}$ $= \frac{1}{2\pi \cdot 1500 \cdot 10\times10}$	10 610,33 Hz

1.5 Based on your calculations, identify the filter response type. Explain how you came to your conclusion

[2]

Chebyshev (2.0 dB) as the values of K_1 & K_2 are closest to this filter response type.

1.6 Calculate the filter's -3 dB cut-off frequency f_c. Explain the rationale behind your conclusion

[3]

$$f_p = f_n f_c \quad \therefore \quad f_c = \frac{f_p}{f_n}$$

Stage 1: $f_{c1} = \frac{f_{p1}}{f_{n1}} = \frac{4822,88}{0,471} = 10,239,66 \text{ Hz}$
$\approx 10,2 \text{ k Hz}$

Stage 2: $f_{c2} = \frac{f_{p2}}{f_{n2}} = \frac{10\,610,33}{0,964} = 11\,006,57 \text{ Hz}$
$\approx 11 \text{ kHz}$

Question 2: Filter Design

You have a friend who is a guitarist. They know that you are an aspiring electronics engineer so they have challenged you to build an analog effect pedal for their guitar.

You decide to try limit the guitar's audio to the frequency range 5kHz-15kHz. From what you know of audio signals, you decide the following specifications are reasonable:

- Attenuate frequencies below 3kHz by more than 25dB
- Attenuate frequencies above 20kHz by more than 5dB

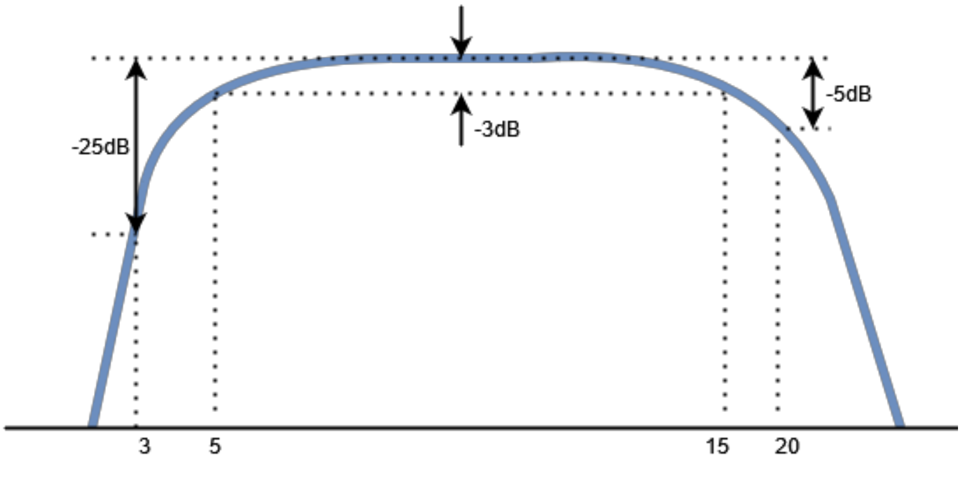


Figure 1: Magnitude Frequency Response Showing Requirements for Designed Filter

You have decided that a reasonable approach to achieving this filter is to cascade a low-pass and a high-pass filter.

2.1 Low Pass Filter: [12]

You have chosen to design a Butterworth low-pass filter with a -3dB cut-off frequency of 15kHz and a roll off that achieves at least -5dB of attenuation for frequencies more than 20khz:

2.1.1

Calculate the minimum order of the low pass filter that will meet the specifications.

[2]

Hint: the closed form expression of a Butterworth low-pass filter is given by

$$H(j\omega) = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2n}}}$$

Q2.1) Low Pass Filter

2.1.1) Now : We have $-5 = 20\log(H(j\omega))$

$$\therefore \quad -\frac{5}{20} = \log(H(j\omega))$$

$$\therefore \quad 10^{-5/20} = H(j\omega)$$

$$\therefore \quad H(j\omega) = 0{,}5623$$

$$H(j\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{20}{15}\right)^{2n}}}$$

If $n = 1$ then $H(j\omega) = 0{,}6$

If $n = 2$ then $H(j\omega) = 0{,}49$

Thus, it is at minimum, a 2nd Order Filter.

2.1.2

Design and draw the circuit for your low-pass filter. Limit your choice of components to those in the E12 series. Tabulate your component values in a table of the following format. You will need to add additional rows to the table for each component calculated. Make sure to include your full working below the table

[4]

Component Label	Value	Unit
R1	1056	Ω
R2	1056	Ω
C1	10	nF
C2	10	nF
Ri	10 000	Ω
Rf	5 870	Ω

2.1.2) 2nd Order Filter :

Repetition 1 : Choose $C = 10nF$ $(F_c = 15k)$

$$R = \frac{1}{2\pi F_c \cdot C} = \frac{1}{2\pi \cdot 15k \cdot 10nF} = 1061\ \Omega$$

$$E_{12}: \quad 1061 \approx 1056\Omega = \overbrace{1000\Omega + 56\Omega}^{E12\ resistors}$$

Repetition 2 : Choose $R = 1056\ \Omega$

$$C = \frac{1}{2\pi \cdot F_c \cdot 1056} = 1{,}005 \times 10^{-8}\ F$$
$$\approx 10nF$$

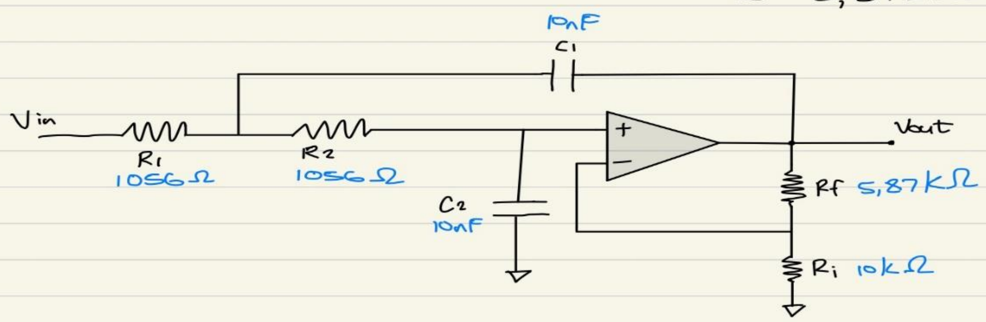
Voltage Gain : $K = 1 + \frac{Rf}{Ri}$ (Non-inverting gain)

Butterworth K is 1,586

$\therefore$ If $Ri = 10k$

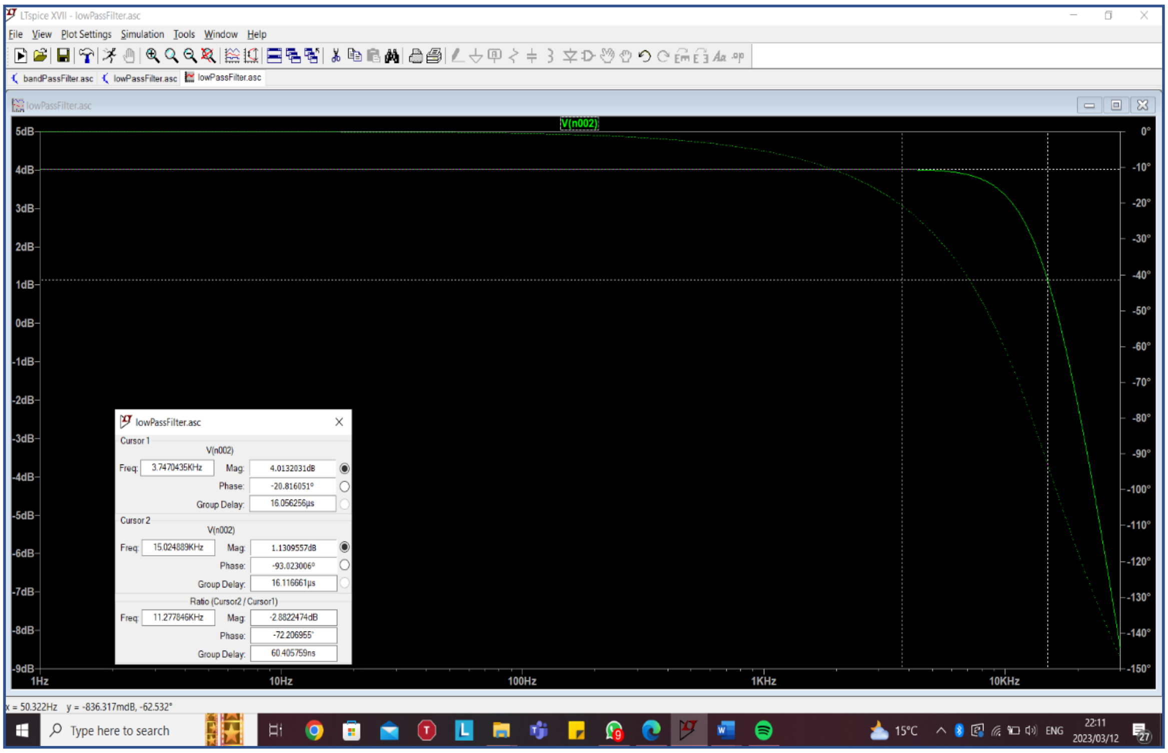
then $Rf = (1{,}586 - 1)\ Ri$

$$\therefore Rf = 5860\Omega \approx \overbrace{5{,}6k\Omega + 270\ \Omega}^{E12}$$
$$\approx 5{,}87\ k\Omega$$



2.1.3

Simulate your circuit in LT Spice and generate the filter’s frequency response. Include a screenshot in your report. Ensure your plot is formatted to display useful information. It is recommended to draw cursor position at relevant points on the plot **[3]**

Frequency Response


2.1.4

Compare the performance of the simulated filter to the specifications. In your discussion make reference to the passband, -3dB point, and roll-off. **[3]**

We know that our voltage gain $K = 1,586$. Then $A_v(dB) = 20\log(K) = 4\,dB$, thus our cutoff freq (15K Hz) should be when we are a 1dB. We have a 2nd order filter thus expect a -20dB/decade rolloff.

From the graph we indeed see that our passband gain is at 4dB, -3dB pt being at $f_c = 5kHz$ and 1dB on Y-axis. We notice the -20dB/decade roll-off.

2.2 High Pass Filter: [12]

You have chosen to design a Butterworth high-pass filter with a -3dB cut-off frequency of 5kHz and a roll off that achieves at least -25dB of attenuation for frequencies less than 3kHz:

2.2.1 Calculate the minimum order of the high pass filter that will meet the specifications. **[2]**

Q2.2.1) $-25 = 20\log(H(j\omega))$

$\therefore -\frac{25}{20} = \log(H(j\omega))$

$\therefore 10^{-25/20} = H(j\omega)$

$\therefore H(j\omega) = 0,0582$

$$H(j\omega) = \frac{\left(\frac{\omega}{\omega_c}\right)^n}{\sqrt{\left(1 + \left(\frac{\omega}{\omega_c}\right)^{2n}\right)}}$$ Sub in for n

If $n=1$ then $H(j\omega) = 0,514$

If $n=2$ then $H(j\omega) = 0,514$

If $n=1$ then $H(j\omega) = 0,514$

If $n=1$ then $H(j\omega) = 0,514$

If $n=1$ then $H(j\omega) = 0,514$

If $n=1$ then $H(j\omega) = 0,514$

$\therefore$ It is at minimum an order 6 filter

2.2.2) Design and draw the circuit for your high-pass filter, showing all calculation steps. Limit your choice of resistors and capacitors to those in the E12 series. Tabulate your component values in a table of the following format. You will need to add additional rows to the table for each component calculated.

Make sure to include your full working below the table **[4]**

Component Label	**Value**	**Unit**
R1	**1180**	**Ω**
R2	**1180**	**Ω**
R3	**1180**	**Ω**
R4	**1180**	**Ω**
R5	**1180**	**Ω**
R6	**1180**	**Ω**
Ri (Stage 1)	**10 000**	**Ω**
Rf (Stage 1)	**680**	**Ω**
Ri (Stage 2)	**10 000**	**Ω**
Rf (Stage 2)	**5870**	**Ω**
Ri (Stage 3)	**10 000**	**Ω**
Rf (Stage 3)	**14 820**	**Ω**
C1	**27x10-9**	**F**
C2	**27x10-9**	**F**
C3	**27x10-9**	**F**
C4	**27x10-9**	**F**
C5	**27x10-9**	**F**
C6	**27x10-9**	**F**

2.2.2) 6th Order Filter ($f_c = 5kHz$)

Repetition 1: Choose $C = 27nF$

$$R = \frac{1}{2\pi f_c \cdot C} = \frac{1}{2\pi \cdot 5k \cdot 27\times10^{-9}} = 1179\,\Omega$$

E_{12}: $1179\,\Omega \approx 1180\,\Omega = \underbrace{1\,k\Omega + 180\,\Omega}_{E12}$

Repetition 2: Choose $R = 1180\,\Omega$

$$\therefore \quad C = \frac{1}{2\pi f_c \cdot R} = \frac{1}{2\pi \cdot 5k \cdot 1180} = 2,7\times10^{-8}\,F \approx 27\,nF$$

Voltage Gain: $K = 1 + \frac{R_f}{R_i}$ (Non-inverting gain)

[Stage 1] Butterworth $K = 1,068$

If $R_i = 10k\Omega$, then $R_f = (1,068 - 1)\,R_i$

$\therefore R_f = \overbrace{680\,\Omega}^{E12}$

[Stage 2] Butterworth $K = 1,586$

If $R_i = 10k\Omega$, then $R_f = (1,586 - 1)\,R_i$

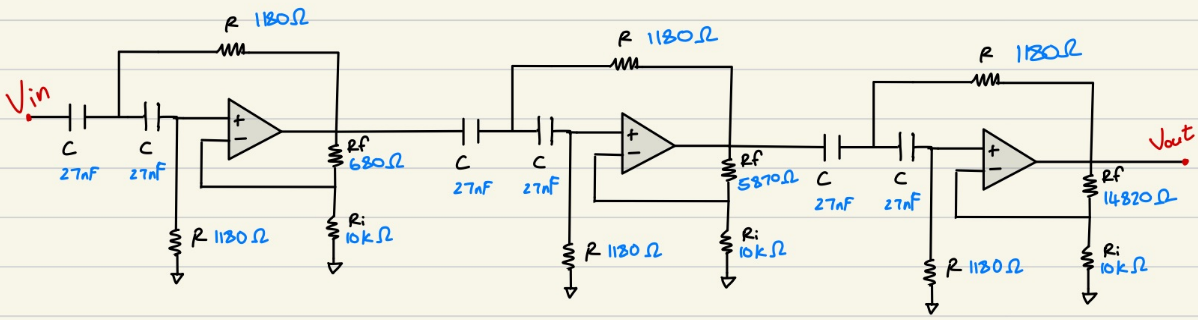
$\therefore R_f = 5860\,\Omega \approx \overbrace{5,6k\Omega + 270\,\Omega}^{E12} \approx 5,87k\Omega$

[Stage 3] Butterworth $K = 2,483$

If $R_i = 10k\Omega$, then $R_f = (2,483 - 1)\,R_i$

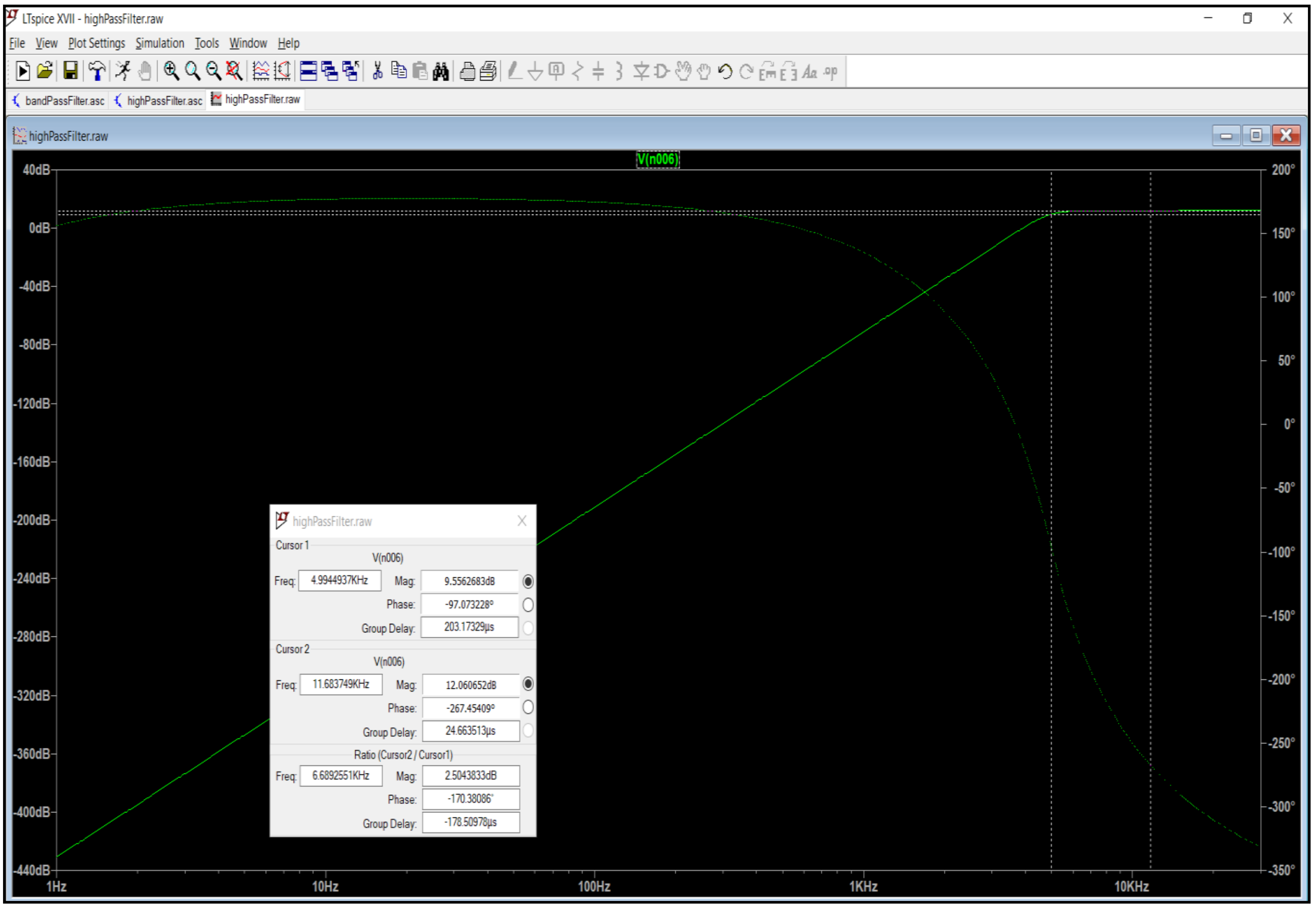
$\therefore R_f = 14830 \approx \overbrace{12k\Omega + 2,7k\Omega + 120\,\Omega}^{E12} = 14820\,\Omega$

High Pass Circuit:



2.2.3

Simulate your circuit in LT Spice and generate the filter's frequency response. Include a screenshot in your report. Ensure your plot is formatted to display useful information **[3]**



2.2.4

Compare the performance of the simulated filter to the specifications. In your discussion, make reference to the passband, -3dB point, and roll-off. **[3]**

We have gains: $K_1 = 1{,}068$ $\quad K_2 = 1{,}586$ $\quad K_3 = 2{,}483$

Passband gain: $A_v(dB) = 20\log(K_1 \cdot K_2 \cdot K_3)$

$= 12{,}48\,dB$

Thus $-3dB$ pt at $f_c = 5kHz$; $9{,}48dB$

It is a 6th order filter so thus $-60dB$/decade rolloff.

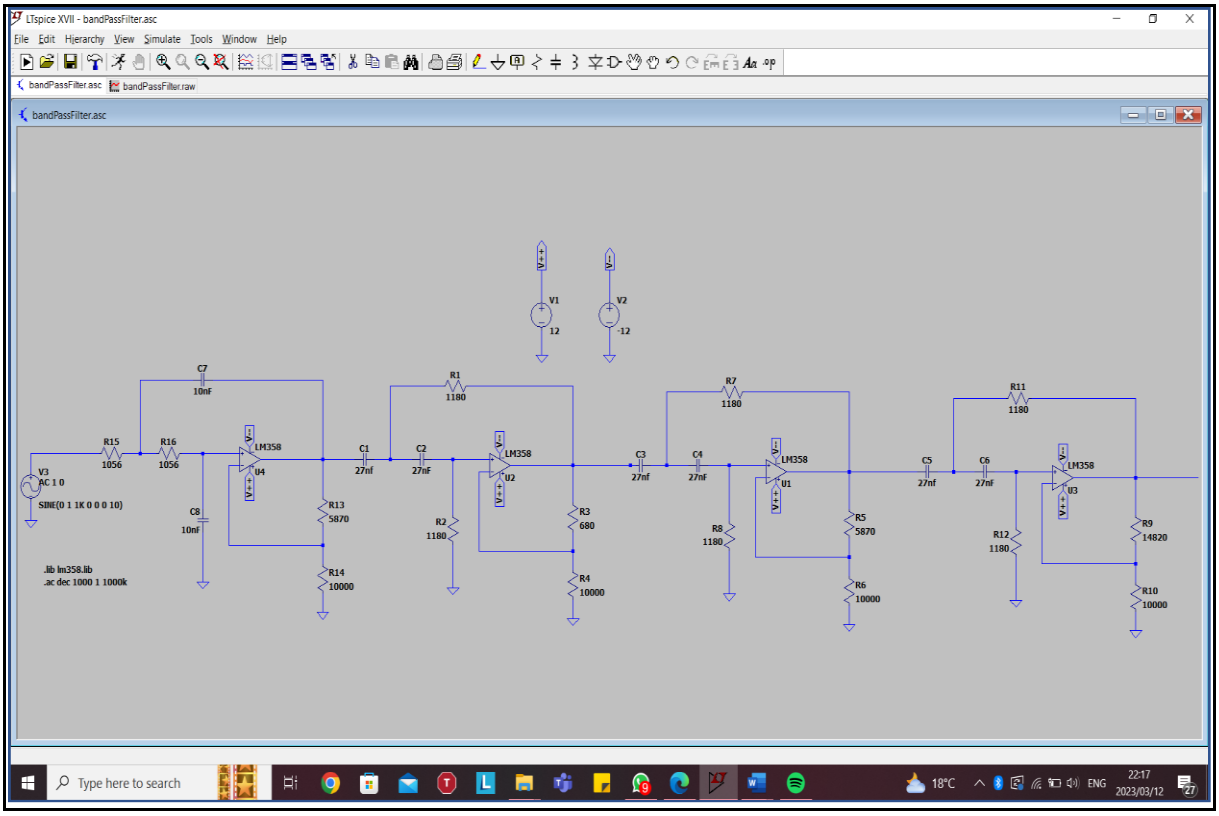
From the graph we see that the passband gain is approx. the same at 12,06dB, with the -3dB pt at 4,99 kHz and 9,55dB, with a steep rolloff rate to match the 6th order filter.

2.3 Interconnection [13]

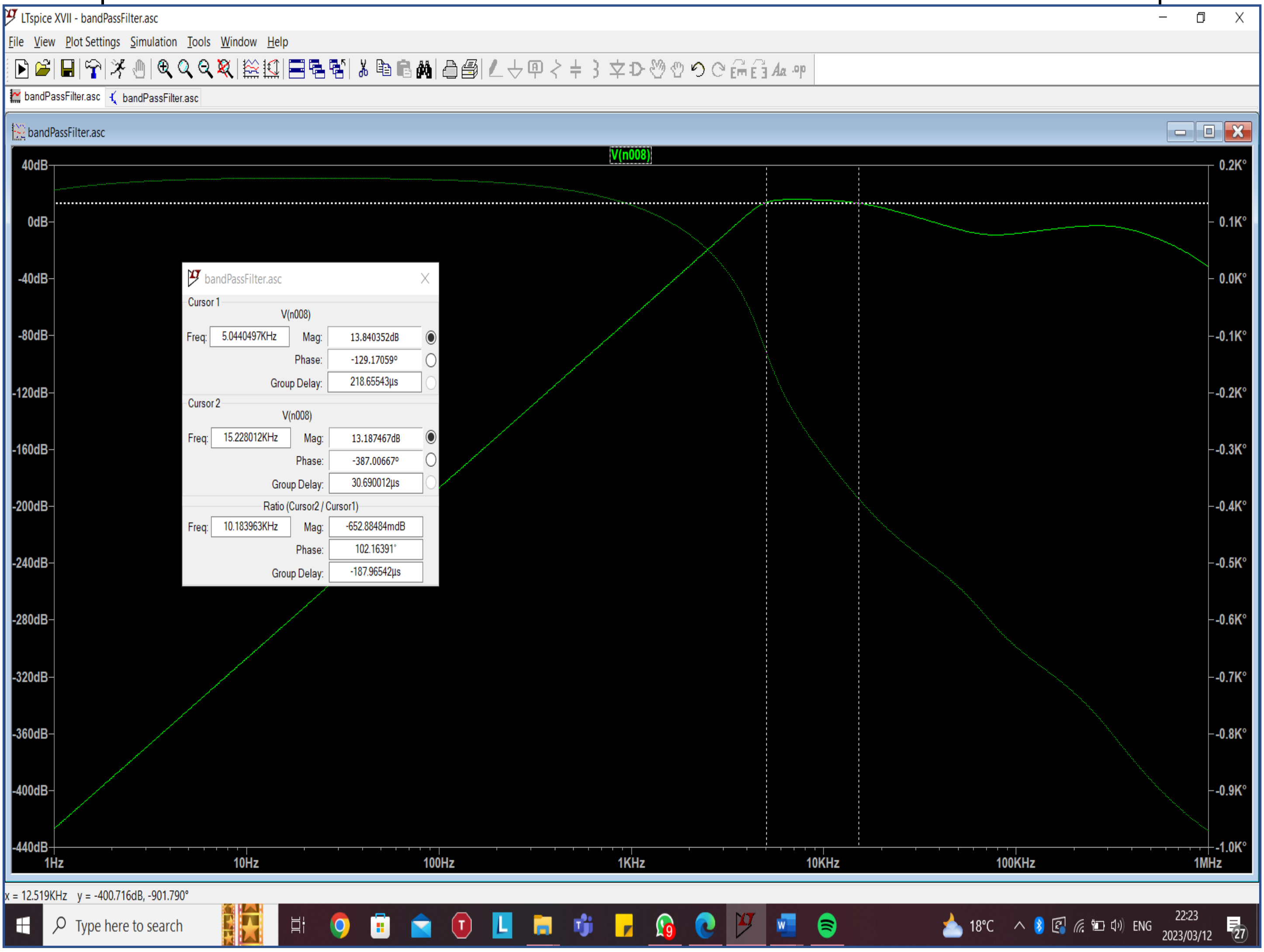
In LTSpice, cascade your low-pass and high-pass filter designs to realise the band-pass filter.

2.3.1

Use an AC analysis to simulate the frequency response of your circuit. Include a screenshot of your circuit as well as your generated magnitude and phase plots. **[4]**



Frequency Response Analysis



2.3.2

Using the simulated results, measure and state the following parameters of the filter

a. The approximate passband gain of the filters. **[1]**

The approx. passband gain is around 15,83 dB (at 6,33 kHz)

b. The filter's actual -3dB frequencies and bandwidth **[3]**

$f_{c1} = 15{,}84$ kHz

$f_{c2} = 4{,}85$ kHz

$BW = f_{c1} - f_{c2} = 15{,}84 - 4{,}85 = 11$ kHz

c. The roll-off rate in each stop-band. **[3]**

Rolloff rate before passband: 20dB/decade

Rolloff rate after passband: -40dB/decade

2.3.3

Analyse the bode plot produced above. Is it as expected? Describe any discrepancies and give explanations as to why these discrepancies exist. **[2]**

The bode plot above only has a brief passband, which is not very pronounced, and this mismatched discrepency is due to us cascading a 2nd order filter with a 6th order filter. If we had used the same order of filters for both the HPF and LPF it would be more balanced and one filter would not dominate the frequency response.

Bonus Question: Alternative BPF Filter Topologies

This question is not for marks and should not be submitted. This being said, completing it will be a very valuable exercise for your own understanding.

Read through the slides on designing band-pass filters.

1) Design a 2^{nd} order Sallen-Key BPF around the peak frequency of 10kHz. Experiment with different Q values to observe how it affects the filter's performance. Simulate your circuits in LTSpice and compare the filter's calculated parameters with simulation results.
2) Use the course notes and the notes here, design a State Variable filter that attempts to meet the system requirements given in question 2. Experiment with different Q values to observe how it affects the filter's performance. Simulate your circuits in LTSpice and compare the filter's calculated parameters with simulation results.