Diffusion models made EZ

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Goals of cognitive models

Cognitive model virtues

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  - Ideally, describing a parameterized process whose parameters have psychological meaning
- Can be used as a lens on the data
  - Focusing on the interesting aspects
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- Transform the data (with a complex distribution) into one or more interpretable quantities

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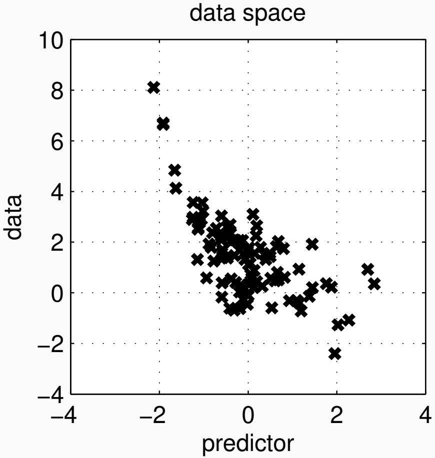
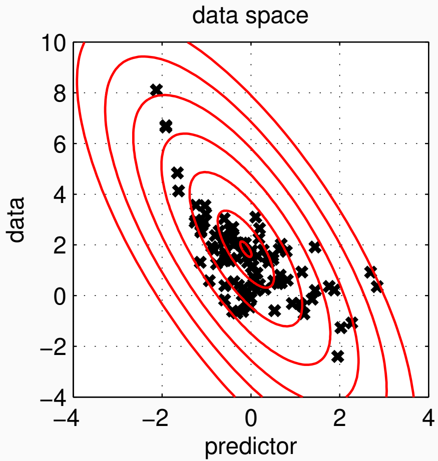


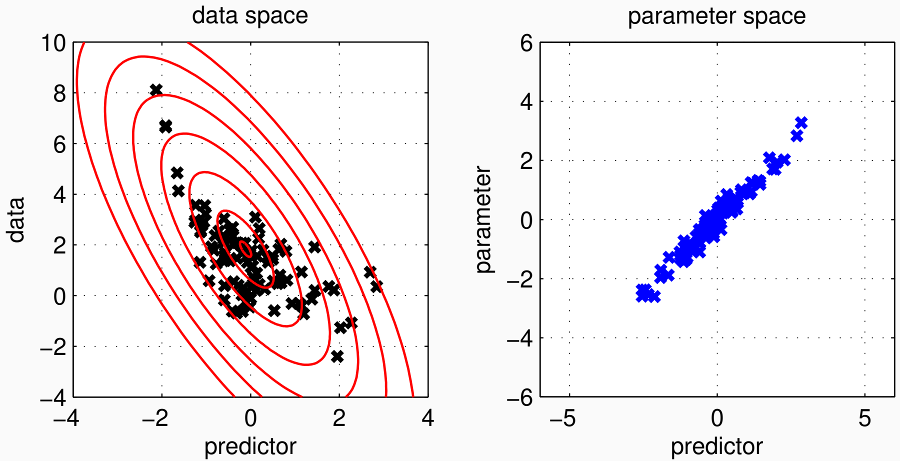
Dual spaces interpretation of cognitive modeling

Often, we do our familiar analyses in this data space.



Dual spaces interpretation of cognitive modeling

But instead, we can map the data into a space of interpretable parameters.



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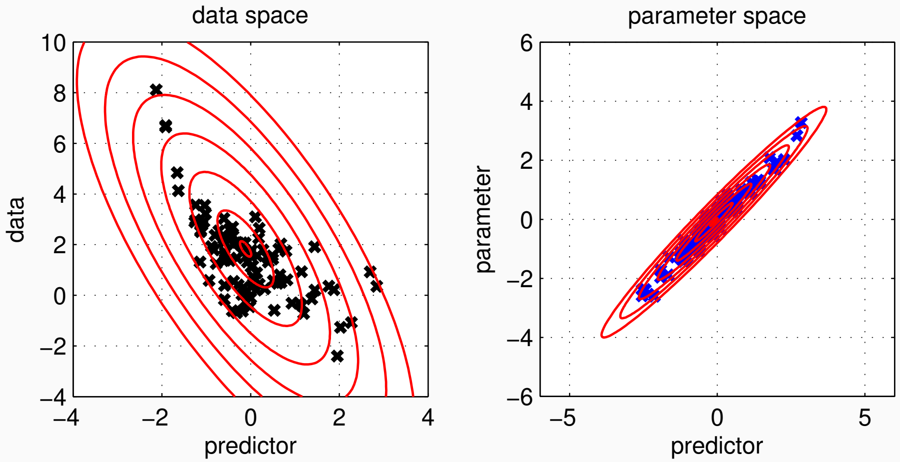
... and do our familiar analyses in that space for added clarity.



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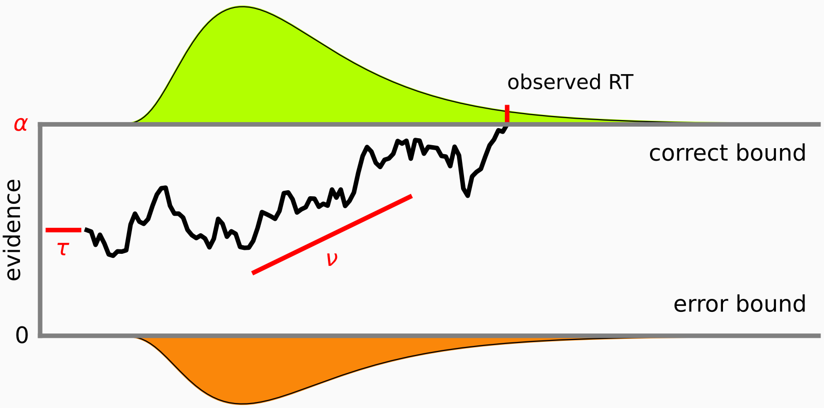

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This type of model allows us to transform complex multidimensional data into interpretable low-dimensional parameter sets.

Drift diffusion – example

Consider these data from a study by Ratcliff and Rouder where participants had to judge the brightness of pixel arrays as "high" or "low":

- 33 stimulus configuration levels controlling the proportion of black vs. white pixels.
- 2 instruction conditions: Speed vs. accuracy.

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Use a nonlinear regression with effects of instruction X_i and brightness Z_s on $\nu_{i,s}$.

$$\nu_{i,s}^{\text{pred}} = \mu_\nu + \beta_0 \Phi(\beta_1 + \beta_2 |Z_s| + \beta_3 X_i |Z_s|) + \beta_4 X_i$$

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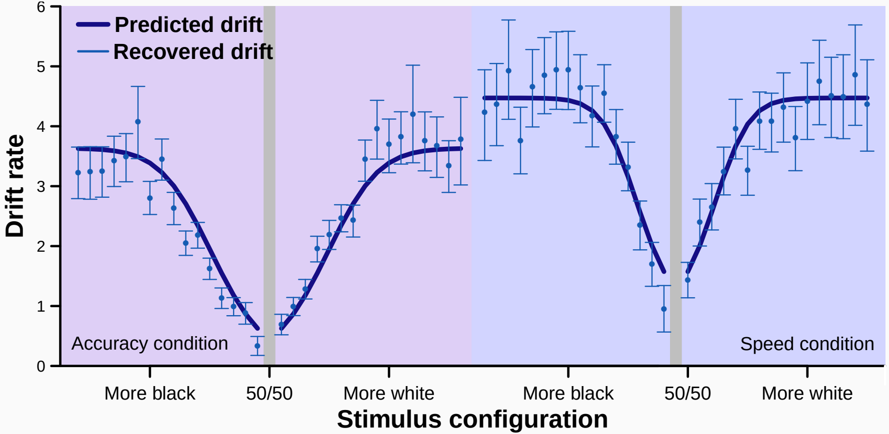

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- The likelihood of the DDM is quite complicated so maximum likelihood estimation can be slow.
- People have written entire PhD dissertations on how to fit the DDM.
- Sampling from the DDM is not so difficult but a little time consuming.

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- EZ diffusion involves two sets of equations, the forward equations give the summary statistics in terms of parameters, and the inverse equations give the parameters in terms of summary statistics.

EZ diffusion – forward

Forward EZ equations:

$$R^{\text{pred}} = \frac{1}{y+1} \tag{1}$$

$$M^{\text{pred}} = \tau + \left(\frac{\alpha}{2\nu}\right)\left(\frac{1-y}{1+y}\right), \tag{2}$$

$$V^{\text{pred}} = \left(\frac{\alpha}{2\nu^3}\right)\left\{\frac{1-2\alpha\nu y-y^2}{(y+1)^2}\right\} \tag{3}$$

with $y = \exp\left(-\alpha\nu\right)$.

EZ diffusion – inverse

Inverse EZ equations:

$$
\begin{aligned}
\nu^{\text{est}} &= \operatorname{sgn}\left(R^{\text{obs}} - \frac{1}{2}\right) \sqrt[4]{\frac{L\left({R^{\text{obs}}}^2 L - R^{\text{obs}} L + R^{\text{obs}} - \frac{1}{2}\right)}{V^{\text{obs}}}} && (4) \\
\alpha^{\text{est}} &= \frac{L}{\nu^{\text{est}}} && (5) \\
\tau^{\text{est}} &= M^{\text{obs}} - \left(\frac{\alpha^{\text{est}}}{2\nu^{\text{est}}}\right) \left[\frac{1 - \exp(-\nu^{\text{est}}\alpha^{\text{est}})}{1 + \exp(-\nu^{\text{est}}\alpha^{\text{est}})}\right]. && (6)
\end{aligned}
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with $L = \log\left(\frac{R^{\text{obs}}}{1-R^{\text{obs}}}\right)$.

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We actually know how to generate "noisy," "observed" accuracy rates from "true" accuracy rates, and we know this for means and variances as well.

EZ diffusion – sampling distributions

If N observations are drawn from a diffusion model whose accuracy rate is R^{pred}, then the sampling distribution of the observed number of correct trials $T^{\text{obs}} = N \times R^{\text{obs}}$ is:

$$T^{\text{obs}} \sim \text{Binomial}\left(R^{\text{pred}}, N\right). \tag{7}$$

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If N observations are drawn from a sample whose mean and variance of the RTs are M^{pred} and V^{pred}, then the sampling distribution of the observed mean RT M^{obs} is:

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And the sampling distribution of the variance of the RTs follows this probability law:

$$V^{\text{obs}} \sim \text{Gamma}\left(\frac{N-1}{2}, \frac{2V^{\text{pred}}}{N-1}\right). \tag{9}$$

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Ideally, b is close to 0 on average, and b^2 should decrease when we increase N. Certainly, b should be 0 when we set $(R^{\text{obs}}, M^{\text{obs}}, V^{\text{obs}}) = (R^{\text{pred}}, M^{\text{pred}}, V^{\text{pred}})$.

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This is a pretty low bar, but not all models clear it!

EZ diffusion – assignment

For your final project, you will program a simulate-and-recover study for EZ diffusion.

- Use the equations in these slides to implement the forward equations, inverse equations, and sampling
- Use all the best practices covered in class, including:
  - Unit testing, including the expected value of b if there is no noise (slide 16)
  - A folder structure like the one recommended in class, and meaningful names for classes, methods, variables, etc. with at least 3 subtle Star Trek references for flavor
  - Readable, maintainable code that works in the class container
  - Appropriate acknowledgment of outside sources

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