Introduction to the Bayes Factor

Joachim Vandekerckhove

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What is the Bayes Factor?

- The Bayes factor (B) is a measure for comparing two models M_1 and M_2.
- It quantifies how much more likely the data are under one model compared to the other:

$$B = \frac{P(D \mid M_1)}{P(D \mid M_2)}$$

- If $B > 1$, evidence favors M_1; if $B < 1$, evidence favors M_2.

Example: Binomial Model

- Suppose we have data $D = k$ successes in n trials, modeled as:

$$k \sim \text{Binomial}(n, \theta)$$

- We compare two different prior beliefs about θ:

$$M_1 : \quad \theta \sim \text{Uniform}(0, 1)$$
$$M_2 : \quad \theta \sim \text{Uniform}(0.2, 0.8)$$

Computing the Marginal Likelihood

- The marginal likelihood (evidence) for each model is:

$$P(D \mid M) = \int_0^1 P(D \mid \theta) P(\theta \mid M) d\theta$$

- Evaluating this integral for both priors gives the model *evidence*.

Computing the Bayes Factor

- Using the uniform priors in M_1 and M_2:

$$\begin{aligned} P(D \mid M_1) &= \int_0^1 P(D \mid \theta) P(\theta \mid M_1) d\theta \\ &= \int_0^1 P(D \mid \theta) d\theta \\ P(D \mid M_2) &= \int_0^1 P(D \mid \theta) P(\theta \mid M_2) d\theta \\ &= 0.6 \int_{0.2}^{0.8} P(D \mid \theta) d\theta \end{aligned}$$

- The Bayes factor is:

$$B = \frac{P(D \mid M_1)}{P(D \mid M_2)}$$

Interpreting the Bayes Factor

- $B > 1$: Evidence favors M_1 (the broader uniform prior model).
- $B < 1$: Evidence favors M_2 (the restricted uniform prior model).
- Strength of evidence (Jeffreys' scale):

B	Interpretation
1-3	Weak evidence
3-10	Moderate evidence
10+	Strong evidence

Conclusion

- The Bayes factor provides a principled way to compare models.
- Prior choices influence results but can be assessed quantitatively.
- Useful for hypothesis testing and model selection in Bayesian inference.

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