Multinomial Processing Tree with JAGS

Joachim Vandekerckhove, Michael D. Lee

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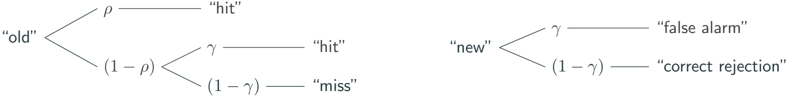

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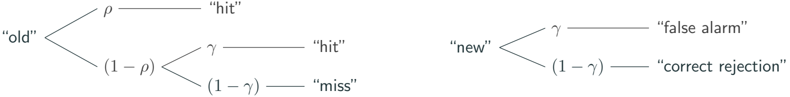

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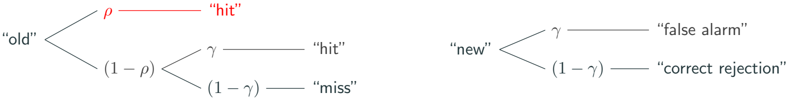

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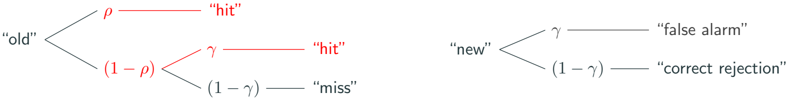

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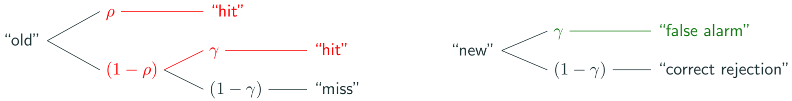

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One-High Threshold Model

The remembering parameter ρ and the old-guessing parameter γ can both take any value with equal likelihood:

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Those rates tell us how often old items lead to hits (the data are k^{h} hits out of n_o old items) and how often new items lead to false alarms (k^{f} false alarms out of n_n new items):

$$\begin{aligned} k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\ k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right) \end{aligned}$$

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Amyloid Status	Hits	False Alarms
negative	13	0
positive	8	4
negative	12	1
negative	14	0
positive	9	4
...	...	...

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positive	14	0
positive	14	1
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\gamma &\sim \text{uniform}\,(0,1) \\
\theta^{\text{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\text{f}} &= \gamma \\
&\quad \forall p \in (1,\ldots,P) \\
k_p^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\
k_p^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right)
\end{aligned}
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```
model{
  rho   ~ dunif(0, 1)
  gamma ~ dunif(0, 1)
  thetaHit = rho + (1-rho)*gamma
  thetaFA  = gamma
  for (p in 1:nPeople){
    hit[p] ~ dbin(thetaHit, nOld)
    fa[p]  ~ dbin(thetaFA , nNew)
  }
}
```

Amyloid Positive Inferences

Patients remember around 60-70% of the items, and guess “old” 5-10% of the time when they do not remember

	Posterior			95% Cred. Int.	
Parameter	Mean	Median	SD	Lower	Upper
gamma	0.075	0.074	0.012	0.053	0.100
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"Convergence of the MCMC procedure was good, with all $\hat{R} < 1.01$."

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