Multinomial Processing Tree with JAGS

Joachim Vandekerckhove, Michael D. Lee

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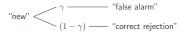
Probability of a correct rejection = correct rejection rate = $1 - \theta^{\rm f}$

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The parameters ρ and γ together determine the hit rate $\theta^{\rm h}$ and false alarm rate $\theta^{\rm f}$

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Those rates tell us how often old items lead to hits (the data are $k^{\rm h}$ hits out of n_o old items) and how often new items lead to false alarms ($k^{\rm f}$ false alarms out of n_n new items):

$$k^{\rm h} \sim {\rm binomial}\left(\theta^{\rm h}, n_o\right)$$

 $k^{\rm f} \sim {\rm binomial}\left(\theta^{\rm f}, n_n\right)$

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\begin{array}{lll} \rho & \sim & \mathrm{uniform} \left( 0, 1 \right) \\ \gamma & \sim & \mathrm{uniform} \left( 0, 1 \right) \\ \theta^{\mathrm{h}} & = & \rho + \left( 1 - \rho \right) \gamma \\ \theta^{\mathrm{f}} & = & \gamma \\ k^{\mathrm{h}} & \sim & \mathrm{binomial} \left( \theta^{\mathrm{h}}, n_o \right) \\ k^{\mathrm{f}} & \sim & \mathrm{binomial} \left( \theta^{\mathrm{f}}, n_n \right) \end{array}
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			Λ
ρ	\sim	uniform $(0,1)$	_
γ	\sim	$\mathrm{uniform}(0,1)$	
$\theta^{\rm h}$	=	$\rho + (1 - \rho) \gamma$	
$ heta^{ m f}$	=	γ	
$k^{\rm h}$	\sim	binomial $(\theta^{\rm h}, n_o)$	
k^{f}	\sim	binomial (θ^{f}, n_n)	

Amyloid Status	Hits	False Alarms
negative	13	0
positive	8	4
negative	12	1
negative	14	0
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Amyloid Status	Hits	False Alarms
positive	8	4
positive	9	4
positive	14	0
positive	14	1
positive	13	2

```
\begin{array}{lll} & \operatorname{model}\{ \\ \rho & \sim & \operatorname{uniform}\left(0,1\right) \\ \gamma & \sim & \operatorname{uniform}\left(0,1\right) \\ \theta^{\mathrm{h}} & = & \rho + (1-\rho)\gamma \\ \theta^{\mathrm{f}} & = & \gamma \\ & & \forall p \in (1,\dots,P) \\ k_p^{\mathrm{h}} & \sim & \operatorname{binomial}\left(\theta^{\mathrm{h}}, n_o\right) \\ k_p^{\mathrm{f}} & \sim & \operatorname{binomial}\left(\theta^{\mathrm{f}}, n_n\right) \\ k_p^{\mathrm{f}} & \sim & \operatorname{binomial}\left(\theta^{\mathrm{f}}, n_n\right) \\ \end{array} \right. \left. \begin{array}{ll} \operatorname{model}\{ \\ \text{rho} & \tilde{} & \operatorname{dunif}(0, 1) \\ \\ \operatorname{dunif}(0, 1) \\ \\ \operatorname{thetaHit} & = \operatorname{rho} + (1-\operatorname{rho}) * \operatorname{gamma} \\ \\ \operatorname{thetaFA} & = \operatorname{gamma} \\ \\ \operatorname{for} & (\operatorname{p in 1:nPeople}) \{ \\ \\ \operatorname{hit}[\operatorname{p}] & \tilde{} & \operatorname{dbin}(\operatorname{thetaHit}, \operatorname{nOld}) \\ \\ \operatorname{fa[p]} & \tilde{} & \operatorname{dbin}(\operatorname{thetaFA}, \operatorname{nNew}) \\ \\ \} \\ \end{array} \right. \right\}
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Patients remember around 60-70% of the items, and guess "old" 5-10% of the time when they do not remember

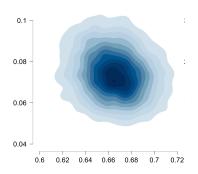
	Posterior			95% Cr	red. Int.
Parameter	Mean	Median	SD	Lower	Upper
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"Convergence of the MCMC procedure was good, with all $\hat{R} < 1.01$."

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