# Multinomial Processing Tree with JAGS

Joachim Vandekerckhove, Michael D. Lee

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# Multinomial Processing Trees

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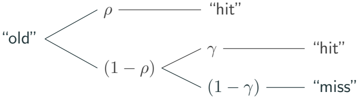
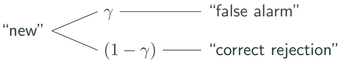
The tree representation shows the "flow" of the process



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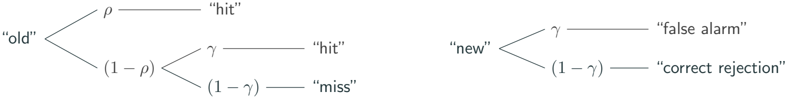


The parameters $\rho$ and $\gamma$ together determine the hit rate $\theta^{\mathrm{h}}$ and false alarm rate $\theta^{\mathrm{f}}$

$$\begin{aligned} \theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\mathrm{f}} &= \gamma \end{aligned}$$

See Matzke et al. (2018) for more like this!

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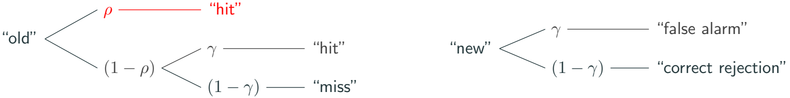


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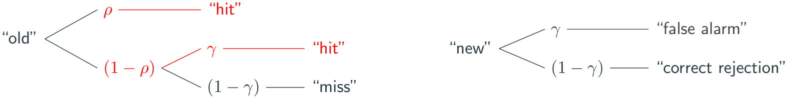


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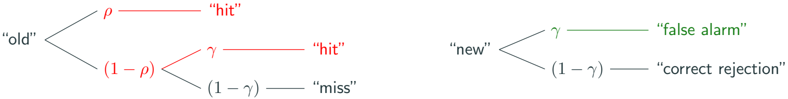


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# One-High Threshold Model

The remembering parameter $\rho$ and the old-guessing parameter $\gamma$ can both take any value with equal likelihood:

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Those rates tell us how often old items lead to hits (the data are $k^{\text{h}}$ hits out of $n_o$ old items) and how often new items lead to false alarms ($k^{\text{f}}$ false alarms out of $n_n$ new items):

$$\begin{aligned} k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\ k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right) \end{aligned}$$

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| Amyloid Status | Hits | False Alarms |
|---|---|---|
| negative | 13 | 0 |
| positive | 8 | 4 |
| negative | 12 | 1 |
| negative | 14 | 0 |
| positive | 9 | 4 |
| ... | ... | ... |

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There were 33 participants. Each saw 15 old and 15 new stimuli.

| AS | Hits | FA |
|---|---|---|
| + | 8 | 4 |
| + | 9 | 4 |
| + | 14 | 0 |
| + | 14 | 1 |
| + | 13 | 2 |
| + | 8 | 0 |
| + | 13 | 3 |
| + | 12 | 1 |
| + | 11 | 3 |
| + | 4 | 0 |
| + | 8 | 0 |
| + | 13 | 1 |
| + | 15 | 0 |
| + | 12 | 0 |
| + | 11 | 0 |
| + | 9 | 0 |
| + | 5 | 1 |

| AS | Hits | FA |
|---|---|---|
| + | 5 | 0 |
| + | 6 | 3 |
| + | 15 | 0 |
| + | 11 | 0 |
| + | 14 | 1 |
| + | 12 | 2 |
| + | 12 | 1 |
| + | 11 | 2 |
| + | 1 | 0 |
| + | 14 | 0 |
| + | 13 | 0 |
| + | 7 | 2 |
| + | 11 | 1 |
| + | 12 | 2 |
| + | 8 | 0 |
| + | 11 | 2 |

# Amyloid Positive Inferences

Patients remember around 60-70% of the items, and guess “old” 5-10% of the time when they do not remember

| | Posterior | | | 95% Cred. Int. | |
|---|---|---|---|---|---|
| Parameter | Mean | Median | SD | Lower | Upper |
| gamma | 0.075 | 0.074 | 0.012 | 0.053 | 0.100 |
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*"Convergence of the MCMC procedure was good, with all* $\hat{R} < 1.01$*."*

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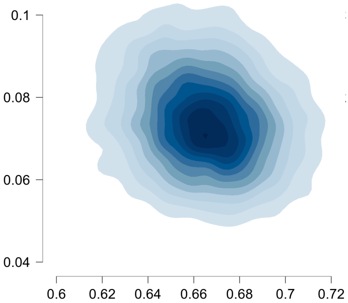

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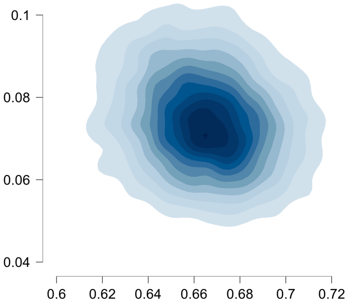


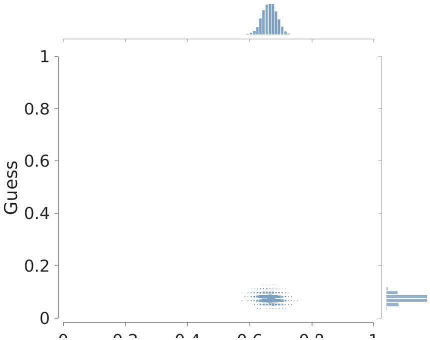

# References

Matzke, D., Boehm, U., & Vandekerckhove, J. (2018). Bayesian inference in psychology, part iii: Bayesian parameter estimation in nonstandard models. *Psychonomic Bulletin & Review*, *25*, 77–101. doi: 10.3758/s13423-017-1394-5