Conditional distribution for jointly normal Gaussian random vectors

Joaquín Rapela

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Theorem 1 *Let* $\mathbf{x}$ *and* $\mathbf{y}$ *be jointly normally-distributed random vectors with*

$$E\left\{\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix}\right\} = \begin{pmatrix}\boldsymbol{\mu}_x\\ \boldsymbol{\mu}_y\end{pmatrix}$$
$$Cov\left\{\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix}\right\} = \begin{pmatrix}\Sigma_{xx} & \Sigma_{xy}\\ \Sigma_{yx} & \Sigma_{yy}\end{pmatrix}$$

where Σ_{yy} *is assumed to be non-singular. Then the conditional distribution of* $\mathbf{x}$ *given* $\mathbf{y}$ *is normal with mean vector*

$$E\{\mathbf{x}|\mathbf{y}\} = \boldsymbol{\mu}_x + \Sigma_{xy}\Sigma_{yy}^{-1}(\mathbf{y} - \boldsymbol{\mu}_y)$$

and covariance matrix

$$Cov\{\mathbf{x}|\mathbf{y}\} = \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}$$