Random Vectors

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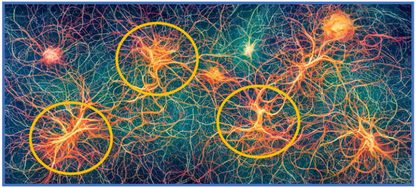
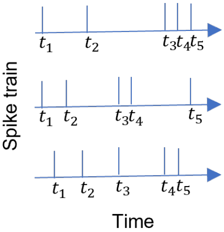
Random Vectors: When are they useful?





$$T_1 = \left(t_1^1, t_2^1, t_3^1, t_4^1, t_5^1\right)^T$$

$$T_2 = \left(t_1^2, t_2^2, t_3^2, t_4^2, t_5^2\right)^T$$

$$T_3 = \left(t_1^3, t_2^3, t_3^3, t_4^3, t_5^3\right)^T$$

Joint Distribution and Densities

$X = (X_1, ..., X_n)^T$: A random vector

$F_x(x)$: Probability *Distribution* Function (*PDF*)

$f_x(x)$: Probability *Density* function (*pdf*)

Joint Distribution and Densities

By definition, probability distribution function (PDF) is:
$F_x(x) = P[X_1 \leq x_1, ..., X_n \leq x_n]$

$x = (x_1, ..., x_n)$ we get:

$F_x(x) = P[X \leq x]$

we associate the events:
$X \leq \infty$ with the certain event, $F_x(\infty) = 1$,and

$X \leq -\infty$ with the impossible event, $F_x(-\infty) = 0$.

Joint Distribution and Densities

The probability *density* function is defined as:

$$f_x(x) = \frac{\partial^n F_x(x)}{\partial x_1 \ldots \partial x_n}$$

Equivalently we could have defined it as:

$$f_x(x) = \lim_{\Delta x_1 \to 0, \ldots, \Delta x_n \to 0} \frac{P[x_1 < X_1 \leq x_1 + \Delta x_1, \ldots, x_n < X_n \leq x_1 + \Delta x_n]}{\Delta x_1 \ldots \Delta x_n}$$

Therefore,

$$f_x(x) \Delta x_1 \ldots \Delta x_n \simeq P[x_1 < X_1 \leq x_1 + \Delta x_1, \ldots, x_n < X_n \leq x_1 + \Delta x_n]$$

Joint Distribution and Densities

pdf is defined as:

$$f_x(x) = \frac{\partial^n F_x(x)}{\partial x_1 \ldots \partial x_n}$$

if we integrate the equation, we obtain:

$$F_x(x) = \int_{-\infty}^{x_1} \ldots \int_{-\infty}^{x_n} f_x(x^{'}) dx_1^{'} \ldots dx_n^{'} = \int_{-\infty}^{x} f_x(x^{'}) dx^{'}$$

more generally:

$P[B] = \int_{x \in B} f_x(x^{'}) dx^{'}$, where $B \subset R^N$

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Joint Distribution and Densities

constraint: $(P[B] \neq 0)$

conditional *PDF*: $F_{x|B}(x|B) = P[X \leq x|B] = \frac{P[X \leq x, B]}{P[B]}$

mixture *distribution* function: $F_x(x) = \sum_{i=1}^{n} F_{x|B_i}(x|B_i)P[B_i]$

conditional *pdf*: $f_{x|B}(x|B) = \frac{\partial^n F_{x|B}(x|B)}{\partial x_1 \ldots \partial x_n}$

mixture *density* function: $f_x(x) = \sum_{i=1}^{n} f_{x|B_i}(x|B_i)P[B_i]$

mixture: a linear combination of marginals

Joint Distribution and Densities

Joint distribution of *two* random vectors:

$X = (X_1, ..., X_n).T$

$Y = (Y_1, ..., Y_M).T$

$F_{XY}(x, y) = P[X \leq x, Y \leq y]$

joint density: $f_{XY}(x, y) = \frac{\partial^{n+m} F_{XY}(x,y)}{\partial x_1 ... \partial x_n \partial y_1 ... \partial y_m}$

marginal density: $f_x(x) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} f_{XY}(x, y) dy_1 ... dy_n$

Expectation Vectors and Covariance Matrices

The expectation of the vector $X = (X_1, ..., X_n)^T$ is a vector μ whose elements are given by

$$\mu_i = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} x_i f_x(x_1, ..., x_n) dx_1 ... dx_n.$$

Properties of Covariance Matrices

This is ...

The Multidimensional Gaussian Law

This is ...

Distribution of the Sample Mean

This is ...

Conditional Gaussian distributions

This is ...

Marginal Gaussian distributions

This is ...