Bayesian linear regression exercises

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1 Decoding reaction time from stimulus contrast

Consider a visual discrimination task where a subject has to decide if two simultaneously presented gratings have the same orientation or not. The experimenter varies the difficulty of the task by changing the contrast of the gratings, x, while she measures the subject reaction times, t. Given a set of N grating contrasts and corresponding subject reaction times, $(x_1, t_1) \ldots, (x_N, t_N)$, your task is to find the posterior distribution of the weights, $\mathbf{w} = [w_0, w_1]^\intercal$, of a linear regression model relating reaction times to grating contrasts

$$\begin{aligned} y(\mathbf{w}, x_n) &= w_0 + w_1 x_n \\ t_n &= y(\mathbf{w}, x_n) + \epsilon_n \end{aligned} \tag{1}$$

(a) Sample N=20 contrast values, $(x_1, \ldots, x_N)$ from a uniform distribution in the $[0, 1]$ range. For each contrast, x_n, sample a corresponding reaction time, $t_n = y(a_0, a_1, x_n) + \epsilon_n$, with $y(a_0, a_1, x_n) = a_0 + a_1 x_n$, $a_0 = -0.3, a_1 = 0.5$ and ϵ_n a sample from a Normal distribution with mean zero and standard deviation $\sigma = 0.2$. Plot $y(a_0, a_1, x)$ as a function of x, for $x \in [0, 1]$. and reaction times, t_n, as a function of contrasts, x_n.

(b) Plot the posterior distribution, $P(\mathbf{w}|t_1, \ldots, t_n)$, corresponding to $n = 1, 2, 10, 20$ observations, using the batch posterior formula.

(c) Reproduced the figure from the lecture illustrating the online calculation of the posterior distribution. That is,

- show a contour plot of the prior distribution,
- draw 10 weights, $\mathbf{w}$, from the prior distribution, and for each weight plot the corresponding regression line; i.e., $y(\mathbf{w}, x)$ as a function of x, for $x \in [0, 1]$.
- for $= 1, 2, 10, 20$ plot
  - contour plot of the likelihood function $p(t_n|\mathbf{w})$, as a function of w_0 and w_1.
  - contour plot of the posterior $p(\mathbf{w}|t_1, \ldots, t_n)$, as a function of w_0 and w_1.
  - draw 10 weights, $\mathbf{w}$, from the above posterior distribution, and for each weight plot the corresponding regression line; i.e., $y(\mathbf{w}, x)$ as a function of x, for $x \in [0, 1]$.

(d) check if the the posterior distributions computed using the batch formula agree with those calculated using the online procedure.