Inference in the Extended Kalman Filter method

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1 Extended Kalman Filter (EKF) model

The next equations provide the EKF model (Durbin and Koopman, 2012, Chapter 10).

$$
\begin{aligned}
\mathbf{x}_{n+1} &= A_n(\mathbf{x}_n) + \mathbf{w}_n \quad \mathbf{w}_n \sim N(\mathbf{w}_n|\mathbf{0}, Q_n(\mathbf{x}_n)) \quad \mathbf{x}_n \in \mathbb{R}^M && (1)\\
\mathbf{y}_n &= C_n(\mathbf{x}_n) + \mathbf{v}_n \quad \mathbf{v}_n \sim N(\mathbf{v}_n|\mathbf{0}, R_n(\mathbf{x}_n)) \quad \mathbf{y}_n \in \mathbb{R}^N \quad n = 1 \ldots T && (2)\\
\mathbf{x}_0 &\sim N(\mathbf{w}_n|\mathbf{m}_0, V_0)
\end{aligned}
$$

where $A_n(\mathbf{x}_n) : \mathbb{R}^M \rightarrow \mathbb{R}^M$ and $C_n(\mathbf{x}_n) : \mathbb{R}^M \rightarrow \mathbb{R}^N$ are differntiable function of $\mathbf{x}_n$, $Q_n(\mathbf{x}_n) : \mathbb{R}^M \rightarrow P_M$ and $R_n(\mathbf{x}_n) : \mathbb{R}^M \rightarrow P_N$[1].

2 Extended Kalman Filter (EKF) inference algorithm

The EKF inference algorithm linearises the state and observation equations of the EKF model (Eqs. 1 and 2) and then applies Kalman filter inference to the resulting linearised model.

Define the Jacobian matrices

$$
\dot{A}_n = \left.\frac{\partial\, A_n(\mathbf{x}_n)}{\partial \mathbf{x}_n}\right|_{\mathbf{x}_n=\mathbf{x}_{n|n}} \qquad \dot{C}_n = \left.\frac{\partial\, C_n(\mathbf{x}_n)}{\partial \mathbf{x}_n}\right|_{\mathbf{x}_n=\mathbf{x}_{n|n-1}}
$$

then the EKF inference algorithm is

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[1] P_M and P_N are the spaces of positive definite matrices of size $M \times M$ and $N \times N$, respectively.

$$
\begin{aligned}
\mathbf{x}_{0|0} &= \mathbf{m}_0 && \text{init filtered mean} \\
P_{0|0} &= V_0 && \text{init filtered covariance} \\
\mathbf{x}_{n+1|n} &= A(\mathbf{x}_{n|n}) && \text{prediction mean} \\
P_{n+1|n} &= \dot{A}_n P_{n|n} \dot{A}_n^\intercal + Q(\mathbf{x}_{n|n}) && \text{prediction covariance} \\
\mathbf{y}_{n|n-1} &= C_n(\mathbf{x}_{n|n-1}) && \text{predicted observation} \\
\tilde{\mathbf{y}}_n &= {\color{red}\mathbf{y}_n} - \mathbf{y}_{n|n-1} && \text{residual} \\
S_n &= \dot{C}_n P_{n|n-1} \dot{C}_n^\intercal + R(\mathbf{x}_{n|n-1}) && \text{residual covariance} \\
\mathbf{x}_{n|n} &= \mathbf{x}_{n|n-1} + K_n \tilde{\mathbf{y}}_n && \text{filtering mean} \\
K_n &= P_{n|n-1} \dot{C}_n^\intercal S_n^{-1} && \text{Kalman gain} \\
P_{n|n} &= (I_M - K_n \dot{C}_n) P_{n|n-1} && \text{filtering covariance}
\end{aligned}
$$

References

Durbin, J. and Koopman, S. J. (2012). *Time series analysis by state space methods*, volume 38. OUP Oxford.