Maximum-likelihood estimation of the combined-controller model in Gillan et al. (2016)

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Abstract

Here I describe the implementation of a maximum-likelihood algorithm to estimate the parameters of the combined-controller model described in pages 19 and 20 of Gillan et al. (2016), and I present parameter fitted to behavioral data from a population of 253 subjects.

1 Introduction

In Section 2 I describe the estimation method, in Section **??** I mention computer implementation details, in Section **??** I show results of the application of this estimation method to behavioral data from a population of 253 subjects, and in Section **??** I comment on future work.

2 Estimation method

I estimated the parameters of the combined-controlled model described in pages 19 and 20 of Gillan et al. (2016) using maximum-likelihood (Eq. 1).

$$
\begin{aligned}
\theta_{ML} &= \arg\max_{\theta} \quad \log P(\{r_t, c_{1,t}, s_t, c_{2,t}\}_{t=1}^{T}|\theta) \\
&= \arg\max_{\theta} \sum_{t=1}^{T} \quad \log P(r_t, c_{1,t}, s_t, c_{2,t}|\theta) \\
&= \arg\max_{\theta} \sum_{t=1}^{T} \{ \log P(r_t|s_t, c_{2,t}) + \\
&\qquad\qquad \log P(c_{2,t}|s_t, \beta^{stage2}, \alpha) + \\
&\qquad\qquad \log P(s_t|c_{1,t}) + \\
&\qquad\qquad \log P(c_{1,t}|\alpha, \beta^{MB}, \beta^{MF0}, \beta^{MF1}, \beta^{stick}) \} \\
&= \arg\max_{\theta} \sum_{t=1}^{T} \{ \log P(c_{2,t}|s_t, \beta^{stage2}, \alpha) + \\
&\qquad\qquad \log P(c_{1,t}|\alpha, \beta^{MB}, \beta^{MF0}, \beta^{MF1}, \beta^{stick}) \} \qquad (1)
\end{aligned}
$$

where $\theta \in \{\alpha, \beta^{MB}, \beta^{MF0}, \beta^{MF1}, \beta^{stick}\}$ and in Eq. 1 $P(c_{2,t}|s_t, \beta^{stage2}, \alpha)$ and $P(c_{1,t}|\alpha, \beta^{MB}, \beta^{MF0}, \beta^{MF1}, \beta^{stick})$ are given in page 19 of (Gillan et al., 2016) as

$$
\begin{aligned}
P(c_{2,t}|s_t, \beta^{stage2}, \alpha) &= K_{2,t} \quad \exp\left(\beta^{stage2} Q_t^{stage2}(s_t, c_{2,t})\right) \\
P(c_{1,t}|\alpha, \beta^{MB}, \beta^{MF0}, \beta^{MF1}, \beta^{stick}) &= K_{1,t} \, (\beta^{MB} Q_t^{MB}(c_{1,t}) + \\
&\qquad \beta^{MF0} Q_t^{MF0}(c_{1,t}) + \beta^{MF1} Q_t^{MF1}(c_{1,t}) + \\
&\qquad \beta^{stick} I(c_{1,t} = c_{1,t-1}) \,)
\end{aligned}
$$

References

Gillan, C. M., Kosinski, M., Whelan, R., Phelps, E. A., and Daw, N. D. (2016). Characterizing a psychiatric symptom dimension related to deficits in goal-directed control. *Elife*, 5:e11305.