Report worksheet 4

Joaquín Rapela

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Exercise 1: Permutation test using locally-weighted log likelihood

I used a sum of the locally-weighted log likelihood function, `lwll`, function values as the test statistic. This function takes three parameters: x, θ_x, κ_x. I considered ten equally-spaced values of x_i in [0, 1]. For each x_i I took the angles $\{\theta_j\}$ of the x_j's in the neighbourhood of x_i (i.e., of the x_js such that $|x_i - x_j| < 0.005$). I passed these angles as inputs to the function `scipy.stats.vonmises.fit` to estimate θ_{x_i} and κ_{x_i}. The test statistic `test_lwll` I used is:

$$\text{lwll\_stat}(\{x_t, \theta_t\}) = \sum_{i=1}^{10} \text{lwll}\left(\{x_t, \theta_t\} | x_i, \theta_{x_i}, \kappa_{x_i}\right) \tag{1}$$

Given a dataset $\{x_i, \theta_i\}, i = 1, \ldots, N$, I employed as shuffled dataset $\{x_i, \theta_{p(i)}\}$, with $\{p(1), \ldots, p(N)\}$ a permutation of $\{1, \ldots, N\}$.

A Python implementation of `lwll_stat` and of `lwll` can be found here and a script calling these functions to perform permutation tests can be found here. To improve runtime, this script uses multiprocessing.

Figure 1 plots the non-independent dataset in the exercise statement and Figure 2 shows the results of a permutation test, with 1000 resamples, on this dataset. The independence null hypothesis was rejected with a p-value of zero.

Figure 3 plots an independent dataset, where all samples of θ were generated from a von Mises distribution with parameters $\theta = 0, \kappa = 1$. Figure 4 shows the results of a permutation test, with 1000 resamples, on this dataset. The independence null hypothesis was not rejected with p-value=0.427

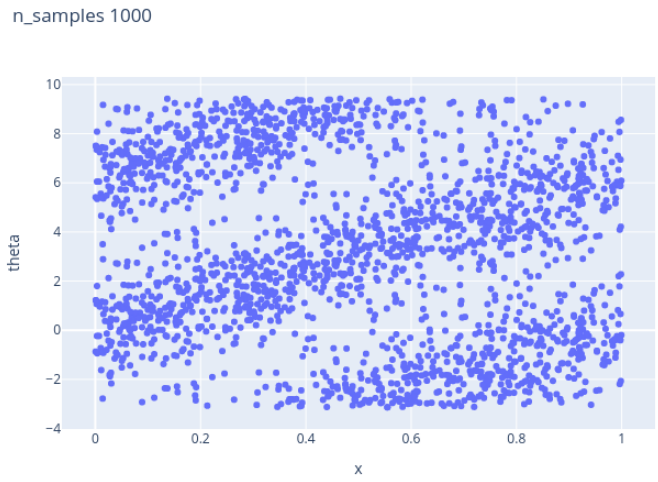

Figure 1: Non-independent dataset in the exercise statement used to calculate the permutation test in Figure 2. To generate this figure, I used this script, with its default parameters. Click on the image to view its interactive version.

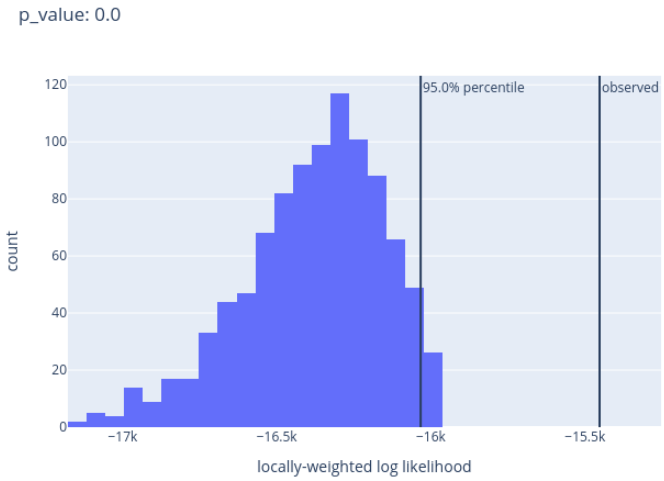

Figure 2: Result from a permutation test for the non-independent dataset in the exercise statement (Figure 1). I used 1000 resamples. The independence null hypothesis was rejected with a p-value of zero. I performed this test using this script with its default parameters. I generated this figures using this script, with its default parameters. Click on the image to view its interactive version.

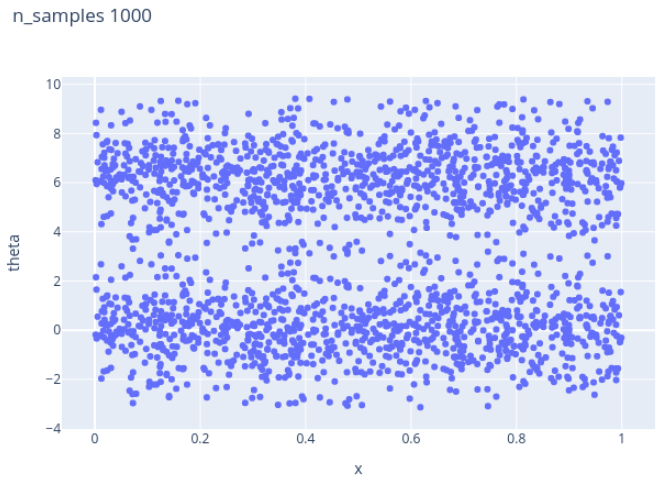

Figure 3: Independent dataset used to calculate the permutation test in Figure 4. To generate this figure, I used this script, with its default parameters, except `--loc_slope=0`. Click on the image to view its interactive version.

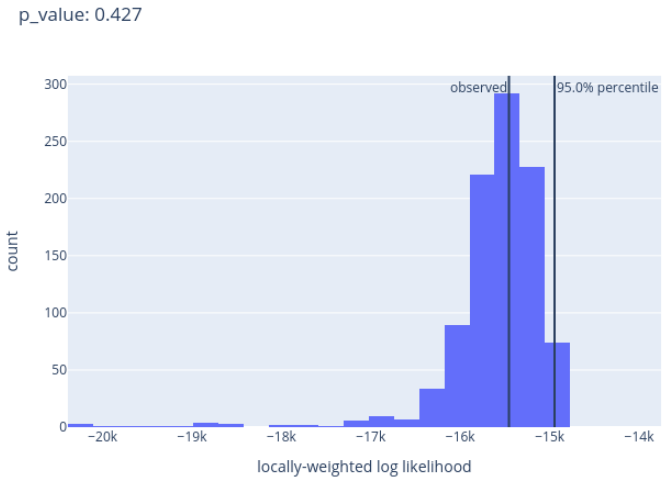

Figure 4: Result from a permutation test for the independent dataset (Figure 3). I used 1000 resamples. The independence null hypothesis was not rejected with a p-value of 0.427. I performed this test using this script, with its default parameters, except by `--loc_slope=0`. I generated this figures using this script with its default parameters, except by `--loc_slope=0`. Click on the image to view its interactive version.