Circular Statistics

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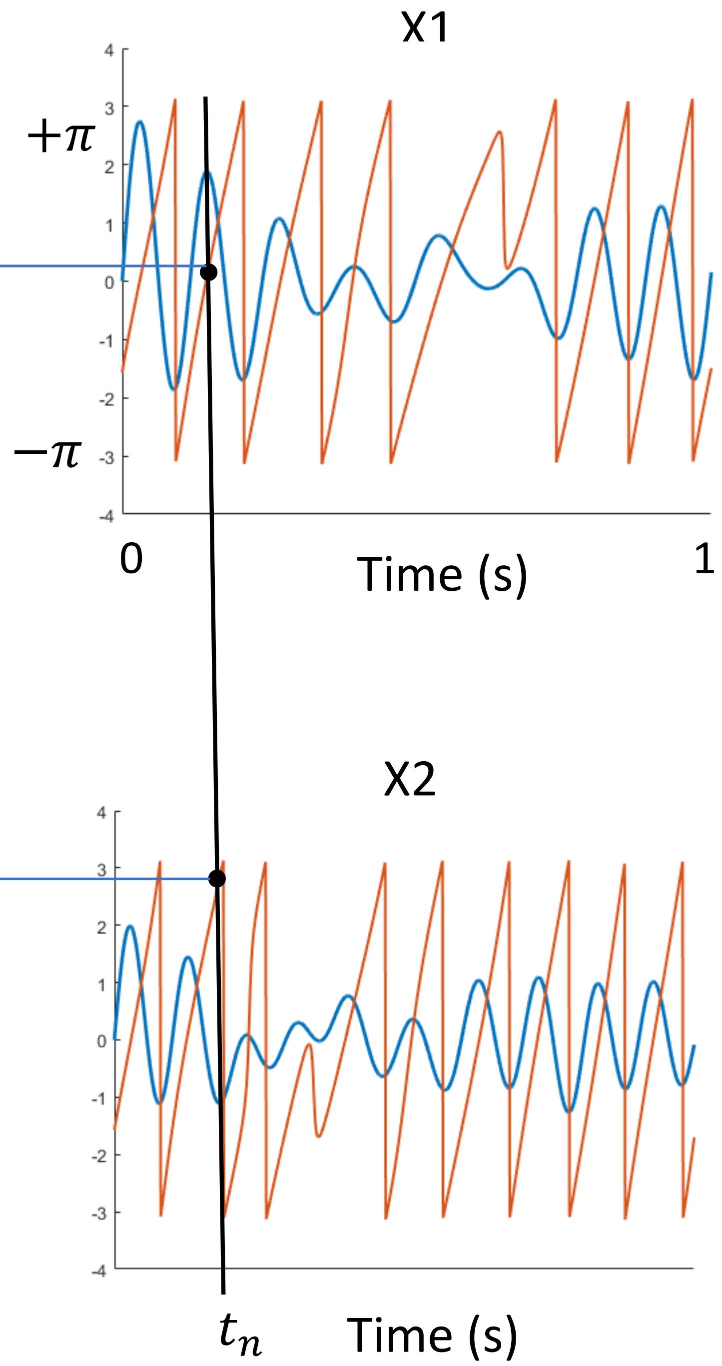
February 1, 2024

ω_1 = 0.30 rad
θ_1 = 18°

$$\theta = \omega * 180/\pi$$

$\omega_2 = 2.8\ rad$
θ_2 = 160°

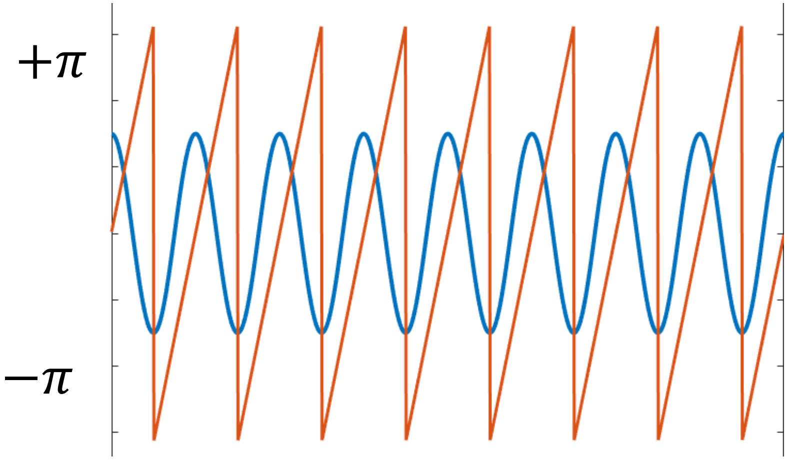


$$e^{i2\pi ft} = \cos(2\pi ft) + i\sin(2\pi ft)$$

$$\theta = 2\pi ft$$

$$\hat{x}(t) = \text{hilbert(x)}$$

Step 1: take fft of signal
Step 2: rotate Fourier coefficients $(+\frac{\pi}{2})$
Step 3: Take $ifft$ of rotated Fourier coefficient



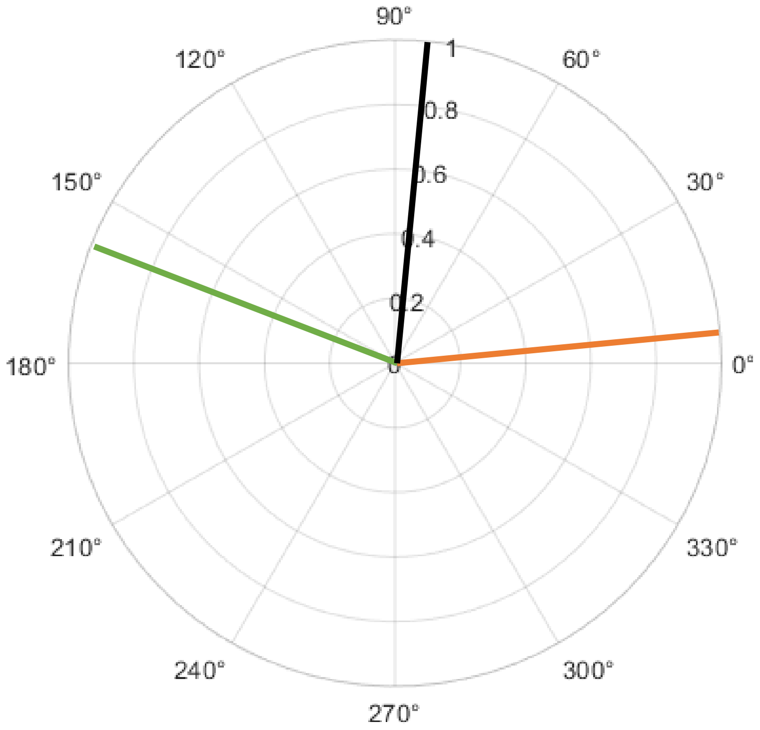
ω_1 = 0.30 rad, 18

ω_2 = 2.8 rad, 160

$\Delta\theta$ = 142°

$\angle\,(\Sigma\, e^{\theta})$= 88°

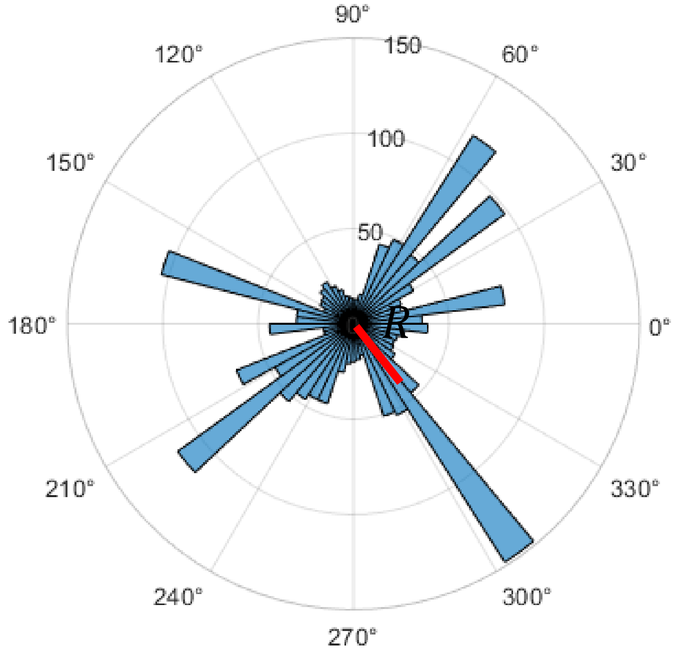
$\Sigma\, e^{\theta_k}$



mean resultant vector length:

$$\bar{R} = \left| \frac{1}{N} \sum_{n=1}^{N} e^{i\theta_n} \right|$$

Variance = 1 - $\bar{R}$



P < 0.001

$$p = e^{\sqrt{1+4N+4(N^2-(N\bar{R})^2)}-(1+2N)}$$

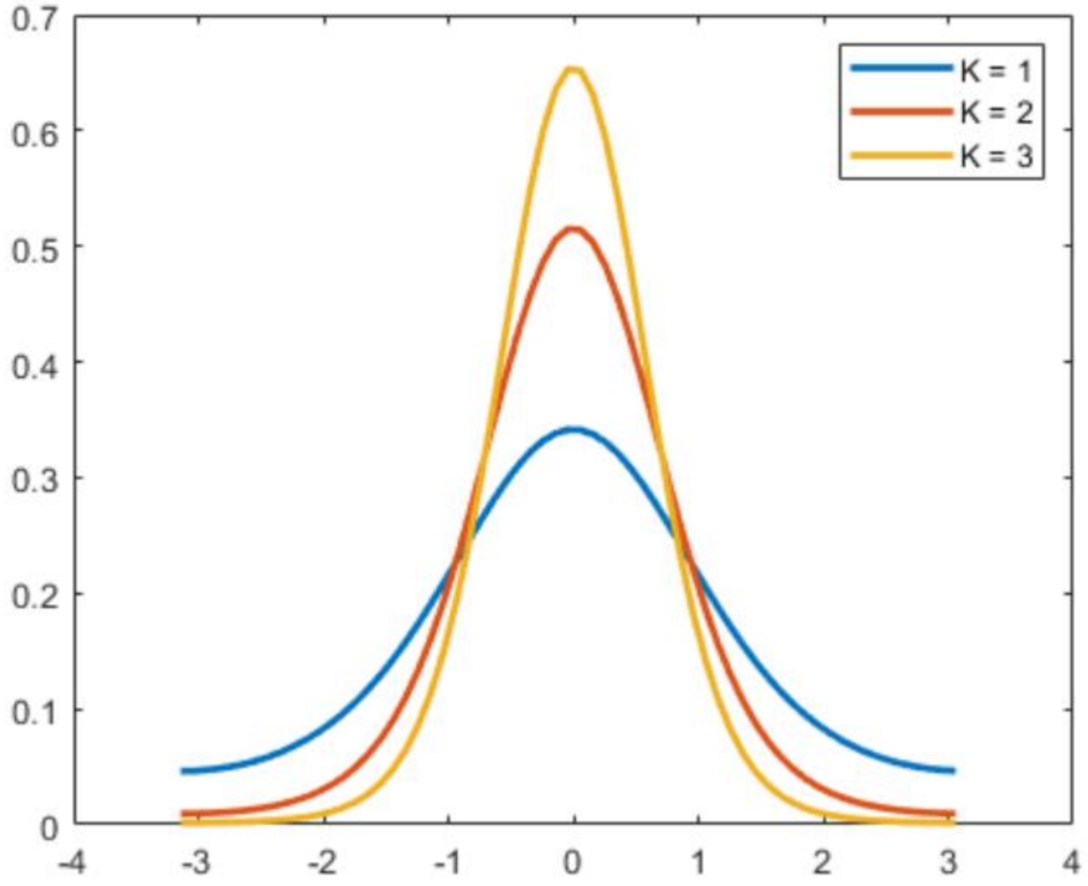
Rayleigh test for non-uniformity:

H0: the population is uniformly distributed around the circle
H1: the population is not distributed uniformly around the circle

Assumption: the distribution has maximally one mode and the data is sampled from a Von Mises distribution!

Von Mises distribution

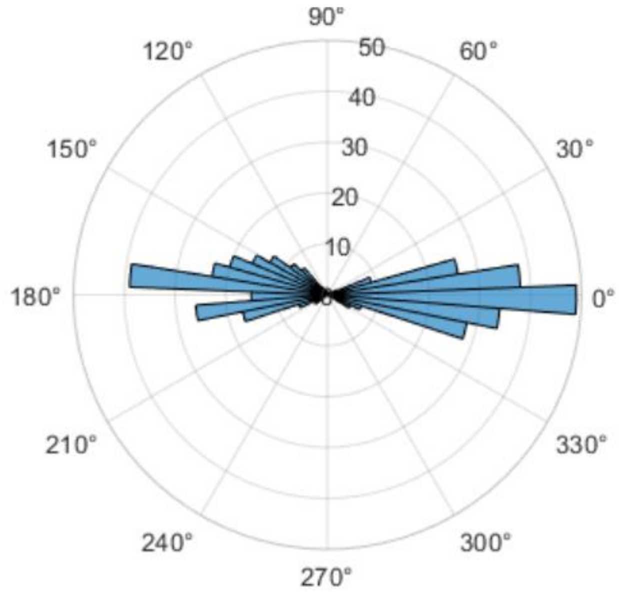
$$\mu = 0$$



IF samples:

- Don’t come from a Von Mises distribution
- Come from bimodal or multimodal distribution

H0: the population is uniformly distributed around the circle
H1: the population is not distributed uniformly around the circle



P = .12
Rayleigh test

P < 0.001
Omnibus test

✓ Non-parametric methods:

‘Omnibus test’:

$$P = \frac{1}{2^{N-1}}(N-2m)\binom{N}{m}$$

Which can for N > 50 be approximated by

$$P \simeq \frac{\sqrt{2\pi}}{A}\exp\left(-\pi^2/(8A^2)\right), \quad A = \frac{\pi\sqrt{N}}{2(N-2m)}.$$

$\boldsymbol{m}$ = the smallest number of degrees that occur within a range of 180 degree.