Worksheet: hypothesis tests

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1. **Detailed hypothesis test for example 2 in discussion notes**

   Identify the null hypothesis $\mathcal{H}_0$: the mean peak visual ERP in medicated subjects is 2 mV

   Identify the alternative hypothesis $\mathcal{H}_a$: the mean peak visual ERP in medicated subjects is different from 2 mV.

   Select a test statistic: standarized sample mean Z.

   Calculate the observed value of the test statistic: $z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{1.3-2}{2.6/\sqrt{50}} = -1.9$

   Calculate the p-value: $\text{p\_value} = P(-|z| > Z) + P(Z > |z|) = 2P(Z > |z|) = 0.57$.

   Draw our conclusion: $\text{p\_value} = 0.57 > 0.05$ then **do not reject $\mathcal{H}_0$**.

2. (a) $\mathcal{H}_0$**:** the population mean is $\mu_0 = 2.3$
   $\mathcal{H}_a$**:** the population mean is $\mu_0 > 2.3$

   (b) Because $n > 30$ it is reasonable to assume that $Z \sim \mathcal{N}(0, 1)$. Then the rejection region is $z > z_\alpha$, with $\alpha = 0.05$.

   (c) Roughly, for $Y \sim \mathcal{N}(\mu, \sigma^2)$ there is a considerable probability of obtaining a sample in the range $[\mu, \mu + 2\sigma]$. Because under the null hypothesis $\bar{X} \sim \mathcal{N}(\mu_0, s/\sqrt{n})$, there is a considerable probabability of obtaining a sample of $\bar{X}$ in the range $[\mu_0, \mu_0 + 2s/\sqrt{n}]$. $s/\sqrt{n} \sim 0.3/6 \sim 0.05$. Thus, there is a considerable probability of obtaining by chance a sample of $\bar{X}$ in the range $[2.3, 2.3{+}2\;0.05] = [2.3, 2.4]$. Because 2.4 is in the boundary of this interval, it is not obvious if a hypothesis test will reject or not the null hypothesis. Lets do the test. We first compute the observed test statistic:

   $$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.4 - 2.3}{0.29/\sqrt{35}} = 2.04$$

   The p-value corresponding to this observed statistic is $p = 0.02$, so we reject the null hypothesis with a confidence level $\alpha = 0.05$.

3. **Potency of an antibiotic**

   (a) $\mathcal{H}_0$: the mean potency of the antibiotic is $\mu_0 = 80\%$.

   (b) $\mathcal{H}_a$: the mean potency of the antibiotic is $\mu_0 < 80\%$.

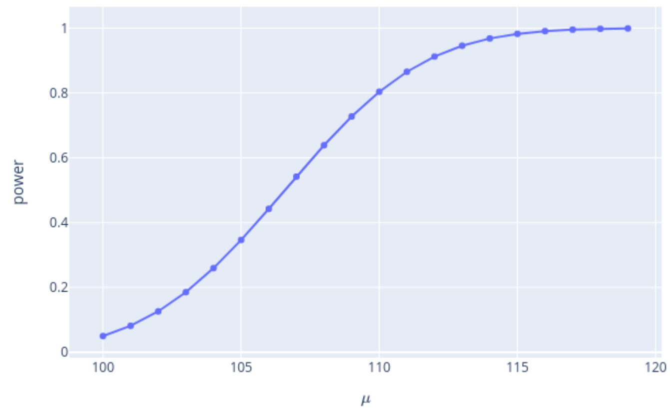

Figure 1: Power versus effect size.

(c) because $n = 100$ it is sensible to assume $Z \sim \mathcal{N}(0, 1)$. I will perform a right-tailed z-test with $\bar{x} = 79.7\%, \mu_0 = 80.0\%, n = 100, s = 0.8$ and $\alpha = .05$. Lets compute the observed statistic.

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{29.7 - 80}{0.8/\sqrt{100}} = -3.75$$

The p-value corresponding to this observed statistic is $p < 0.0001$, so we reject the null hypothesis with a confidence level $\alpha = 0.05$.

4. **Smoking and lung capacity** Because $n = 20$ it is not safe to assume $\bar{Z}\ \mathcal{N}(0, 1)$. We will perform a right-tailed t-test instead. Lets compute the observed statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{89.85 - 100}{14.53/\sqrt{20}} = -3.12$$

The p-value corresponding to this observed statistic is $p < 0.003$, so we reject the null hypothesis with a confidence level $\alpha = 0.01$.

5. **Power of a test**

(a) the calculated powers are:

$\mu_{\mathbf{Ha}} = 108$**:** power=0.64

$\mu_{\mathbf{Ha}} = 112$**:** power=0.91

$\mu_{\mathbf{Ha}} = 116$**:** power=0.99

Power versus effect size is shown in Figure 1.

(b) Figure 2 shows power plots for $\alpha = 0.01$ and $\alpha = 0.05$.

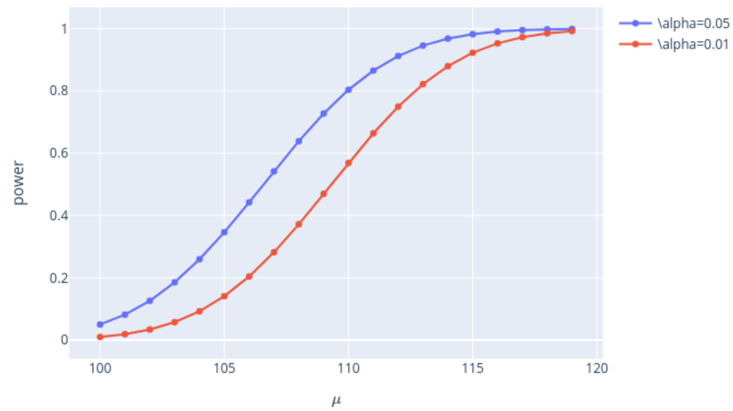

Figure 2: Power plots for significance levels $\alpha = 0.01$ and $\alpha = 0.05$.

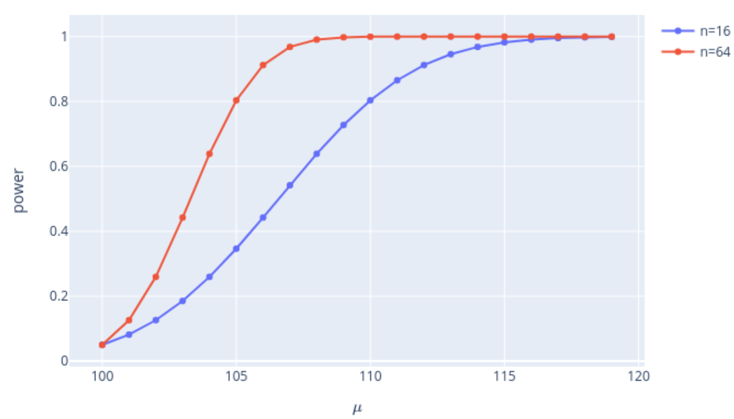

Figure 3: Power plots for $n = 16$ and $n = 64$.

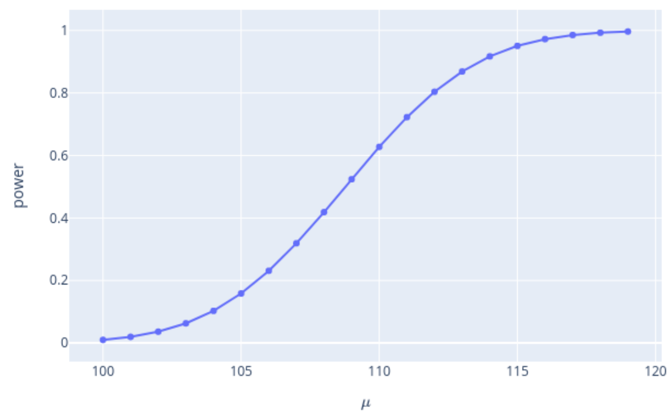

Figure 4: Power plots for $n = 18$, $\alpha = 0.01$, and $\beta = 0.2$. As required for $\mu_a = 112$ the power of the test is $1 - \beta = 0.8$.

(c) Figure 3 shows power plots for $n = 16$ and $n = 64$.

(d) the minimum sample size to achieve significance level of $\alpha = 0.01$ and power of $1 - \beta = 0.8$ for $\mathcal{H}_a : \mu_a = 112$ is $n = 18$. Figure 4 shows the power plot for this sample size.