Worksheet: LFPs and spectral analysis

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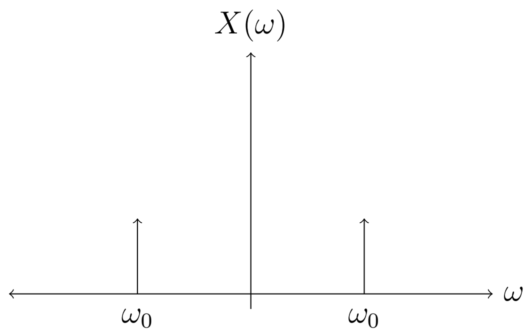
Please show all your calculations required for your answers.

1. We measure the LFP in human motor cortex with an Utah array. It is known that this LFP only has an oscillation at 11 Hz (i.e., $LFP(t) = \cos(\omega_0\, t)$ with $\omega_0 = 2\pi\, f_0$ rad/sec, $f_0 =$ 11 Hz). However, when we sample this LFP at a frequency of 10 Hz (i.e., $\omega_s = 2\pi\, f_s$ rad/sec, $f_s = 10$ Hz) we only observe an oscillation at 1 Hz (Figure 1).

   (a) explain the appearance of the 1 Hz oscillation using the sampling therem.
   Hints:

   - the Fourier transform of a cosine is $\mathcal{FT}\{\cos(\omega_0 t)\} = \frac{1}{2}[\delta(-\omega_0) + \delta(\omega_0)]$ and has the spectrum in the figure below.

   - replicate the above spectrum, as indicated by the sampling theorem, with replicas at multiples of the sampling frequency $\omega_s = 2\pi\, f_s$ ($f_s = \frac{1}{T_s} = 10\ Hz$).
   - check if any of the above replicates adds signal at 1 Hz (i.e., $\omega = 2\pi\, f$ rad/sec, $f = 1$ Hz).

   Note: to avoid this type of problems of low-frequency oscillations appearing due to frequencies in the signal above the Nyquist frequency (i.e., half of the sampling frequency), before sampling signals are low pass filtered with an analog filter at the Nyquist frequency. This filter is called an **antialiasing filter**. The sampling theorem only applies to digital filters and therefore analog ones do not generate aliasing.

   (b) use a sampling frequency above the Nyquist rate (i.e., $f_s > 2f_N$ where f_N is the largest frequency in the signal) and check that oscillations at 11 Hz are present in the sampled signal. You may want to use this code.

   (c) build another example of an LFP having an oscillation at a high frequency that when sampled at a frequency below the Nyquist rate generates an oscillation at a lower frequency. You can use this code to verify that with your values of the LFP frequency and the sampling frequency an oscillation at a low frequency emereges.

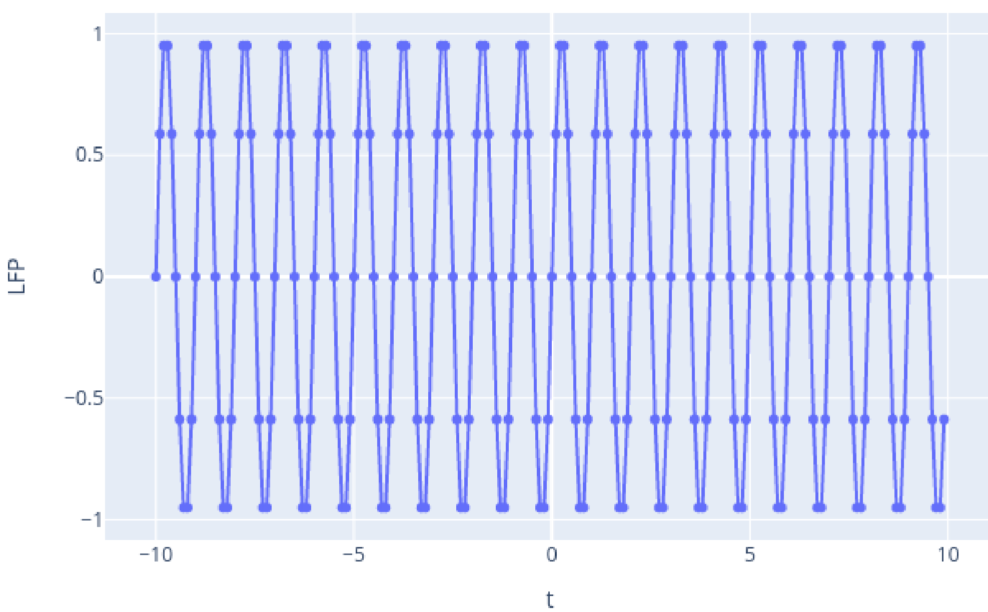


Figure 1: An LFP oscillating at 11 Hz (i.e., $LFP(t) = \cos(\omega_0 t)$ with $\omega_0 = 2\pi f_0$ rad/sec, $f_0 = 11$ Hz) when sampled at a frequency of 10 Hz (i.e., $\omega_s = 2\pi f_s$ rad/sec, $f_s = 10$ Hz) only displays an oscillation at 1 Hz. Use the sampling theorem to explain this observation. Code to generate this figure appears here.

2. solve Ken's worksheet on power spectra.