Worksheet: hypothesis tests

Joaquin Rapela

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Please show all your calculations required for your answers.

1. Perform a detailed hypothesis test for example 2 in the discussion notes.

2. A random sample of $n = 35$ observations from a quantitative population produced a mean $\bar{x} = 2.4$ and a standard deviation $s = 0.29$. Suppose your research objective is to show that the population mean μ exceeds 2.3.

   (a) Give the null and alternative hypotheses for the test.

   (b) Locate the rejection region for the test using a 5% significance level.

   (c) Before you conduct the test, use your intuition to decide whether the sample mean $\bar{x} = 2.4$ is likely or unlikely, assuming that $\mu = 2.3$. Now conduct the test. Do the data provide sufficient evidence to indicate that $\mu > 2.3$?

3. **Potency of an Antibiotic** A drug manufacturer claimed that the mean potency of one of its antibiotics was 80%. A random sample of $n = 100$ capsules were tested and produced a sample mean of $\bar{x} = 79.7\%$ with a standard deviation of $s = 0.8\%$. Do the data present sufficient evidence to refute the manufacturer's claim? Let $\alpha = .05$.

   (a) State the null hypothesis to be tested.

   (b) State the alternative hypothesis.

   (c) Conduct a statistical test of the null hypothesis and state your conclusion.

4. **Smoking and Lung Capacity** It is recognized that cigarette smoking has a deleterious effect on lung function. In a study of the effect of cigarette smoking on the carbon monoxide diffusing capacity (DL) of the lung, researchers found that current smokers had DL readings significantly lower than those of either exsmokers or nonsmokers. The carbon monoxide diffusing capacities for a random sample of $n = 20$ current smokers are listed here:

   103.768 92.295 100.615 102.754 88.602 61.675 88.017 108.579 73.003 90.677 71.210 73.154 123.086 84.023 82.115 106.755 91.052 76.014 89.222 90.479

   Do these data indicate that the mean DL reading for current smokers is significantly lower than 100 DL, the average for nonsmokers? Use a $\alpha = .01$.

5. **Power of a test** Let X be a random variable representing the IQ of a randomly selected British. Assume that X is normally distributed with unknown mean and standard deviation 16. Take a random sample of $n = 16$ individuals so that at a significance level $\alpha = 0.01$ we can test the null hypothesis $H_0 : \mu = 100$ versus the alternative one $H_a : \mu > 100$.

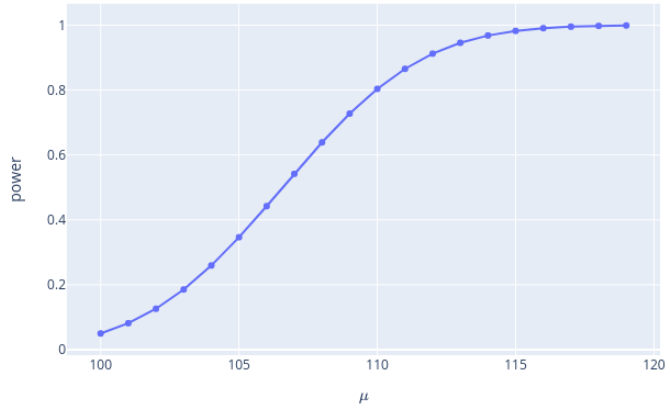

Figure 1: Power versus effect size.

(a) what is the power of the test if the population mean is $\mu = 108$? (Hint: slide "Calculation of β" from the Hypothesis Test discussion session).

what is the power if $\mu = 112$? what if $\mu = 116$?

Generate a plot of power as a function of effect size, i.e., power plot, as in Figure 1.

In this plot you should see that power increases with the effect size.

(b) (optional) show in the same figure power plots for significance levels $\alpha = 0.05$ and $\alpha = 0.01$.

In this plot you should see that power decreases as our demands on type I error become more stringent.

(c) (optional) show in the same figure power plots for $n = 16$ vs $n = 65$.

In this plot you should see that, for a fixed significance level and effect size, power increases with sample size.

(d) (optional) find the minimum sample size required to achieve a power of 0.8, for a significance level $\alpha = 0.05$, and for an alternative hypothesis $H_a : \mu = 112$. Show a power plot for the obtained minimum sample size and check that the power for $\mu = 112$ is above 0.8.

Hints:

i. derive an expression for the critical sample mean, $\bar{x}_c$ (i.e., the value of $\bar{x}$ separating the non-reject and reject regions) only using the type I error. This expression should depend on n

ii. derive a second expression for $\bar{x}_c$ only using the type II error. This expression should also depend on n

iii. equate the above expressions and solve for n.

6. (optional) solve problems from Prof. Harris' worksheet on t-test and randomization test and/or on making a raster plot.