Worksheet: temporal time series analysis

Joaquin Rapela

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1. Is the x_t random process in Eq. 1 wide-sense stationary (WSS)?

$$x_t = \phi x_{t-1} + w_t \tag{1}$$

   where $\{w_t\}$ is a white noise random process with variance σ_w^2. For this random process to be WSS, can the parameter ϕ take any value? Why or why not?

2. Write code to generate the figures in the lecture slide titled *Analytical and estimated autocovariance function for AR(1)*. Provide the code and the generated figures.

   Hint: you may want to modify the code in the solution of the lecture slide titled *Analytical and estimated autocovariance function for MA*.

3. (optional) For the random walk with drift model:

   (a) Calculate the covariance function $\gamma(s,t)$ and use it to derive the variance function $var(t)$.

   (b) To check that your variance function is correct, complete and execute this python script. It plots 100 samples of the random walk with drift model (colored traces), with the mean (solid line) and 95% confidence bands (dashed lines). See Figure 1. At any time point (abscissa) you should observe that 95% of the traces (5 traces) are above the upper line or below the lower line.

   (c) Is the random walk with drift process wide-sense stationary? Why or why not?

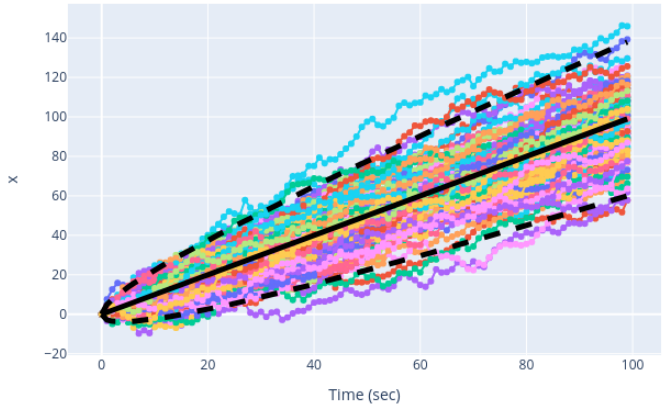

Figure 1: One hundread samples of a random walk with drift process (color traces). The solid line is the mean of the random process and the dotted lines mark the 95% confidence interval. At any time point 95% of the samples (i.e., 5 samples) should lie above or below the dotted lines.