Worksheet: temporal time series analysis

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January 14, 2025

1. Is the X_t random process in Eq. 1 wide-sense stationary (WSS)?

$$X_t = \phi X_{t-1} + W_t \tag{1}$$

   where $\{W_t\}$ is a white noise random process with variance σ_w^2. For this random process to be WSS, can the parameter ϕ take any value? Why or why not?

2. Write code to generate the figures in the lecture slide titled *Analytical and estimated autocovariance function for AR(1)*. Provide the code and the generated figures.

   Hint: you may want to modify the code in the solution of the lecture slide titled *Analytical and estimated autocovariance function for MA*.

3. (optional) Calculate the covariance function $\gamma(s,t)$ for the random walk with drift random process? Is this process wide-sense stationary?

4. (optional) Forecasting an AR(5) random process.

   a Simulate $n = 500$ samples from an AR(5) random process. Set the coefficients, ϕ_i, of the AR(5) model to values of your choice.

   Hint: use $|\phi_i| < 1$ to ensure stationary.

   b Forecast $X_{n+1}, \ldots, X_{n+h}$, with $h = 25$ using the last $m = 20$ simulated values.

   Hint: you may want to complete the example code provided here.

   Provide your completed code and the generated figures.