

# Worksheet: temporal time series analysis

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1. Is the  $x_t$  random process in Eq. 1 wide-sense stationary (WSS)?

$$x_t = \phi x_{t-1} + w_t \quad (1)$$

where  $\{w_t\}$  is a white noise random process with variance  $\sigma_w^2$ . For this random process to be WSS, can the parameter  $\phi$  take any value? Why or why not?

2. Write code to generate the figures in the [lecture](#) slide titled *Analytical and estimated autocovariance function for AR(1)*. Provide the code and the generated figures.

Hint: you may want to modify the code in the solution of the [lecture](#) slide titled *Analytical and estimated autocovariance function for MA*.

3. (optional) For the random walk with drift model:

- (a) Calculate the covariance function  $\gamma(s, t)$  and use it to derive the variance function  $var(t)$ .
- (b) To check that your variance function is correct, complete and execute [this](#) python script. It plots 100 samples of the random walk with drift model (coloured traces), with the mean (solid line) and 95% confidence bands (dashed lines). See Figure 1. At any time point (abscissa) you should observe that 95% of the traces (5 traces) are above the upper line or below the lower line.
- (c) Is the random walk with drift process wide-sense stationary? Why or why not?

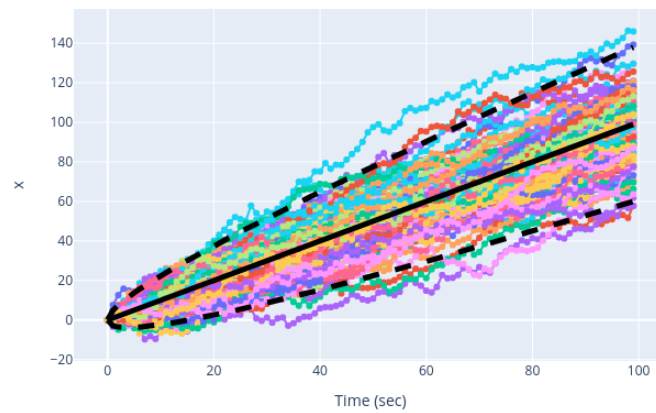


Figure 1: One hundred samples of a random walk with drift process (colour traces). The solid line is the mean of the random process and the dotted lines mark the 95% confidence interval. At any time point 95% of the samples (i.e., 5 samples) should lie above or below the dotted lines. Click on the figure to see its interactive version.