NLL

$$P(D|\mu,k) = \prod_{i=1}^{n} \frac{e^{k\cos(\theta_i-\mu)}}{2\pi I_0(k)}$$

Log likelihood function is:

$$\log P(D|\mu,k) = \sum_{i=1}^{n} \log\left(\frac{e^{k\cos(\theta_i-\mu)}}{2\pi I_0(k)}\right)$$

$$= \sum_{i=1}^{n}\left[\log\left(e^{k\cos(\theta_i-\mu)}\right) - \log(2\pi I_0(k))\right]$$

$$= \sum_{i=1}^{n} k\cos(\theta_i-\mu) - \sum_{i=1}^{n}\log(2\pi I_0(k))$$

$$= k\sum_{i=1}^{n}\cos(\theta_i-\mu) - n\log(2\pi I_0(k))$$

Negative:

$$-\log P(D|\mu,k) = -k\sum_{i=1}^{n}\cos(\theta_i-\mu) + n\log(2\pi I_0(k))$$

$$= -k\sum_{i=1}^{n}\cos(\theta_i-\mu) + n\log(2\pi) + n\log(I_0(k)).$$

the only dependent on μ

$$\frac{\partial}{\partial\mu}\left(-k\sum\cos(\theta_i-\mu)\right) = \frac{\partial}{\partial\mu}\left(-k\sum\left(\cos\theta_i\cos\mu + \sin\theta_i\sin\mu\right)\right)$$

$$= +k\sum{}^{+}\cos\theta_i\sin\mu - \sin\theta_i\cos\mu.$$

$$= k \sin\mu \sum_{i=1}^{n} \cos\theta_i - k\cos\mu \sum_{i=1}^{n} \sin\theta_i$$

$$= k\left(\sin\mu \sum_{i=1}^{n} \cos\theta_i - \cos\mu \sum_{i=1}^{n} \sin\theta_i\right)$$

$$= k \sum_{i=1}^{n} \sin(\mu - \theta_i)$$

$$k \sum_{i=1}^{n} \sin(\mu - \theta_i) = 0$$

$$\sum_{i=1}^{n} \sin\theta_i \cos\mu - \cos\theta_i \sin\mu = 0$$

$$\cos\mu \sum_{i=1}^{n} \sin\theta_i - \sin\mu \sum_{i=1}^{n} \cos\theta_i = 0$$

$$\cos\mu \sum_{i=1}^{n} \sin\theta_i = \sin\mu \sum_{i=1}^{n} \cos\theta_i$$

$$\frac{\sin\mu}{\cos\mu} = \frac{\sum_{i=1}^{n} \sin\theta_i}{\sum_{i=1}^{n} \cos\theta_i}$$

$$\tan\mu = \frac{\sum_{i=1}^{n} \sin\theta_i}{\sum_{i=1}^{n} \cos\theta_i}$$

$$\mu = \arctan\left(\frac{\sum_{i=1}^{n} \sin\theta_i}{\sum_{i=1}^{n} \cos\theta_i}\right)$$