svGPFA analysis 30190367 of MC_MAZE_SMALL with LDA latents

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December 15, 2025

Contents

1 Hyper-parameters

number of latents: 15

number of inducing points: 20

number of trials: 100

number of clusters: 142

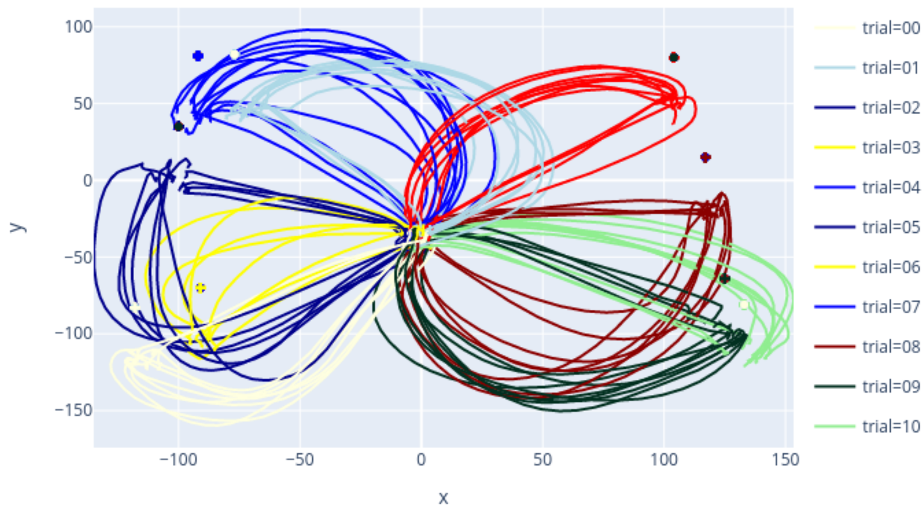

Figure 1: Hand reaching behavior of a monkey. Each trace correspond to a different trial. Trials are colored by target location. There are nine target locations, three to the upper left quadrants, and two to the other quadrants. Click on the figure to get its interactive version.

2 Behavioral data

Figure 1 plots the behavioral data. In every trial monkeys reach to one of the nine target locations.

3 Data for LDA

We provided LDA a matrix of data, $X \in \mathbb{R}^{L \times N}$, and a vector of labels, $\mathbf{y} \in \mathbf{R}^N$. The matrix of data X contains all 15 latents, of all trials, from a 200 ms segment starting at movement onset. A column of X contains the value of the 15 latents corresponding to one sample time point of one trial; thus $L = 15$. The number of columns of X is the number of trials times the number of sample points in the 200 ms segment. The length of $\mathbf{y}$ is the same as the number of columns of X, and the value of $y[i]$ is the target location corresponding to the trial of the ith column of X.

4 Methods

Our implementation of LDA estimated eight (number of target locations minus one) non-orthogonal directions that maximize a criterion of separability, $J_1(A)$, of the projected data Y, where $Y = AX$, $J_1(A) = \mathrm{Trace}(S_b\, S_w^{-1})$, and S_b, S_w and the between and within group scatter matrices of Y. Custom code implementing LDA can be found here. Our LDA implementation produced almost identical results as the scikit-learn one.

The rows of matrix A span the LDA space of dimension number of reach targets minus one. We orthonormalized this space and projected the estimated latents on the orthonormalized space.

5 Results

5.1 First discriminatory direction separates left from right trials

Figure 2 plots histograms of projections of columns of X onto the first discriminatory directions obtained from LDA. There are nine histograms, as many of reaching directions. The histogram for the ith reaching direction contains projections of all columns of X corresponding to trials where the subject reached to the ith direction. The color of the histogram corresponds to the color of the reaching direction in Figure 1. The title of the figure shows the eigenvalue corresponding to this discriminatory direction, which indicates the contribution of this direction to the optimized separability criterion $J_1(A)$ (Section 4). $J_1(A) = \mathrm{Trace}(S_b\, S_w^{-1}) = \mathrm{eigval}_1 + \mathrm{eigval}_2 + \cdots + \mathrm{eigval}_{\mathrm{nTargetLocs}-1}$ (Fukunaga, 1990, Eq. 10.19). This discriminatory direction separates well right (red and green, Figure 1) from left (blue and yellow, Figure 1) trials.

Also, latents corresponding to right and left trials are well separated in the 0.0-0.2 time interval when projected onto the first dimension of the orthonormlised LDA space (see Section 4), as show in Figure 3.

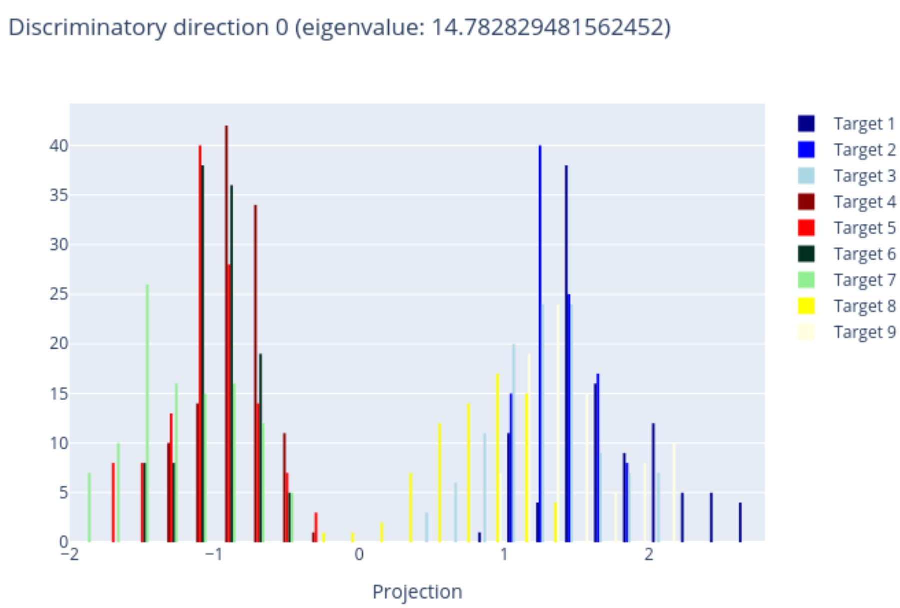

Figure 2: Histograms of projections onto the first LDA direction. Different histograms correspond to the different reach target locations show in Figure 1. This LDA direction separates right (red and green) from left (blue and yellow) trials. Click on the figure to get its interactive version.

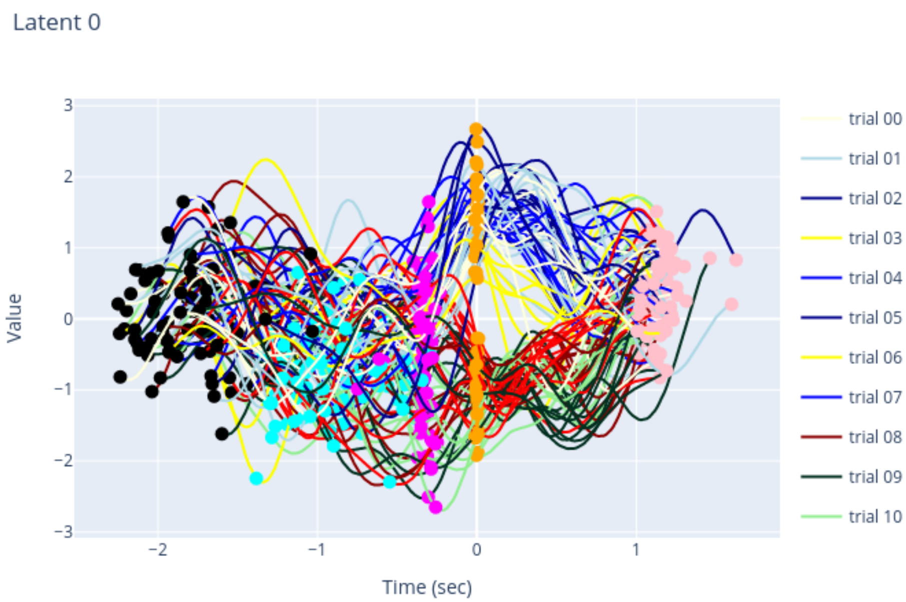

Figure 3: Latents projected onto the first direction of the orthonormalized LDA space. Latents corresponding to trials where the target was on the left (blue and yellow) are well separated in the interval 0.0-0.2 sec from those of trials where the target was on the right (red and green). Click on the figure to get its interactive version.

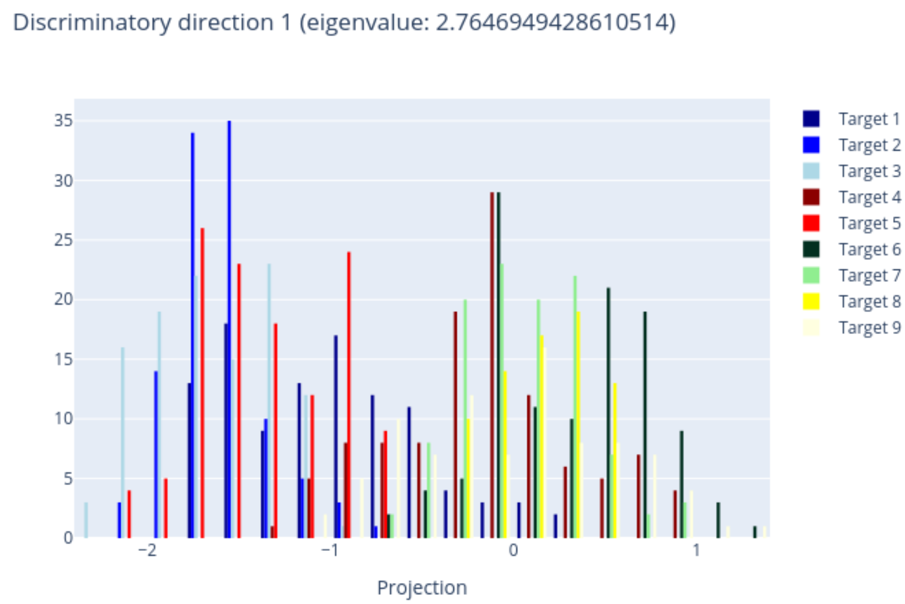

Figure 4: Histograms of projections onto the second LDA direction. Same format as in Figure 2. This LDA direction separates top (red and blue) from bottom (green and yellow) trials. Click on the figure to get its interactive version.

5.2 Second discriminatory direction separates top from bottom trials

Figure 4 plots histograms of projections of columns of X onto the second discriminatory directions obtained from LDA. Note that the eigenvalue corresponding to this discriminatory direction is almost 80% smaller than that for the first discriminatory direction, indicating that this direction contributes almost 80% less to the discriminatory criterion $J_1(A)$. This discriminatory direction separates top (red and blue, Figure 1) from bottom (green and yellow, Figure 1) trials. However, this separation is weaker than that with the first discriminative direction (Figure 2).

Also, latents corresponding to top and bottom trials are well separated in the 0.0-0.2 time interval when projected onto the second dimension of the orthonormlised LDA space (see Section 4), as show in Figure 5. However,

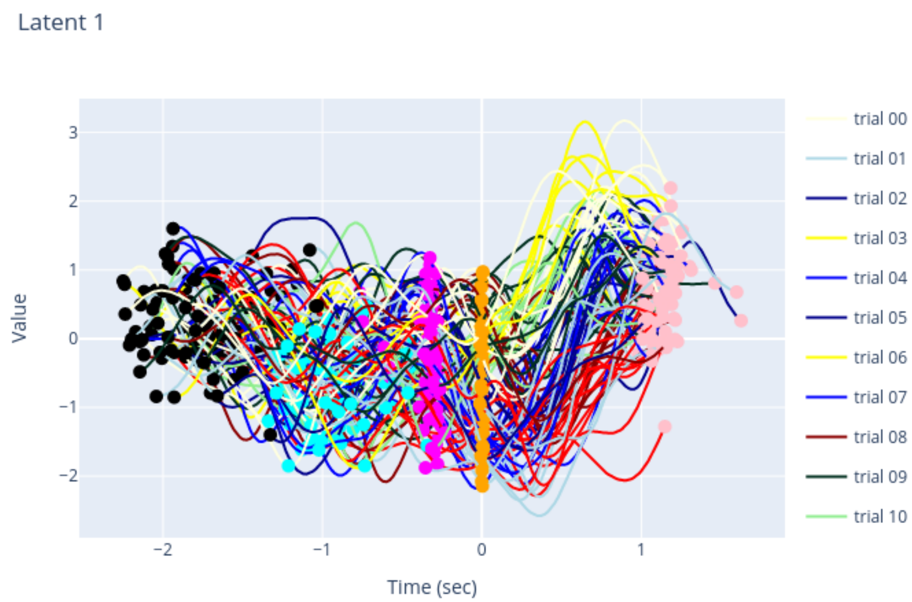

Figure 5: Latents projected onto the second direction of the orthonormalized LDA space. In the interval 0.0-0.2 sec, latents corresponding to top trials (red and blue) are separated from those corresponding to bottom trials (yellow and green), but to a lower degree than in Figure 7. Click on the figure to get its interactive version.

this separation is weaker than that in Figure 3.

5.3 Third discriminatory direction separates reaches with positive and negative slope

Figure 6 plots histograms of projections of columns of X onto the third discriminatory directions obtained from LDA. The eigenvalue corresponding to this discriminatory direction is 90% smaller than that for the first discriminatory direction, indicating that this direction contributes 90% less to the discriminatory criterion $J_1(A)$. This discriminatory direction separates reaches with positive slopw (red and blue, Figure 1) from those with negative slope (green and yellow, Figure 1).

Also, latents corresponding to top and bottom trials are well separated

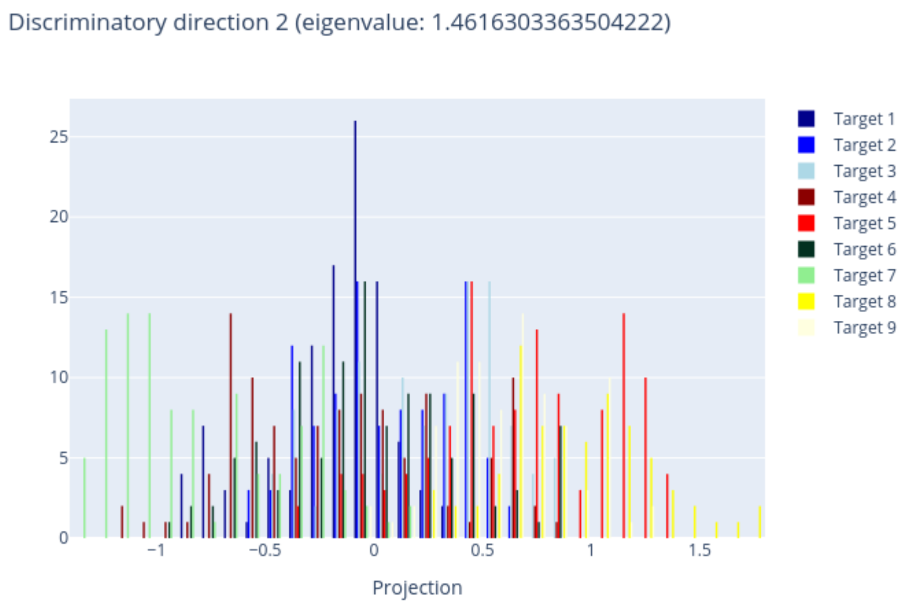

Figure 6: Histograms of projections onto the second LDA direction. Same format as in Figure 2. This LDA direction separates reaches with positive (red and yellow) and negative (blue and green) slopes. Click on the figure to get its interactive version.

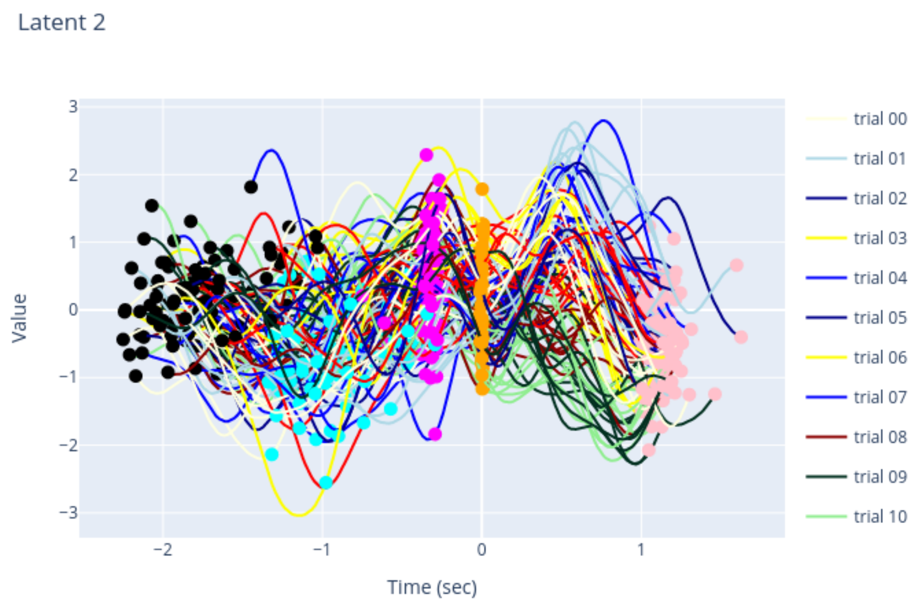

Figure 7: Latents projected onto the third direction of the orthonormalized LDA space. In the 0.0-0.2 sec interval, latents corresponding to reaches with positive slope (red and yellow) are separated from those corresponding to reaches with negative slope (blue and green). Click on the figure to get its interactive version.

in the 0.0-0.2 time interval when projected onto the second dimension of the orthonormlised LDA space (see Section 4), as show in Figure 7. However, this separation is weaker than that in Figure 3.

List of Figures

References

Fukunaga, K. (1990). *Introduction to statistical pattern recognition.* Academic Press.