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Math 18 B00

Exercise 4.1

a.)

- det(A+B) = 0
- det(A-B) = 1038
- det(AB) = 3.6480e+03
- det(A^-1) = 0.0132
- det(B') = 48

```
>> A=[8 11 2 8; 0 -7 2 -1; -3 -7 2 1; 1 1 2 4]

A =

     8    11     2     8
     0    -7     2    -1
    -3    -7     2     1
     1     1     2     4

>> B=[1 -2 0 5; 0 7 1 5; 0 4 4 0; 0 0 0 2]

B =

     1    -2     0     5
     0     7     1     5
     0     4     4     0
     0     0     0     2
```

```
>> det(A+B)

ans =

     0

>> det(A-B)

ans =

        1038
```

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```
>> det(A*B)

ans =

   3.6480e+03

>> det(A^-1)

ans =

    0.0132

>> det(B')

ans =

    48
```

b.) The matrix formed by A+B is not invertible because its determinant is 0 and according to theorem 4, a square matrix A is invertible if and only if det A doesn't equal 0.

c.) If I didn't know the entries of A and B, but I did know their determinants, then I would still be able to compute the determinants of B' because according to theorem 5, if A is an n x n matrix, then det A' = det A, so it will be equal to det B in this case. I would also be able to compute det(AB) because according to the Multiplicative Property If A and B are n x n matrices, then det AB = (det A) (det B). We would also be able to compute det (A^-1) because we know that it is equal to 1/det(A). We can't do this with addition or subtraction.

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Exercise 4.2

```
>> N=[0.003 0.02 0; 0.1 1 0; 0 0 0.015]

N =

    0.0030    0.0200         0
    0.1000    1.0000         0
         0         0    0.0150

>> det(N^100)

ans =

 -1.7211e-201
```

```
>> (det(N))^100

ans =

     0
```

Based on this information I might think that N^100 is not invertible because it shows me that it equals 0. After calculating the determinant of N^100 by hand I would reconsider my answer to the previous question because I didn't get a 0, so the computer had a rounding error.

$$N = \begin{bmatrix} 0.003 & 0.02 & 0 \\ 0.1 & 1 & 0 \\ 0 & 0 & 0.015 \end{bmatrix}$$

$$\det N = (-1^{1+3})\, 0 \det N_{13} + (-1^{2+3})\, 0 \det N_{23} + (-1^{3+3})\, 0.015 \det N_{33}$$

$$\det N \quad 0 + 0 + (1)\, 0.015 \begin{vmatrix} 0.003 & 0.02 \\ 0.1 & 1 \end{vmatrix}$$

$$\det N = 0.015((0.003)(1) - (0.1)(0.02)) = 1.5 \times 10^{-5}$$

$$\det N^{100} = (1.5 \times \underline{10^{-5}})^{100} = 5.7665\,e\text{-}49$$

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Exercise 4.3

a.)

```
>> V = [-8 6 -6 -30; 3 9 12 -10; 3 -6 -1 18; 3 0 4 7]

V =

    -8     6    -6   -30
     3     9    12   -10
     3    -6    -1    18
     3     0     4     7

>> [P,D] = eig(V)

P =

   -0.8874    0.8660    0.0605    0.8706
    0.2662   -0.2887    0.8812    0.0725
    0.2662   -0.2887   -0.4036   -0.4353
    0.2662   -0.2887    0.2388   -0.2176


D =

    1.0000         0         0         0
         0    2.0000         0         0
         0         0    1.0000         0
         0         0         0    3.0000
```

b.) By looking at the eigenvalues I know V is invertible because I can compute the determinant of V by calculating the product of the eigenvalues which are the diagonal entries of D. det(V) = 6

c.) When using MATLAB to evaluate P^-1VP, I noticed that it is equal to D.

```
>> P^-1*V*P

ans =

    1.0000    0.0000    0.0000    0.0000
   -0.0000    2.0000   -0.0000    0.0000
    0.0000   -0.0000    1.0000   -0.0000
   -0.0000    0.0000    0.0000    3.0000

>>
```

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Exercise 4.4

a.)

```
>> [P,D] = eig(F)

P =

   -0.8507    0.5257
    0.5257    0.8507


D =

   -0.6180         0
         0    1.6180

>> PDP^-1
Unrecognized function or variable 'PDP'.

>> P*D*P^-1

ans =

   -0.0000    1.0000
    1.0000    1.0000
```

b.)

```
>> F^10

ans =

    34    55
    55    89

>> P*D^10*P^-1

ans =

   34.0000   55.0000
   55.0000   89.0000
```

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c.) The pattern I see is that we start with the [1; 2] vector and in order to get the next vector we take the last bottom entry(row 2) as the new top entry(row 1) and the new bottom entry is the sum of the two entries of the previous vector. We can continue doing this in order to compute the vectors that come next.

d.) f_30 = 1346269

```
>> F^30*f

ans =

     1346269
     2178309
```

Exercise 4.5

a.)

```
>> P = [0.8100 0.0800 0.1600 0.1000;
0.0900 0.8400 0.0500 0.0800;
0.0600 0.0400 0.7400 0.0400;
0.0400 0.0400 0.0500 0.7800]

P =

    0.8100    0.0800    0.1600    0.1000
    0.0900    0.8400    0.0500    0.0800
    0.0600    0.0400    0.7400    0.0400
    0.0400    0.0400    0.0500    0.7800

>> x0 = [0.5106; 0.4720; 0.0075; 0.0099]

x0 =

    0.5106
    0.4720
    0.0075
    0.0099
```

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```
>> [Q,D] = eig(P)

Q =

    0.6656    0.7676    0.5432   -0.4641
    0.6165   -0.2841   -0.8148   -0.1254
    0.2946   -0.5682    0.1811   -0.2508
    0.3001    0.0848    0.0905    0.8402


D =

    1.0000         0         0         0
         0    0.6730         0         0
         0         0    0.7600         0
         0         0         0    0.7370

>> Q*D*Q^-1

ans =

    0.8100    0.0800    0.1600    0.1000
    0.0900    0.8400    0.0500    0.0800
    0.0600    0.0400    0.7400    0.0400
    0.0400    0.0400    0.0500    0.7800
```

b.)
By looking at D I can see that:

```
L =

    1    0    0    0
    0    0    0    0
    0    0    0    0
    0    0    0    0
```

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c.)

```
>> Pinf = Q*L*Q^-1

Pinf =

    0.3547    0.3547    0.3547    0.3547
    0.3285    0.3285    0.3285    0.3285
    0.1570    0.1570    0.1570    0.1570
    0.1599    0.1599    0.1599    0.1599
```

d.)

My answer is equal to the results I saw in the second part of the exercise from last lab when k got big and x_k started to have less significant changes and looked the same in the computer's output.

```
>> Pinf*x0

ans =

    0.3547
    0.3285
    0.1570
    0.1599
```

e.) My result for P∞y and my result to P∞x0 are equal. The initial distribution vector y of the electorate doesn't seem to affect the distribution in the long term because it will end up approaching those specific values we computed as it goes to infinity.

```
>> y = [0.1; 0.2; 0.3; 0.4]

y =

    0.1000
    0.2000
    0.3000
    0.4000

>> Pinf*y

ans =

    0.3547
    0.3285
    0.1570
    0.1599
```

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Exercise 4.6

a.)

- e_10 =

```
>> L^10*e0

ans =

    1.0625
    1.1250
    1.1875
    1.1250
    1.1719
    0.5781
    0.5938
    1.1562
```

- In order for each entry of e_n to change by less than 1% when we increase n by 1, n must be at least 26.

```
>> L^26*e0

ans =

    1.1410
    1.1416
    1.1440
    1.1420
    1.1469
    0.5695
    0.5706
    1.1444
```

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```
>> L^27*e0

ans =

    1.1469
    1.1444
    1.1414
    1.1425
    1.1415
    0.5705
    0.5708
    1.1420

>> L^100*e0

ans =

    1.1429
    1.1429
    1.1429
    1.1429
    1.1429
    0.5714
    0.5714
    1.1429
```

b.)
In the graph of the network of webpages represented by L, the vertices that have an edge going out and pointing toward website C are B and G. The vertices that the edges coming out of C point to are D and E.