COSC 522 HOMEWORK 2

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PROBLEM TWO

a. THE OR GATE TABLE

X_1	X_2	Y
0	0	0
0	1	1
1	0	1
1	1	1

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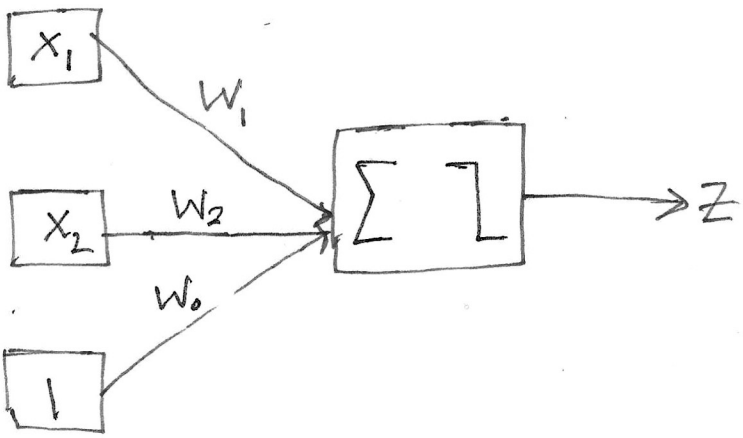

Initial parameters

$W_1 = 0.1$

$W_2 = 0.2$

$W_0 = 0.5$

Learning rate (LR) = 1

Activation Function

$Z = W_1X_1 + W_2X_2 + W_0$

Decision Rule

$\hat{y} = 1$ if $Z \geq 0$; otherwise $\hat{y} = 0$ ($\hat{y}$ is the Predicted label)

Weight and Biase update rule:

$$\begin{pmatrix} W_1 \\ W_2 \\ W_0 \end{pmatrix}^{K+1} = \begin{pmatrix} W_1 \\ W_2 \\ W_0 \end{pmatrix}^{K} + LR \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} (y - \hat{y})$$

First Epoch For Online Learning (Stochastic Gradient Descent)

Sample 1

$(x_1\ x_2) = (0, 0)$, $y = 0$

$Z = W_1 x_1 + W_2 x_2 + W_0 x_0 = 0.1 \times 0 + 0.2 \times 0 + 0.5 \times 1 = 0.5$

So $\hat{y} = 1$ since $Z = 0.5 > 0$

Weight update

$$\begin{pmatrix} W_1 \\ W_2 \\ W_0 \end{pmatrix}^{K+1} = \begin{pmatrix} W_1 \\ W_2 \\ W_0 \end{pmatrix}^{K} + LR \begin{pmatrix} x_1 \\ x_2 \\ x_0 \end{pmatrix} (y - \hat{y})$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.5 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0 - 1) = \begin{pmatrix} 0.1 \\ 0.2 \\ -0.5 \end{pmatrix}$$

So $W_1 = 0.1$, $W_2 = 0.2$, $W_0 = -0.5$

Sample 2

$(x_1\ x_2) = (0, 1)$, $y = 1$

$Z = 0.1 \times 0 + 0.2 \times 1 + -0.5 \times 1 = -0.3$

So $\hat{y} = 0$ since $Z = -0.3 < 0$

Weight update

$$\begin{pmatrix} W_1 \\ W_2 \\ W_0 \end{pmatrix}^{2} = \begin{pmatrix} 0.1 \\ 0.2 \\ -0.5 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} (1 - 0) = \begin{pmatrix} 0.1 \\ 1.2 \\ 0.5 \end{pmatrix}$$

So $W_1 = 0.1$, $W_2 = 1.2$ $W_0 = 0.5$

Sample 3

$(x_1\ x_2) = (1\ 0)$ $y = 1$

$Z = 0.1 \times 1 + 1.2 \times 0 + 0.5 = 0.6$

$\hat{y} = 1$ since $0.6 > 0$

$y = \hat{y} = 1$ hence no weight update as weight will remain the same even if updated.

Sample 4

$(x_1\ x_2) = (1\ 1)$, $y = 1$

$Z = w_1 x_1 + w_2 x_2 + w_0 x_1 = 0.1 \times 1 + 1.2 \times 1 + 0.5 \times 1 = 1.8$

$\hat{y} = 1$ since $Z = 1.8 > 0$

No weight update since $y = \hat{y} = 1$

Hence Final Weight for Epoch are:

$w_1 = 0.1$, $w_2 = 1.2$, $w_0 = 0.5$

First Epoch For Batch Learning (Batch Gradient Descent). Here, we pass all the samples Through the network Independently then we get averages For w_1, w_2 and w_0.

Sample One

$(x_1\ x_2) = (0, 0)$, $y = 0$

$Z = w_1 x_1 + w_2 x_2 + w_0 x_1 = 0.1 \times 0 + 0.2 \times 0 + 0.5 \times 1 = 0.5$

$\hat{y} = 1$ Since $Z = 0.5 > 0$

Weight update

$$\begin{pmatrix} w_1 \\ w_2 \\ w_0 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.5 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0 - 1) = \begin{pmatrix} 0.1 \\ 0.2 \\ -0.5 \end{pmatrix}$$

Hence $w_1 = 0.1$, $w_2 = 0.2$, $w_0 = -0.5$

Sample 2

$(x_1\ x_2) = (0\ 1)$ $y = 1$

$Z = w_1 x_1 + w_2 x_2 + w_0 x_1 = 0.1 \times 0 + 0.2 \times 1 + 0.5 \times 1 = 0.7$

$\hat{y} = 1$ Since $Z = 0.7 > 0$

Weights remain the same since $y = \hat{y} = 1$, Hence $w_1 = 0.1$ $w_2 = 0.2$ $w_0 = +0.5$

Sample 3

$(x_1\ x_2) = (1, 0)$ $y = 1$

$Z = w_1 x_1 + w_2 x_2 + w_0 x_1 = 0.1 \times 1 + 0.2 \times 0 + 0.5 \times 1 = 0.6$

$\hat{y} = 1$ Since $Z = 0.6 > 0$

No weight update since $y = \hat{y} = 1$

$w_1 = 0.1$, $w_2 = 0.2$, $w_0 = 0.5$

Sample 4

$(x_1, x_2) = (1, 1)$, $y = 1$

$z = w_1 x_1 + w_2 x_2 + w_0 x_1 = 0.1 \times 1 + 0.2 \times 1 + 0.5 \times 1 = 0.8$

$\hat{y} = 1$ Since $z = 0.8 > 0$

Since $y = \hat{y} = 1$, No weight update

Hence $w_1 = 0.1$, $w_2 = 0.2$ $w_0 = 0.5$

Therefore

$$w_1 = \frac{0.1 + 0.1 + 0.1 + 0.1}{4} = 0.1$$

$$w_2 = \frac{0.2 + 0.2 + 0.2 + 0.2}{4} = 0.2$$

$$w_0 = \frac{-0.5 + 0.5 + 0.5 + 0.5}{4} = \frac{1}{4} = 0.25$$

So for batch learning Epoch 1

$w_1 = 0.1$

$w_2 = 0.2$

$w_0 = 0.25$