VM Placement Optimization

Decision Variables

- $x_{v,c}$: Binary variable, 1 if VM v is placed in cluster c, 0 otherwise.
- z: Continuous variable representing the minimum utilization across all resources and clusters.

Objective Function

The objective is to maximize the minimum utilization across all resources and clusters:

$$\text{maximize} \quad z \tag{1}$$

Constraints

1. **Each new VM must be placed exactly once:**

$$\sum_{c \in \text{clusters}} x_{v,c} = 1 \quad \forall v \in \text{new\_vms} \tag{2}$$

2. **Resource capacity constraints for each cluster and resource:**

$$\sum_{v \in \text{new\_vms}} \left(\frac{\text{vm\_demand}[v][r] \cdot x_{v,c}}{\text{cluster\_capacity}[c][r]} \right) + \frac{\text{current\_usage}[c][r] \cdot \text{cluster\_capacity}[c][r]}{\text{cluster\_capacity}[c][r]} \leq 1 \quad \forall c \in \text{clusters}, \forall r \in \text{resources} \tag{3}$$

3. **Minimum utilization constraints:**

$$\frac{\sum_{v \in \text{new\_vms}} \text{vm\_demand}[v][r] \cdot x_{v,c} + \text{current\_usage}[c][r] \cdot \text{cluster\_capacity}[c][r]}{\text{cluster\_capacity}[c][r]} \geq z \quad \forall c \in \text{clusters}, \forall r \in \text{resources} \tag{4}$$