Comparing and Intereting Machine Learning Algorithms Estimating Technical Prices

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This document provides a practical example of applications of machine learning algorithms to car insurance data using the `mlr3` package. A popular model used in pricing of non-life insurance policies is the frequency-severity model, where the price is decomposed into the product of the probability of a claim arrises and the expected claim size given a claim occurs. This paper argues that a treebased model gives a particular well fit to both the frequency and severity model. Furthermore, the estimate is interpreted using shapley values to give an insight into the risk factors of car insurance. Lastly, the price model is debiassed using a decomposition techique. The result is a pricing model that does not discriminate based on gender.

Keywords: mlr3, machine learning, regression, non-life insurance, estimating technical price, XGBoost, debiassing, bias, SHAP-values, interpretation of ML models

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Getting familiar with the data

The data we use in this project is on the form of a table where out main objective is model the claim size Y_i given the explanatory variables X_i in the table. In particular, we want to construct an estimator that predicts an expecected claim size given the information available i.e. the quantity $\mathbb{E}[Y \mid X]$.

Missing values. As with any statistical modelling we start by doing some exploratory analysis of the data `freMPL1`. As it was seen in the previous section, the number of variables (columns) missing datapoints were only one. The one in question is `RecordEnd` which has 14143 out of 30595 missing values. This does not bather us since we have the variable `Exposure` giving the time difference between `RecordBeg` and `RecordEnd` in calendar years. Furthermore we have that `RecordBeg` ranges from 2004-01-01 to 2004-12-31 and `RecordBeg + 365.25*Exposure` being at most 2004-12-31 meaning that all contracts span within the year 2004. Assuming no seasonality trends this would incentivice us to remove the two variables `RecordBeg` and `RecordEnd`.

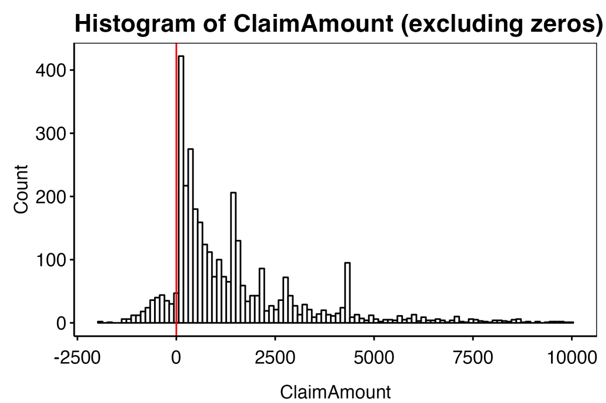
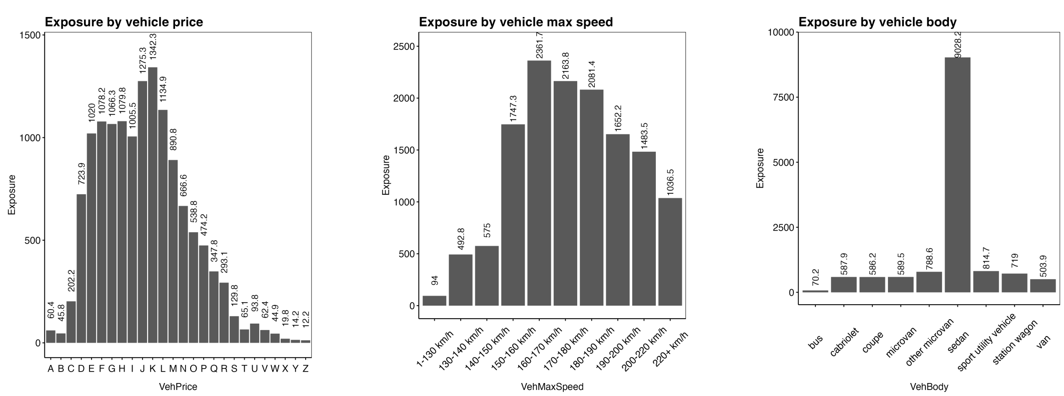


Figure 1: Histogram of the variable ‘ClaimAmount‘.

ClaimAmount and ClaimInd. The variable of interest, `ClaimAmount`, excibits a strange behaviour as it contains 285 strictly negative values. This is seen in figure 1. As this does not intuitively makes sense we will set these values as zero and ensure that the `ClaimInd` reflects this change.

VehEngine and VehEnergy. *(Vehicle specific, 1)* Regarding the categorical variables we notice that some levels is has sparse data. The variables `VehEngine` and `VehEnergy` has both the levels `GPL` and `electric`. We do however not have any substantial datapoints as in total these levels contain 8 observations. As the total dataset has 30595 observations in total we choose to remove these observations.



(a) (b) (c)

Figure 2: Exposure by respectively vehicle price (a), max speed (b) and body (c).

VehPrice, VehMaxSpeed and VehBody. *(Vehicle specific, 2)* In figure 2 we see that `VehPrice` is well represented in most price categories. We do however har a shorter supply of data in the tails. We will therefore combine the lowest three categories A through C, the levels R through T and lastly U through Z. Regarding to the vehicle max speed we see that a very few number of observations are in the lowest category `1-130 kmh`. We therefore combine the lowest level with the level `130-140 kmh`. The only vehicle body with very few observations is `bus` with only 159 observations. We do however see that `bus` act much like the category `sedan` with respect to frequency and severity and so these are combined under `sedan`.

VehAge, VehUsage and VehClass. *(Vehicle specific, 3)* The remaining three vehicle specific variable is well represented throughout all levels. The only scarce observation is `VehUsage` being `Professional run`. We therefore combine `Proffesional` and `Professional run` under `Professional`. We leave the remaining levels as is.

SocioCateg and Gender. *(Socails)* When constructing a statistical model, one does not simply have to consider which variables have the most explanatory value but one also have to take into consideration the lawfullness of discriminating customors based on covariates as gender, race and so forth. It is common knowledge that insurance companies cannot discriminate based on gender and so we will not use the variable `gender` as explanatory variable even though it might improve the model predictions. The variable `SocioCateg` representing the socioeconomic status of the insured ranges between category 1 and 99. It is not at the moment clear whether this is a covariate that may be used in pricing, so we will prefer not using it. However, if it does indeed improve the fit without overfitting, we may get some additional information from this variable. The data is indicate that the custumors in general are in the category 50 with a few other levels having significant more observations than others. For this reason we combine the catagories from 1 to 49 into `A` and the catagories 51 to 99 into `C` and keep the category `B` as is.

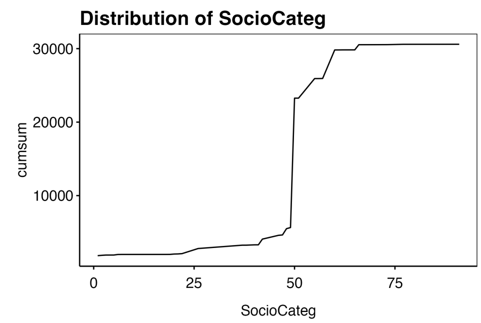


Figure 3: Cummulative distribution of the variable 'SocioCateg'.

LicAge, DrivAge and MariStat. *(Customer specific)* In general in France, one can acquire a drivers license at the age of 18 and so we have the obvious restriction `LicAge <= DrivAge - 18` and we will in general have that the license age will be approximately 18 years less than the drivers license. It is therefore reasonable to discuss whether to include both variables. We would however assume that for an older person the license age would be more important than for youngsters. Furthermore, we would assume that the `ClaimInd` and the `LicAge` are negative correlated. We will therefore include both. Both `Alone` and `Other` is well represented in `MariStat`.

HasKmLimit, RiskVar, Garage and BonusMalus *(Policy related and others)* The variables remaining HasKmLimit, RiskVar, Garage and BonusMalus are well represented throughout all levels and so no action is taken here.

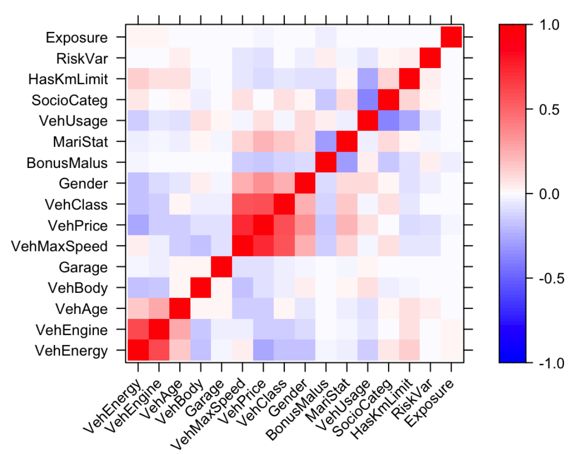


Figure 4: Spearman correlation matrix.

Correlations. As discussed previously the age of the driver and the license age i very correlated and so one may consider whether or not both variables are needed. In practice we do however see that age and experience are used when modelling the premium. Secondly, we see that the covariates vehicle engine and energy are very correlated and also the three variables vehicle max speed, price and class are well correlated. Thirdly, we see that vehicle usage and socio category are correlated. This is likely because wealthier people more often have a car for professional use.

Modelling the technical premium

The technical premium is based upon a frequency/severity model. We have the base model assumptions as: Let $Y := Y_{t+\Delta t}$ be the claim risen by a policy during the interval $[t, t + \Delta t)$ with Δt representing the exposure. Notice that in principle Δt is a stopping time defined as

$$\Delta t := \min\big(1, \inf\{s \geq t : Y_s > 0\}\big),$$

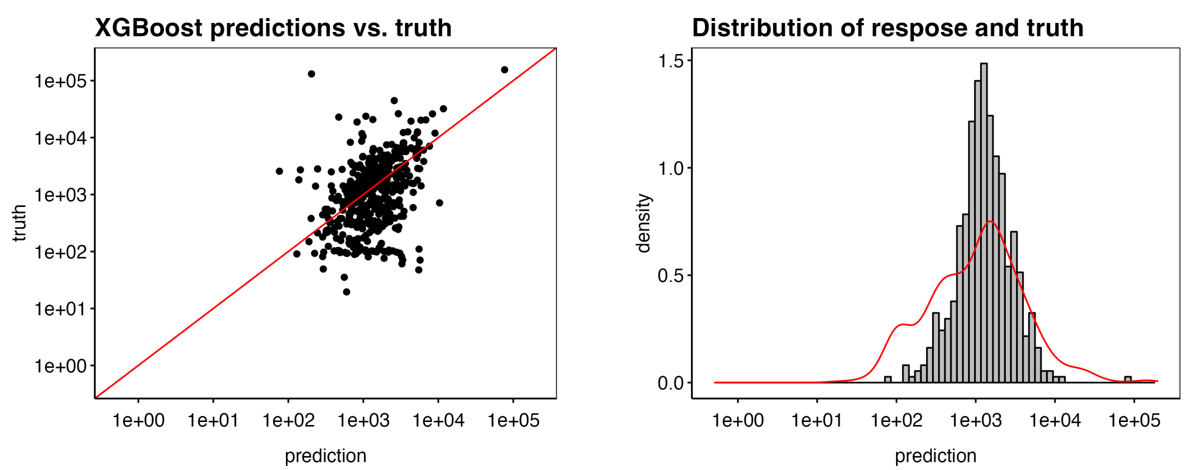
meaning the policy is terminated at the time $t + \Delta t$ with $t + \Delta t = t + 1$ if $Y = 0$. Notice that the exposure is censored if $\Delta t_i < 1$ and $Y_i = 0$. We furthermore have the covariates $X \in \mathcal{X}$ with $\mathcal{X}$ being a p-dimensional space. We are interested in the object $\mathbb{E}[Y \mid X]$ being the expected claim risen given the covariates X. Our main assumption is that this expectation is decomposed into

$$\mathbb{E}[Y \mid X] = \mathbb{E}[Y 1_{Y>0} \mid X] = \mathbb{E}[Y \mid X, 1_{Y>0}] \cdot \mathbb{E}[1_{Y>0} \mid X] = \mu_X \cdot p_X,$$

where μ_X is the expected claim in the event, that a claim arises i.e. the severity and p_X is the probability that a claim arises i.e. the frequency. However since the data is censored with exposure $\tilde{\Delta t} \leq \Delta t$ we need to consider how we may translate a predictive model into a price function for new policies. In practice, we include exposure as a covariate.

We will search for the most efficient estimators for both the severity and frequency. We will denote these estimators by $m_\mu(X)$ and $m_p(X)$ where we will denote the model by a superscript $m_j^{(*)}$ with $*$ denoting the model for $j = \mu, p$.

Severity



(a)

(b)

Figure 5: (a): Predictions from the model with y being the actual value and x being the estimate. The red line represent the mapping $y = x$. (b): Distribution of the response and actual values. The red density function represents the empirical density of the test data set.

We fit a XGBoost estimator to the dataset where we only consider the non-zero claims. The distribution of the claim is rather heavy tailed as most claims are relative small but some claims have the potential of being large. If we fit a gamma distribution to the entire dataset, we see that the claim amount appear to be gamma distributed with shape 0.51 and scale 4681. As such it would be appropriate to use the log link function when fitting the parameters by setting `objective = "reg:gamma"`. This means that the estimator $m_\mu(X)$ takes the form

$$\log(\mathbb{E}[Y \mid X, Y > 0]) = m_\mu(X).$$

Since XGBoost is a tree based algorithm with estimator $\hat{m}_\mu(X) = \sum_{i=1}^k \hat{f}_k(X)$ we have the decomposition

$$\mathbb{E}[Y \mid X, Y > 0] \approx \exp\left(\sum_{i=1}^k \hat{f}_k(X)\right) = \prod_{i=1}^k \exp\left(\hat{f}_k(X)\right),$$

where k is the weak leaner (`nrounds`). By using the package `mlr3` and the root mean squared loss we can estimate the hyperparameters: leaning rate η, depth and number og trees k. This is done using 5-fold cross validation with 1000 evals. Using the hyperparameters the model is fitted to the dataset.

In the figure above it can be seen how the model does on the training set using a model fitted to 85% randomly chosen datapoints. The model seem to predict well and we see a symmetry around $y = x$. I does however seem as though the estimates have less variance than the actual claim amounts (see the (b) figure).

Frequency

The frequency is modeled using the `xgboost` packages. The claims is assumed to arrive poisson distributed given the covariates X and so the XGBoost estimator is fitted with `objective = "count:poisson"`. This implements the log-link function and optimizes using the gradient of the poisson deviance.

Like in the case of the severity model we have the same log link function and so for trees $1, ..., l$ we will have the estimate

$$\mathbb{E}[1_{Y>0} \mid X] \approx \prod_{i=1}^l \exp(g_i(X)).$$

In the figure on the right, we see that the algorithm fit higher probabilities (frequencies) to policies that actually happened to raise a claim.

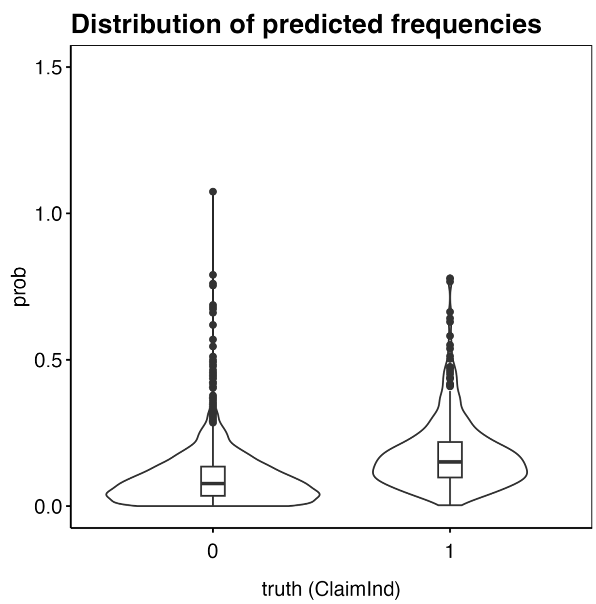


Figure 6: A combined boxplot and violin plot of the distribution of the predicted probability for respectively the non claim (left) and claim (right) policies.

Initial estimate of the technical premium

The technical price is given by the product

$$\pi = \mu_X \cdot p_X.$$

Notice that the length of the contract i.e. the exposure is used as a covariate in the frequency/severity model, and so the effect of longer contracts is already priced in. By default new contracts are sold with a yearly length, however in the case of estimating on the dataset we use the given `Exposure`. Up untill now we have used the protected feature gender in the estimate, so we start by fitting a biased estimate

$$\hat{\pi}^{(biased)} = \hat{m}_\mu^{(biased)}(X) \cdot \hat{m}_p^{(biased)}(X).$$

We can apply the technical price onto the dataset `df` by running the following code:

```r
# Estimating mu
sev_covariates <- df[,sev_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount,-Exposure)
sev_covariates <-  as.matrix(sev_covariates)
m_mu <- predict(xgb_regr, newdata = sev_covariates)
# Estimating p
freq_covariates <- df[,freq_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount)
freq_covariates <- as.matrix(freq_covariates)
m_p <- predict(xgb_freq_train, newdata = freq_covariates)
freq_covariates[,"Exposure"] <- 1
m_p_2 <- predict(xgb_freq_train, newdata = freq_covariates)
# Inserting estimates
df[,"pi_hat"] <- m_mu * m_p
df[,"mu_hat"] <- m_mu
df[,"p_hat"] <- m_p
df[,"p_hat_new"] <- m_p_2
df[,"pi_hat_new"] <- m_mu * m_p_2
```

We can summarize the models predictions on the policies below.

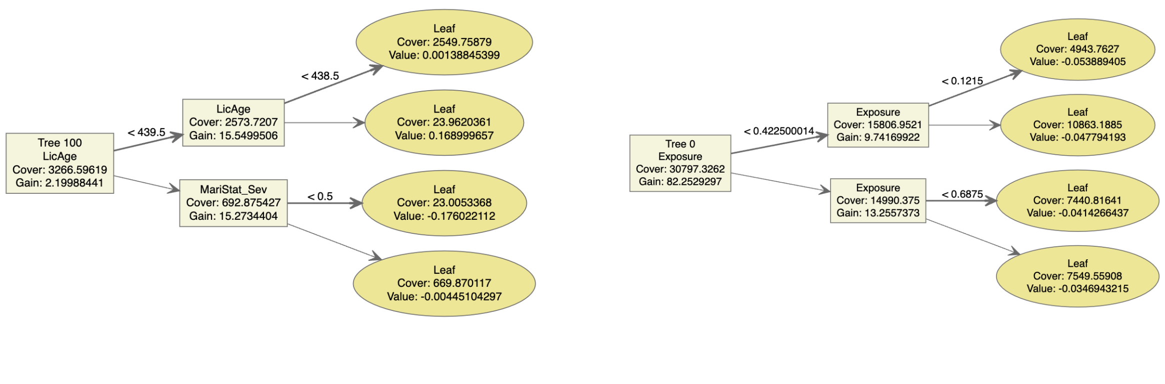
Table 1: Comparing premiums

		Actual				Extrapolated			
Policy	Gender	Exposure	p(Y>0)	E(Y|Y>0)	Price	Exposure	p(Y>0)	E(Y|Y>0)	Price
6254	Female	0.083	0.0465	1051	48.9	1	0.154	1051	161.5
24123	Male	0.633	0.1677	917	153.7	1	0.191	917	175.1
25010	Female	0.208	0.1097	1226	134.4	1	0.499	1226	611.8
25188	Male	0.095	0.0524	2468	129.4	1	0.223	2468	550.8
30495	Female	0.751	0.1034	755	78.1	1	0.113	755	85.5
30555	Male	0.163	0.0338	617	20.9	1	0.145	617	89.6

In the table above we have altered the dataset with exposure set to 1 for all rows, to predict the price of a new contract i.e. with exposure of one year. One sees that the prices for the new contracts are larger than the technical price for the historical as the exposure is larger. Furthermore as expected we see that the estimate $\hat{\mu}_X$ does not depend on the exposure.

Decomposing the estimates

When fitting a algorithm such as XGBoost it can be difficult to interpret how, when it comes to pricing, the model is affected by the risk factors X. Certainly, one could try to uncover the dynamics of the algorithm by studying the structure of the trees and making an outline of all leaves. However since most models consist of hundreds if not thousand of trees this task would be tidies and even though this is possible one would probably be left with more new questions than the ones answered.



(a) (b)

Figure 7: (a): Tree number 101 in $\hat{m}_\mu^{(biased)}(X)$. (b): Tree number 1 in $\hat{m}_p^{(biased)}(X)$.

Above one can see how one selected tree from either one of the decision trees of $\hat{m}_\mu^{(biased)}$ and $\hat{m}_p^{(biased)}$ contribute to the estimate. For instance for the severity model we see that the space $\mathcal{X}$ is separated into four leaves where the estimate up until tree number 100 will be increased with a factor of $\exp(\hat{f}_k(X))$. To be specific we see that

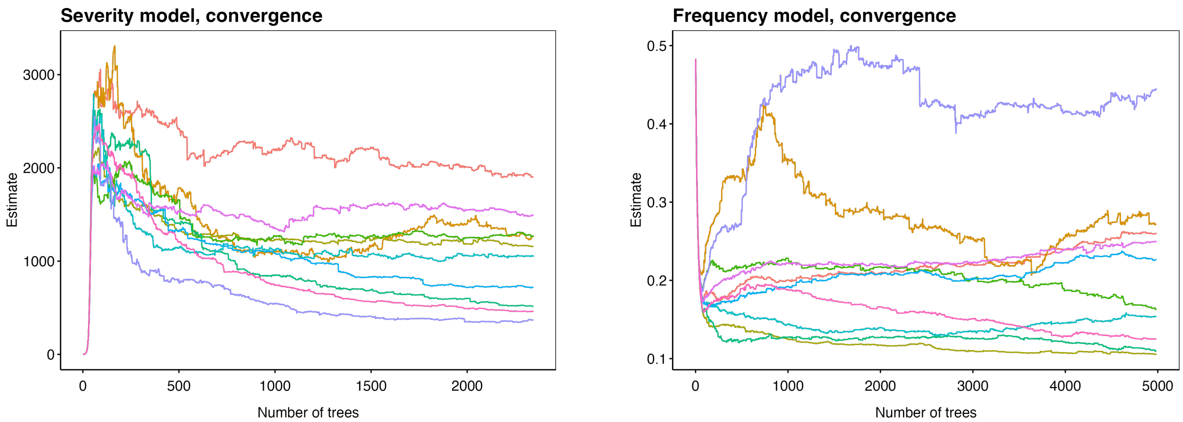
$$\exp(\hat{f}_k(X)) = \begin{cases} 1 + 1.39 \cdot 10^{-3} & \text{if} \quad \text{LicAge} < 438.5 \\ 1 + 184.12 \cdot 10^{-3} & \text{if} \quad 438.5 \leq \text{LicAge} < 439.5 \\ 1 - 161.40 \cdot 10^{-3} & \text{if} \quad \text{LicAge} \geq 439.5 \text{ and MariStat\_Sev} = 0 \\ 1 - 4.44 \cdot 10^{-3} & \text{if} \quad \text{LicAge} \geq 439.5 \text{ and MariStat\_Sev} = 1 \end{cases}$$

Likewise we see that the frequency model changes the estimates with a factor of

$$\exp(\hat{g}_k(X)) = \begin{cases} 1 - 52.46 \cdot 10^{-3} & \text{if} \quad \text{Exposure} < 0.1215 \\ 1 - 46.67 \cdot 10^{-3} & \text{if} \quad 0.1215 \leq \text{Exposure} < 0.4225 \\ 1 - 40.58 \cdot 10^{-3} & \text{if} \quad 0.4225 \leq \text{Exposure} < 0.6875 \\ 1 - 34.10 \cdot 10^{-3} & \text{if} \quad 0.6875 \leq \text{Exposure} \end{cases}$$

The reader will notice that exposure is the first variable used in improving the estimate indicating the the probability of an insurance event is highly correlated with the amount of exposure. This is consistent with the assumption that the number of claims is poisson distributed and claims arrives as a poisson count process.

Running through every step we can plot how the model estimate approaches a value for each added tree. Above we see how the algorithm predicts the first 10 policies by applying the improvement $\exp(\hat{f}_k(X))$ for



(a) (b)

Figure 8: Convergence of estimator for the first 10 policies.

$k = 1, ..., K$ and $\exp(\hat{g}_l(X))$ for $l = 1, ..., L$ for the two estimator. This picture gives us an indication of how robust the estimates are, but if we want to decompose the final estimate into contributions from each covariates we will have to turn to another tool.

Breakdown of the severity estimator

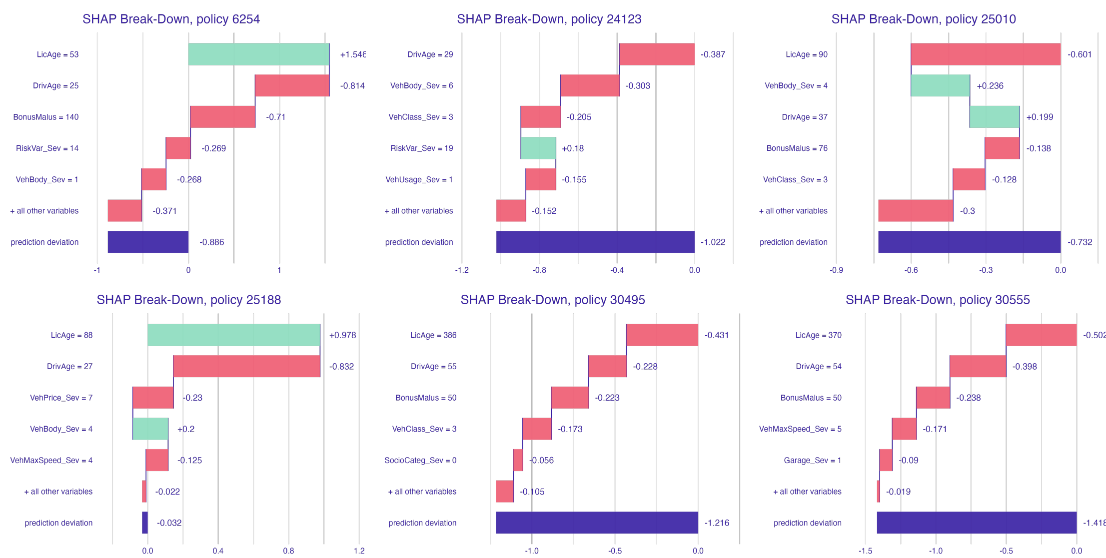


Figure 9: Decomposition of the severity estimates for the policies 6254, 24123, 25010, 25188, 30495 and 30555. The intercept of the model is approximatly 2550. The prediction deviation gives the logarithm of the factor multiplier. That is the estimate for the policy 6254 is $2550 \cdot \exp(-0.886) \approx 1050$ as seen in table 1.

Breakdown of the frequency estimator

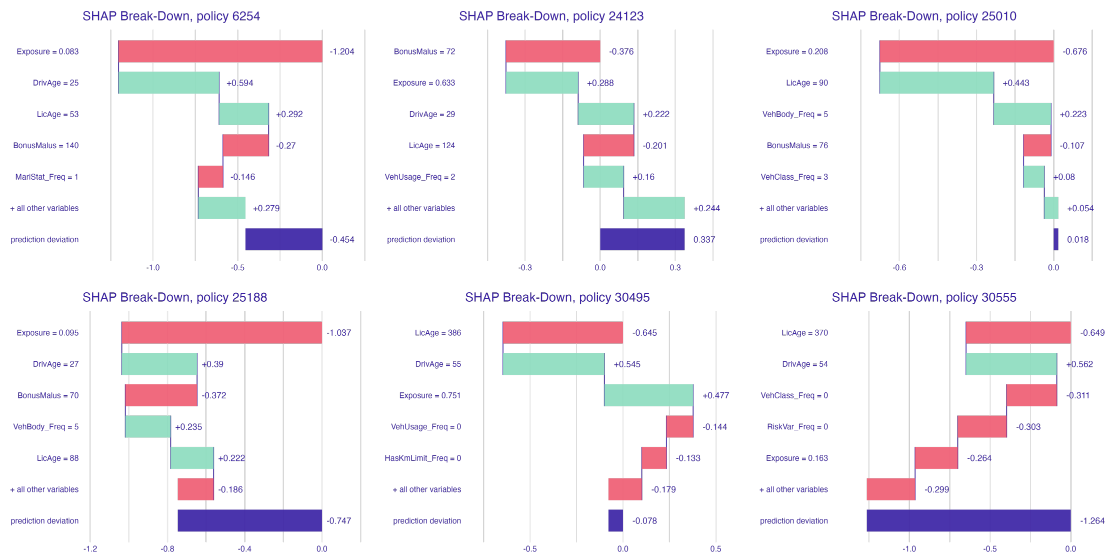


Figure 10: Decomposition of the frequency estimates for the policies 6254, 24123, 25010, 25188, 30495 and 30555. The prediction deviation gives the logarithm of the factor multiplier.

Discrimination and debaissing of the model

Table 2: Comparing premiums

		Biased			Debiased		
Policy	Gender	p(Y>0)	E(Y|Y>0)	Price	p(Y>0)	E(Y|Y>0)	Price
6254	Female	0.0465	1051	48.9	0.0550	1007	55.4
24123	Male	0.1677	917	153.7	0.1469	932	136.9
25010	Female	0.1097	1226	134.4	0.1074	1189	127.7
25188	Male	0.0524	2468	129.4	0.0502	2518	126.4
30495	Female	0.1034	755	78.1	0.0976	804	78.4
30555	Male	0.0338	617	20.9	0.0296	618	18.3

Repreducing statement

Packages

```
library(CASdatasets)
library(lattice)
library(evmix)
library(ggplot2)
library(mlr3)
library(mlr3learners)
library(mlr3extralearners)
library(dbarts)
library(mlr3mbo)
library(mlr3measures)
library(mlr3tuning)
library(ranger)
library(mlr3viz)
library(fastDummies)
library(dplyr)
library(patchwork)
library(xgboost)
library(glex)
library(treeshap)
```

Statistical data

The data we will be working with is the `freMPL1` (*French Motor Personal Line datasets*) dataset from the package `CASdatasets` (*Computational Actuarial Science datasets*). The author write the following regarding the dataset

> This collection of ten datasets comes from a private motor French insurer. Each dataset includes risk features, claim amount and claim history of around 30,000 policies for year 2004.

A detailed description of the variables may be read in the below table.

Variable	Description
Exposure	The exposure, in years.
LicAge	The driving licence age, in months.
RecordBeg	Beginning date of record.
RecordEnd	End date of record.
VehAge	The vehicle age, in years.
Gender	The gender, either "Male" or "Female".
MariStat	The marital status, either "Alone" or "Other".
SocioCateg	The social category known as CSP in France, between "CSP1" and "CSP99".
VehUsage	The vehicle usage among "Private", "Private+trip to office" "Professional", "Professional run".
DrivAge	The driver age, in years (in France, people can drive a car at 18).
HasKmLimit	A numeric, 1 if there is a km limit for the policy, 0 otherwise.
BonusMalus	A numeric for the bonus/malus, between 50 and 350: <100 means bonus, >100 means malus in France.
VehBody	The vehicle body, among "bus","cabriolet","coupe","microvan","othermicrovan",

Variable	Description
VehPrice	The category of the vehicle price from “A” (cheapest) to “Z” (most expensive).
VehEngine	The vehicle engine, among “carburation”,“directinjectionoverpowered”,“electric”, “GPL”, “injection”, “injection overpowered”.
VehEnergy	The vehicle energy, among “diesel”, “eletric”, “GPL”, “regular”.
VehMaxSpeed	The VehMaxSpeed, among “1-130km/h”,“130-140km/h”,“140-150km/h”,“150-160 km/h”, “160-170 km/h”, “170-180 km/h”, “180-190 km/h”, “190-200 km/h”, “200-220 km/h”, “220+ km/h”.
VehClass	The vehicle class (unknown categories), among “0”, “A”, “B”, “H”, “M1”, “M2”.
ClaimAmount	Total claim amount of the guarantee.
RiskVar	Unkonw risk variable between 1 and 20, possibly ordered.
Garage	The garage, if any, among “Collective garage”, “None”, “Private garage”.
ClaimInd	Claim indicator of the guarantee. (this is not the claim number)

Feature formatting

The following changes has been made to the data `freMPL1`:

1. The columns `RecordBeg` and `RecordEnd` are discarded.
2. The column `ID` is added to remember policynumbers.
3. The negative `ClaimAmount` entries is overwritten with zeroes and the `ClaimIndicator` is changed accordingly.
4. The observations `VehEngine` being equal to either `electric` or `GPL` is removed.
5. The levels A-C, R-T and U-Z are combined in `VehPrice`.
6. The lowest two `VehMaxSpeed` are combined.
7. The category `bus` is layed under `sedan` in the column `VehBody`.
8. The column `SocioCateg` is combined into three layers: `A`, `B` and `C`.

These changes may be read in the script below.

```
## Feature selection and formatting
data("freMPL1",package = "CASdatasets")

#1: RecordBeg and RecordEnd discarded
freMPL1 <- freMPL1 %>%
  select(-RecordBeg,-RecordEnd)

#2: Add id's
freMPL1[,"ID"] <- 1:dim(freMPL1)[1]

#3: Claim amount and indicator
freMPL1$ClaimAmount[freMPL1$ClaimAmount<0] <- 0
freMPL1$ClaimInd <- ifelse(freMPL1$ClaimAmount>0,1,0)

#4: Electric or GPL vehicals
freMPL1 <- freMPL1 %>%
  filter(!(VehEngine %in% c("electric","GPL")))
```

```r
#5: Combining price categories
levels(freMPL1$VehPrice)[1:3] <- "A-C"
n <- length(levels(freMPL1$VehPrice))
levels(freMPL1$VehPrice)[(n-5):n] <- "U-Z"
n <- length(levels(freMPL1$VehPrice))
levels(freMPL1$VehPrice)[(n-3):(n-1)] <- "R-T"

#6: Combining max speed levels
levels(freMPL1$VehMaxSpeed)[1:2] <- "1-140 kmh"

#7: Bus set to sedan
levels(freMPL1$VehBody)[levels(freMPL1$VehBody) == "bus"] <- "sedan"

#8: SocioCateg change levels
freMPL1 <- freMPL1 %>%
  #Get numerical value of SocioCateg
  mutate(helper = as.numeric(substr(SocioCateg,4,5))) %>%
  #Overwrite SocioCateg
  mutate(SocioCateg = factor(ifelse(helper > 50, "C",
                                    ifelse( helper < 50, "A",
                                            "B")),
                             levels = c("A","B","C"))) %>%
  select(-helper)
```

Preparing the dataset

We lastly format the data by creating a data frame called `data` which we will henceforth be referring to. The data frame is created with splitting the catagorical variables into sorted numerical variables by ordering on both severity and frequency. For instance the `RiskVar` column are translated via the table:

Table 4: RiskVar transformation

Before	Metrics		After	
RiskVar	Frequency	Severity	RiskVar_Freq	RiskVar_Sev
1	0.259	1751	16	4
2	0.231	1597	6	2
3	0.203	1811	2	5
4	0.283	3259	17	19
5	0.243	1820	9	6
6	0.228	1589	4	1
7	0.244	2170	10	11
8	0.237	2860	7	16
9	0.294	1730	19	3
10	0.257	1854	14	7
11	0.249	2107	12	8
12	0.194	1438	1	0
13	0.230	2560	5	13
14	0.185	2158	0	9
15	0.204	2611	3	14
16	0.258	2480	15	12
17	0.248	2918	11	17

Table 4: RiskVar transformation *(continued)*

Before	Metrics		After	
RiskVar	Frequency	Severity	RiskVar_Freq	RiskVar_Sev
18	0.252	2165	13	10
19	0.241	2619	8	15
20	0.290	3109	18	18

The code is implemented below.

```
#Create df
df <- freMPL1
#Catagorical variables
cat_variables <- c("Gender","VehAge","MariStat","SocioCateg",
                   "VehUsage","HasKmLimit","VehBody","VehPrice","VehEngine",
                   "VehEnergy","VehMaxSpeed","VehClass","RiskVar","Garage")
for (col in cat_variables) {
  #Get ordering
  ord <- freMPL1 %>%
    #Group by distinct values
    group_by(!!as.name(col)) %>%
    #Calculate frequency and mean claim
    summarise(Frequency = sum(ClaimInd)/sum(Exposure),
              Severity = sum(ClaimAmount)/sum(ClaimAmount > 0)) %>%
    ungroup() %>%
    #Order Frequency
    arrange(Frequency) %>%
    #Insert 0,...,n
    mutate(Frequency = 0:(length(unique(!!as.name(col)))-1)) %>%
    #Order Severity
    arrange(Severity) %>%
    #Insert 0,...,n
    mutate(Severity = 0:(length(unique(!!as.name(col)))-1))
  #Prepare ord for merging
  colnames(ord)[2:3] <- paste0(col, c("_Freq","_Sev"))

  #Merge new columns into df
  df <- df %>%
    merge(., ord, all.x = TRUE)
}
```

Modelling the technical premium

Severity model

We work with the data `df_sev` which is a subset of `df` with the filter `ClaimAmount > 0`.

```
#Get relevant columns
sev_columns <- colnames(df)[!grepl("_Freq",colnames(df)) &
                              !(colnames(df) %in% cat_variables)]
df_sev <- df[,sev_columns] %>%
  filter(ClaimInd == 1) %>%
  select(-ClaimInd,-Exposure)
#Save ID in row names
row.names(df_sev) <- df_sev$ID
df_sev <- df_sev %>%
  select(-ID)%>%
  mutate_if(is.integer, as.numeric)
```

```

#Start a task
task_sev <- df_sev %>%
  #Start tast with target ClaimAmount
  as_task_regr(.,
               target = "ClaimAmount",
               id= "Severity")

### XGBoost
sev_xgb_learner <- lrn("regr.xgboost",
                   eta = to_tune(0, 0.5),
                   nrounds = to_tune(75, 3000),
                   max_depth = to_tune(1, 2))

### XGBoost
set.seed(20230328) #We choose a seed
# 1: Estimating hyperparameters with 5-fold cross validation
sev_xgb_learner_instance = tune(
  #method = tnr("random_search"), ### tuning method
  method = mlr3tuning::tnr("mbo"), ### tuning method
  task = task_sev,
  learner = sev_xgb_learner,
  resampling = rsmp("cv", folds = 5), #### resampling method: 5-fold cross validation
  measures = msr("regr.rmse"), #### Root Mean Squared Log Error
  terminator = trm("evals", n_evals = 1000) #### terminator
)
#Save the instance
saveRDS(sev_xgb_learner_instance, file = "rds/sev_xgb_learner_instance.rds")
```

Estimating hyperparameters

Table 5: Hyperparameters

		Respone = Y			Respone = log(Y)	
Hyperparameter	Search domain	regr.rmsle	regr.rmse	regr.mse	regr.rmse	regr.mse
eta	[0,0.5]	0.010857	0.2113299	0.318963	0.3076677	0.1096531

Table 5: Hyperparameters *(continued)*

		Respone = Y			Respone = log(Y)	
Hyperparameter	Search domain	regr.rmsle	regr.rmse	regr.mse	regr.rmse	regr.mse
nrounds	[75,3000]	221.000000	2345.0000000	2313.000000	1574.0000000	2996.0000000
max_depth	[1,2]	1.000000	2.0000000	2.000000	2.0000000	2.0000000

```
#Partition for training
indicies <- function(size,ratio){
  set.seed(20230328) #We choose a seed
  train <- sort(sample(1:size,ceiling(size*ratio),replace = FALSE))
  test <- (1:size)[!(1:size %in% train)]
  return(list(train = train, test = test))
}
splits <- indicies(dim(df_sev)[1],0.85)
#Split covariates and truth
regr_data_train <- list(
  covariates = as.matrix(df_sev[splits$train,] %>% select(-ClaimAmount)),
  truth = as.numeric(df_sev$ClaimAmount[splits$train])
)
regr_data_test <- list(
  covariates = as.matrix(df_sev[splits$test,] %>% select(-ClaimAmount)),
  truth = as.numeric(df_sev$ClaimAmount[splits$test])
)
#Fit model using hyperparameters
xgb_regr_train <- xgboost(
  data = regr_data_train$covariates,
  label = regr_data_train$truth,
  eta = 0.2113299,
  nrounds = 2345,
  max_depth = 2,
  objective = "reg:gamma"
)
#Save model
saveRDS(xgb_regr_train, file = "rds/xgb_regr_train.rds")
```

```
#Predictions vs. truth
pred <- predict(xgb_regr_train, newdata = regr_data_test$covariates)
truth <- regr_data_test$truth
mean(pred)
```

Fitting the model

```
## [1] 1855.073
```

```
mean(truth)
```

```
## [1] 2899.917
```

```
#Mean Squared Error
MSE <- mean((pred - truth)^2)
MSE
```

```
## [1] 62934280
```

```r
sqrt(MSE)
```

```
## [1] 7933.113
```

```r
#Split covariates and truth
regr_data <- list(
  covariates = as.matrix(df_sev %>% select(-ClaimAmount)),
  truth = as.numeric(df_sev$ClaimAmount)
)
#Fit model using hyperparameters
xgb_regr <- xgboost(
  data = regr_data$covariates,
  label = regr_data$truth,
  eta = 0.2113299,
  nrounds = 2345,
  max_depth = 2,
  objective = "reg:gamma"
)
mean(regr_data$truth) #2388.372
mean(predict(xgb_regr, newdata = regr_data$covariates)) #1889.226
#Save model
saveRDS(xgb_regr, file = "rds/xgb_regr.rds")
```

Frequency model

We work with the data `df_freq` which is a copy of `df` with the transformed columns `._Freq`.

```
#Get relevant columns
freq_columns <- colnames(df)[!grepl("_Sev",colnames(df)) &
                                !(colnames(df) %in% cat_variables)]
df_freq <- df[,freq_columns] %>%
  select(-ClaimAmount) %>%
  mutate(ClaimInd = factor(ClaimInd, levels = c(0,1)))
#Save ID in row names
row.names(df_freq) <- df_freq$ID
df_freq <- df_freq %>%
  select(-ID) %>%
  mutate_if(is.integer, as.numeric)
```

```
#Start a task
task_freq <- df_freq %>%
  #Start tast with target ClaimInd
  as_task_classif(.,
             target = "ClaimInd",
             id= "Frequency")
#XGB
freq_xgb_learner <- lrn("classif.xgboost",
                        eta = to_tune(0, 0.5),
                        nrounds = to_tune(3000, 10000),
                        max_depth = to_tune(1, 2),
                        predict_type = "prob")
freq_xgb_learner_instance = tune(
    #method = tnr("random_search"), ### tuning method
    method = mlr3tuning::tnr("mbo"), ### tuning method
    task = task_freq,
    learner = freq_xgb_learner,
    resampling = rsmp("cv", folds = 5), #### resampling method: 5-fold cross validation
    measures = msr("classif.logloss"), #### log loss
    terminator = trm("evals", n_evals = 500) #### terminator
  )
saveRDS(freq_xgb_learner_instance, file = "rds/freq_xgb_learner_instance.rds")
```

Estimating hyperparameters

Hyperparamater	Search space	Estimate
XGBoost		
`eta`	$[0, 0.5]$	0.1127201
`nrounds`	$[75, 5000]$	4986
`max_depth`	$[1, 2]$	2

```
#Partition for training
set.seed(20230328) #We choose a seed
splits <- indicies(dim(df_freq)[1],0.85)
#Split covariates and truth
freq_data_train <- list(
```

```
  covariates = as.matrix(df_freq[splits$train,] %>% select(-ClaimInd)),
  truth = as.numeric(as.character(df_freq$ClaimInd[splits$train]))
)
freq_data_test <- list(
  covariates = as.matrix(df_freq[splits$test,] %>% select(-ClaimInd)),
  truth = as.numeric(as.character(df_freq$ClaimInd[splits$test]))
)
#Fit model using hyperparameters
xgb_freq_train <- xgboost(
  data = freq_data_train$covariates,
  label = freq_data_train$truth,
  eta = 0.1127201,
  nrounds = 4986,
  max_depth = 2,
  objective="count:poisson"
)
#Save model
saveRDS(xgb_freq_train, file = "rds/xgb_freq_train.rds")
```

```
#Predictions vs. truth
pred <- predict(xgb_freq_train, newdata = freq_data_test$covariates)
truth <- freq_data_test$truth
mean(pred)
```

Fitting the model

```
## [1] 0.1040023
```

```
mean(truth)
```

```
## [1] 0.1096338
```

```
#log loss
ll_loss <- function(y,p) {
  p <- ifelse(p == 1, 1-0.0000001,p)
  p <- ifelse(p==0, 0.0000001, p)
  ll <- -( y * log(p) + (1 - y)*log(1-p))
  ll <- ifelse(is.na(ll),0,ll)
  return(ll)
}
mean(ll_loss(truth,pred))
```

```
## [1] 0.3083776
```

```
#Split covariates and truth
freq_data <- list(
  covariates = as.matrix(df_freq %>% select(-ClaimInd)),
  truth = as.numeric(as.character(df_freq$ClaimInd))
)
#Fit model using hyperparameters
xgb_freq <- xgboost(
  data = freq_data$covariates,
  label = freq_data$truth,
  eta = 0.1127201,
  nrounds = 4986,
  max_depth = 2,
```

```
  objective="count:poisson"
)
mean(freq_data$truth) # 0.106712
mean(predict(xgb_freq, newdata = freq_data$covariates)) # 0.1067164
#Save model
saveRDS(xgb_freq, file = "rds/xgb_freq.rds")
```

Estimating the biased technical premium

```
# Estimating mu
sev_covariates <- df[,sev_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount,-Exposure)
sev_covariates <-  as.matrix(sev_covariates)
m_mu <- predict(xgb_regr, newdata = sev_covariates)
# Estimating p
freq_covariates <- df[,freq_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount)
freq_covariates <- as.matrix(freq_covariates)
m_p <- predict(xgb_freq_train, newdata = freq_covariates)
freq_covariates[,"Exposure"] <- 1
m_p_2 <- predict(xgb_freq_train, newdata = freq_covariates)
# Inserting estimates
df[,"pi_hat"] <- m_mu * m_p
df[,"mu_hat"] <- m_mu
df[,"p_hat"] <- m_p
df[,"p_hat_new"] <- m_p_2
df[,"pi_hat_new"] <- m_mu * m_p_2
```

Extracting SHAP values

```
#### Severity
unified_xgb_regr <- xgboost.unify(xgb_regr,
                                  data=sev_covariates)
treeshap_xgb_regr <- treeshap(unified_xgb_regr,
                              sev_covariates,
                              verbose = 0)
saveRDS(treeshap_xgb_regr, file = "rds/treeshap_xgb_regr.rds")
#### Frequency
unified_xgb_freq <- xgboost.unify(xgb_freq,
                                  data=freq_covariates)

treeshap_xgb_freq <- treeshap(unified_xgb_freq,
                              freq_covariates,
                              verbose = 0)
saveRDS(treeshap_xgb_freq, file = "rds/treeshap_xgb_freq.rds")
```

Debiassing the premium

```
#### Severity
sev_covariates <- df[,sev_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount,-Exposure) %>%
  mutate_if(is.integer, as.numeric) %>%
```

```r
  as.matrix()
#Decomposing the effects from all interactions
glex_sev <- glex(xgb_regr, sev_covariates)
saveRDS(glex_sev, file = "rds/glex_sev.rds")

# Recalibrating
## Somehow log(prob) - glex = -1.19 for all predictions
## We shift the intercept
xgb_sev_pred <- predict(xgb_regr, newdata = sev_covariates)
glex_sev_pred <- glex_sev$intercept + rowSums(glex_sev$m)
shift <- mean(log(xgb_sev_pred) - glex_sev_pred)
glex_sev$intercept <- glex_sev$intercept + shift

## Debiasing
# Remove gender effects
calibrated_sum <- sum(exp(glex_sev$intercept + rowSums(glex_sev$m)))

glex_sev$m <- glex_sev$m %>%
  select(-contains("Gender_Sev"))
# Construct unbiased estimate of mu
unbias_sev <- exp(glex_sev$intercept + rowSums(glex_sev$m))
# Scale up estimates
unbias_sev <- unbias_sev * calibrated_sum / sum(unbias_sev)
df[,"mu_hat_debiased"] <- unbias_sev

#### Frequency
freq_covariates <- df[,freq_columns] %>%
  select(-ClaimInd,-ID,-ClaimAmount) %>%
  mutate_if(is.integer, as.numeric) %>%
  as.matrix()
#Decomposing the effects from all interactions
glex_freq <- glex(xgb_freq, freq_covariates)
saveRDS(glex_freq, file = "rds/glex_freq.rds")
# Version with exposure 1
freq_covariates_2 <- freq_covariates
freq_covariates_2[,"Exposure"] <- 1
glex_freq_2 <- glex(xgb_freq, freq_covariates_2)
saveRDS(glex_freq_2, file = "rds/glex_freq_2.rds")

# Recalibrating
## Somehow log(prob) - glex = -1.19 for all predictions
## We shift the intercept
xgb_freq_pred <- predict(xgb_freq, newdata = freq_covariates)
glex_freq_pred <- glex_freq$intercept + rowSums(glex_freq$m)
shift <- mean(log(xgb_freq_pred) - glex_freq_pred)
glex_freq$intercept <- glex_freq$intercept + shift
xgb_freq_pred <- predict(xgb_freq, newdata = freq_covariates_2)
glex_freq_pred <- glex_freq_2$intercept + rowSums(glex_freq_2$m)
shift <- mean(log(xgb_freq_pred) - glex_freq_pred)
glex_freq_2$intercept <- glex_freq_2$intercept + shift

## Debiasing
# Remove gender effects
```

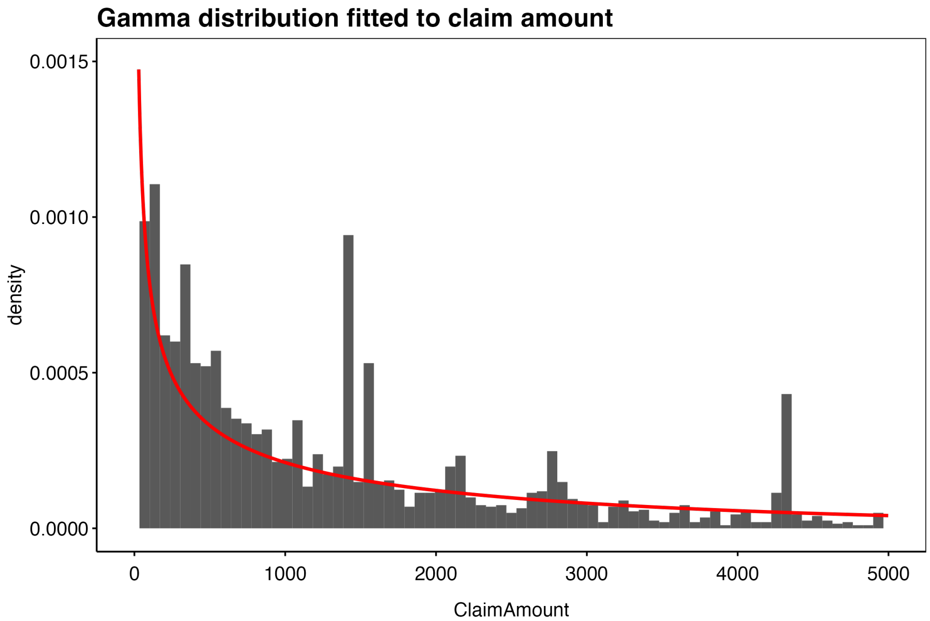
```
calibrated_sum <- sum(exp(glex_freq$intercept + rowSums(glex_freq$m)))
calibrated_sum_2 <- sum(exp(glex_freq_2$intercept + rowSums(glex_freq_2$m)))
glex_freq$m <- glex_freq$m %>%
  select(-contains("Gender_Freq"))
glex_freq_2$m <- glex_freq_2$m %>%
  select(-contains("Gender_Freq"))

# Construct unbiased estimate of mu
unbias_freq <- exp(glex_freq$intercept + rowSums(glex_freq$m))
unbias_freq_2 <- exp(glex_freq_2$intercept + rowSums(glex_freq_2$m))
# Scale up estimates
unbias_freq <- unbias_freq * calibrated_sum / sum(unbias_freq)
unbias_freq_2 <- unbias_freq_2 * calibrated_sum_2 / sum(unbias_freq_2)
df[,"p_hat_debiased"] <- unbias_freq

# Calculate premiums
unbias <- unbias_freq*unbias_sev
df[,"pi_hat_debiased"] <- unbias
df[,"pi_hat_new_debiased"] <- unbias_freq_2*unbias_sev
```

Appendicies

Distributions of severity



One may fit samples $X_1, ..., X_N$ of iid $\Gamma(\alpha, \beta)$ distributed variables using MLE methods. In this notation we have $\alpha > 0$ being the shape parameter and $\beta > 0$ being the rate. One may calculate an estimate of α and β using:

$$\hat{\alpha}_N = \frac{N \sum_{i=1}^{N} X_i}{N \sum_{i=1}^{N} \log(X_i) X_i - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} \log(X_i)\right)}$$

and

$$\frac{1}{\hat{\beta}_N} = \frac{1}{N^2}\left(N \sum_{i=1}^{N} \log(X_i) X_i - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} \log(X_i)\right)\right).$$