Non-Markov Modelling

Conditional Nelson-Aalen and Aalen-Johansen estimation

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Questions

Main question:

How costly is the Markov assumption when modelling occupation probabilities and transition rates for Non-Markov processes?

Sub questions:

1. What choices are there for modelling Non-Markov processes?
2. What are the practical consideration between Markov and Non-Markov modelling?
   2.1 How severe consequences may occur under a Markov assumption?
   2.2 Can one measure the error for each model? Inherent (model) risk and sample size (approximation) risk?

Methods

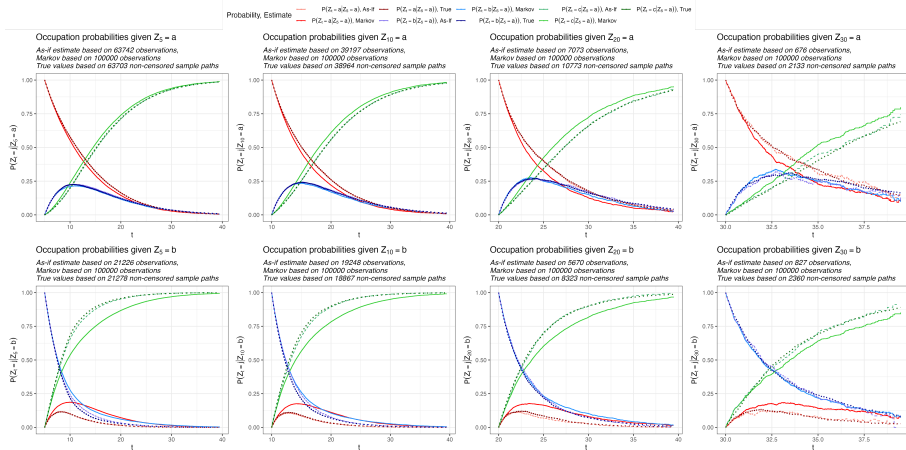1. Establish, under right-censoring, consistent estimator for the occupation probability and transition rates

$$p^k(t|Z_s = j) = \mathbb{E}\left[\mathbb{1}_{\{Z_t = k\}} \middle| Z_s = j\right], \quad M^{ki}(t|Z_s = j) = \mathbb{E}\left[N^{ki}(t) - N^{ki}(s) \middle| Z_s = j\right].$$

2. Study, through examples, the effect of modelling estimators under a Markov assumption or As-If Markov model on the cash-flow.

3. Simulate different sample sizes to study the Model risk and approximation risk of the Markov model.

4. Compare with Monte Carlo estimate for the true theoretical values.

Insights

1. The conditional Aalen-Johnasen and Nelson-Aalen estimator is a consistent estimators for the occupation probabilities and cumulative transition rates and cash-flows may be calculated conventionally.

2. When the Markov property is reasonable the approximation risk of the As-If model may be severe.

3. In general, the As-If approach leads to significantly lower model risk but is costly due to sub-sampling

4. On low probability sets $\{Z_s = j\}$ high approximation risk may make the As-If approach inferior.

5. In the studied Semi-Markov model, at least $L = 50,000$ samples may be needed for comparable approximation risk for $\hat{p}_k^{(L)}(t|Z_5 = b)$.

Occupation probabilities Semi-Markov model

Cash-Flows Semi-Markov model