

Trigonometry

Hertzberg, Joakim D.

April 10, 2024

MAT03c: Mathematics 3c

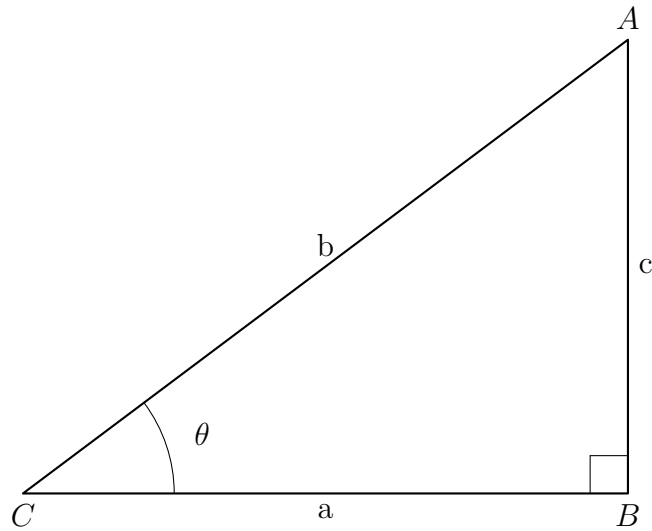


Figure 1: A right angle triangle $\triangle ABC$

1 Trigonometric Functions

1.1 The *sin* & *cos* functions

The sin function is defined as *the ratio between the opposite side and hypotenuse of a right angle triangle, at a certain angle θ* . The cos function, on the other hand is defined as *the ratio between the adjacent side and hypotenuse in a right triangle, defined at a certain angle θ* .

In the above example, $\sin(\theta)$ is defined as follows:

$$\sin(\theta) = \frac{c}{b}$$

And $\cos(\theta)$ as such:

$$\cos(\theta) = \frac{a}{b}$$

1.1.1 On a graph

1.2 The *tan* function

The tan function is defined as *the ratio between the opposite, and adjacent side of a right triangle, defined at a certain angle θ* . See Fig 1. Hence, we may define $\tan(\theta)$ as such:

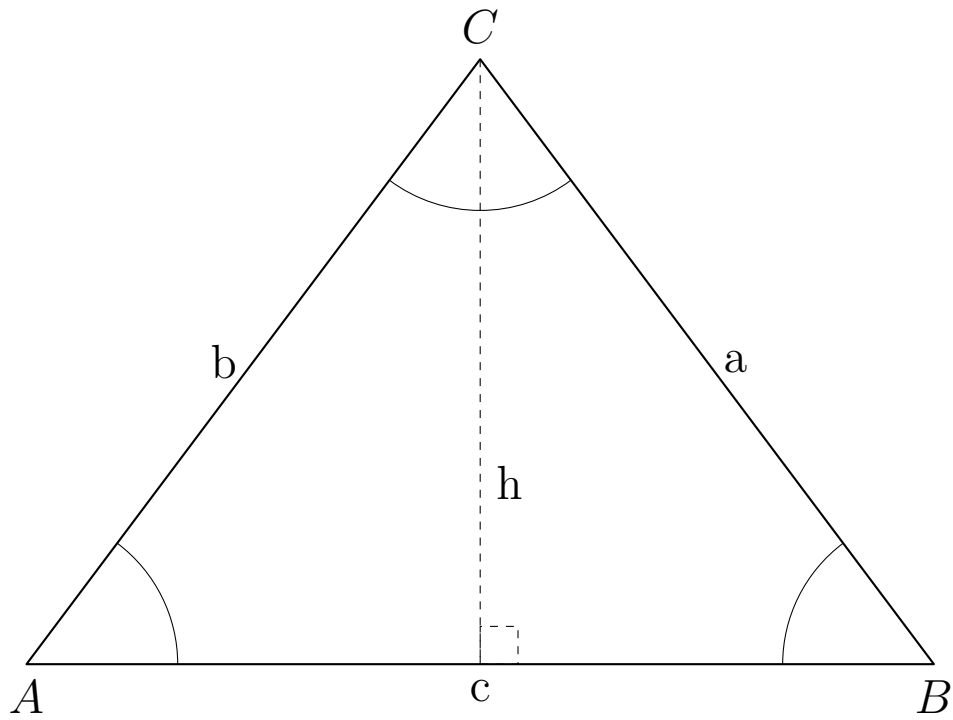
$$\cos(\theta) = \frac{c}{a}$$

1.2.1 On a graph

1.3 On non-right triangles

1.3.1 Finding Areas

Acute triangles



In a triangle like this, $A = \frac{ac \sin(B)}{2}$

Proof:

Assume that $A = \frac{ch}{2}$

$$\frac{ac \sin(\angle B)}{2} = \frac{ch}{2}$$

$$\frac{a \sin(\angle B)}{2} = \frac{h}{2}$$

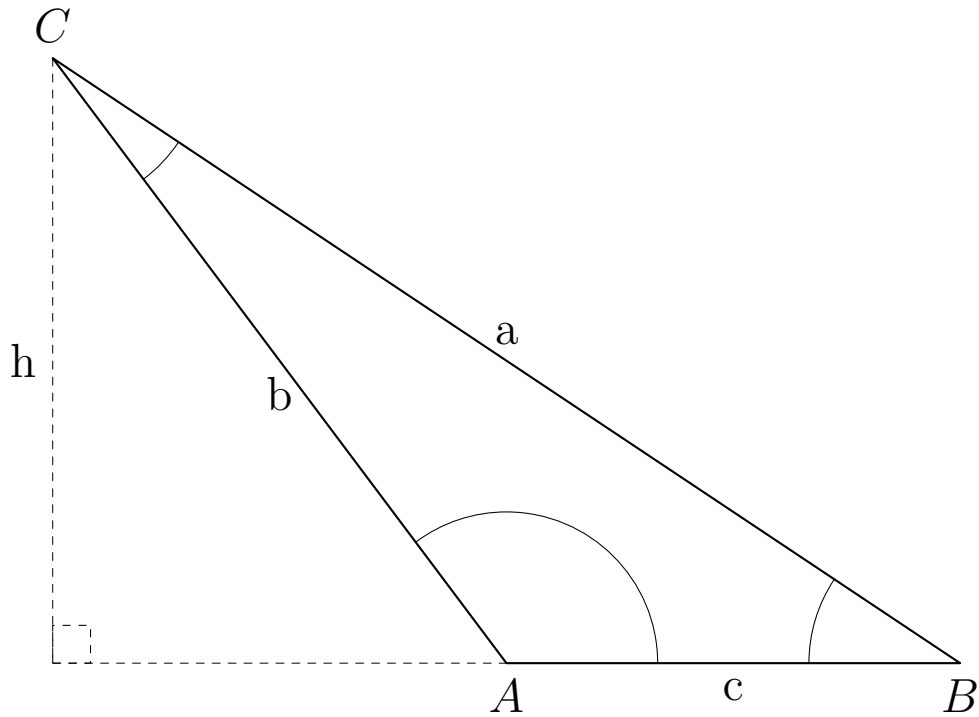
$$a \sin(\angle B) = h$$

$$\therefore \sin(\angle B) = \frac{h}{a} \implies a \sin(\angle B) = h$$

$$h = h$$

$$LHS = RHS \quad \square$$

Obtuse triangles



For triangles like above, it is true that $A = \frac{cb \sin(\angle A)}{2}$

Proof: Assume that $A = \frac{ch}{2}$

$$\frac{cb \sin(\angle A)}{2} = \frac{ch}{2}$$

$$\frac{b \sin(\angle A)}{2} = \frac{h}{2}$$

$$b \sin(\angle A) = h$$

$$\because (\sin(180^\circ - \angle A) = \sin(\angle A)) \wedge \sin(\angle A) = \frac{h}{b} \implies b \sin(\angle A) = h$$

$$h = h$$

$$LHS = RHS \quad \square$$

Summary

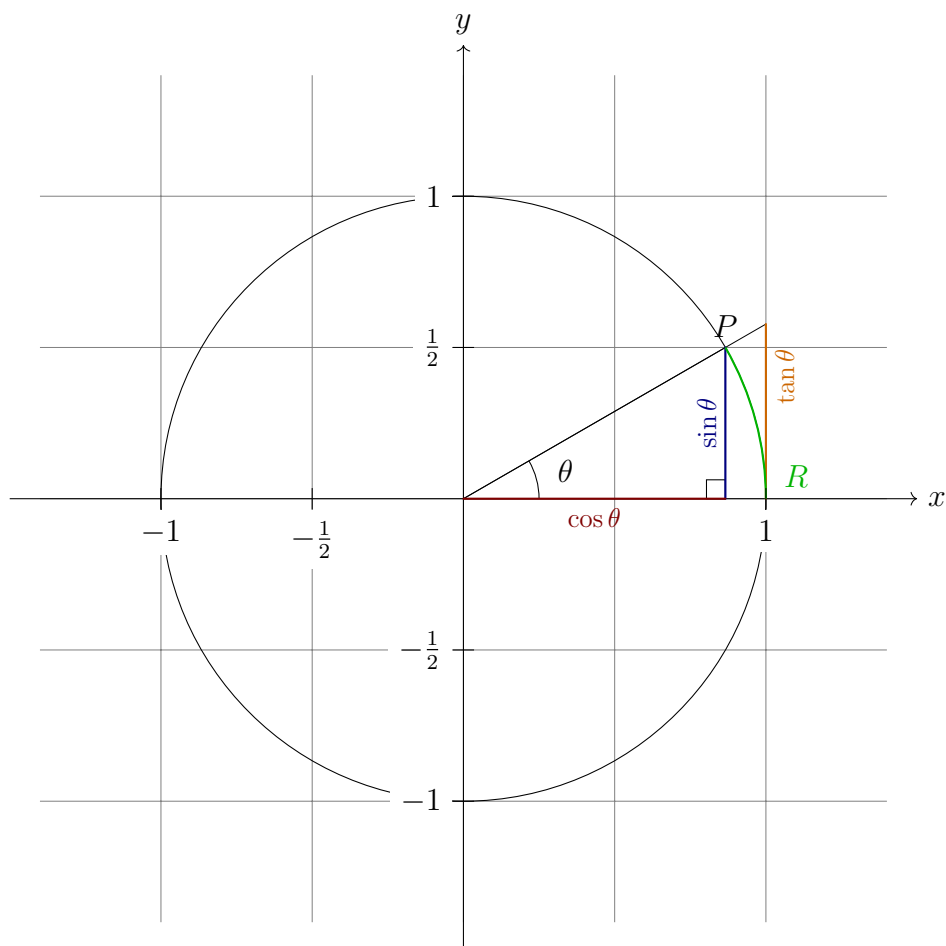
This then implies that *the sine of an interior angle multiplied by it's adjacent sides, all over 2, gives the area of the triangle.*

1.4 The functions *cot*, *csc*, and *sec*

These functions are all reciprocals of their respective trigonometric functions in 1.1 & 1.2.

2 The Unit Circle

The *Unit Circle* is a circle with a radius of 1.



In the Unit Circle, a certain point P , defined as the corner of a right triangle with a certain angle θ , which is lying on the circumference of the unit circle, has following coordinates:

$$P = (\cos \theta, \sin \theta)$$

For a circle for which $r \neq 1$ is true, it can be more generally represented as:

$$P = (r \cos \theta, r \sin \theta)$$

3 The trigonometric Functions on a Graph

