Trigonometry

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MAT03c: Mathematics 3c

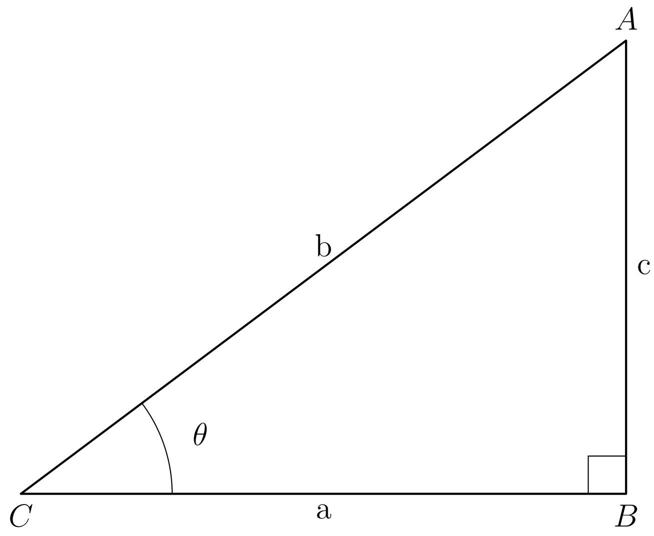


Figure 1: A right angle triangle $\triangle ABC$

1 Trigonometric Functions

1.1 The *sin* & *cos* functions

The sin function is defined as *the ratio between the opposite side and hypotenuse of a right angle triangle, at a certain angle θ.* The cos function, on the other hand is defined as *the ratio between the adjacent side and hypotenuse in a right triangle, defined at a certain angle θ.*

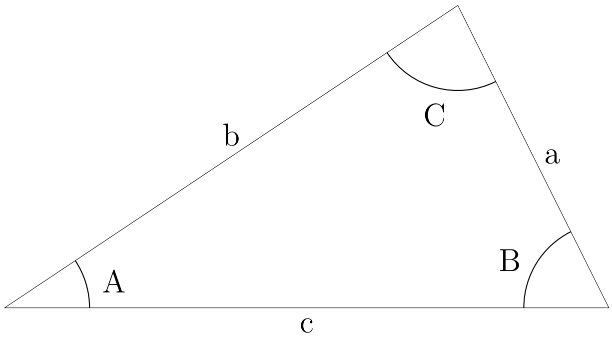
In the above example, $\sin(\theta)$ is defined as follows:

$$\sin(\theta) = \frac{c}{b}$$

And $\cos(\theta)$ as such:

$$\cos(\theta) = \frac{a}{b}$$

1.1.1 The Law of Sines



For this triangle it is true that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

1.2 The *tan* function

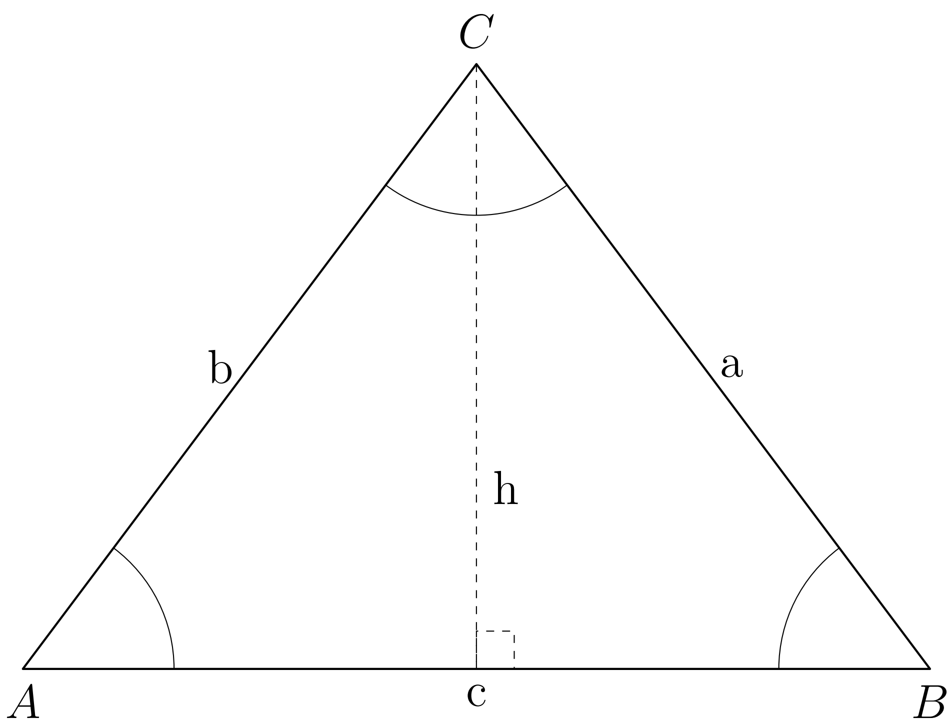
The tan function is defined as *the ratio between the opposite, and adjacent side of a right triangle, defined at a certain angle θ.* See *Fig 1.* Hence, we may define $\tan(\theta)$ as such:

$$\cos(\theta) = \frac{c}{a}$$

1.3 On non-right triangles

1.3.1 Finding Areas

Acute triangles



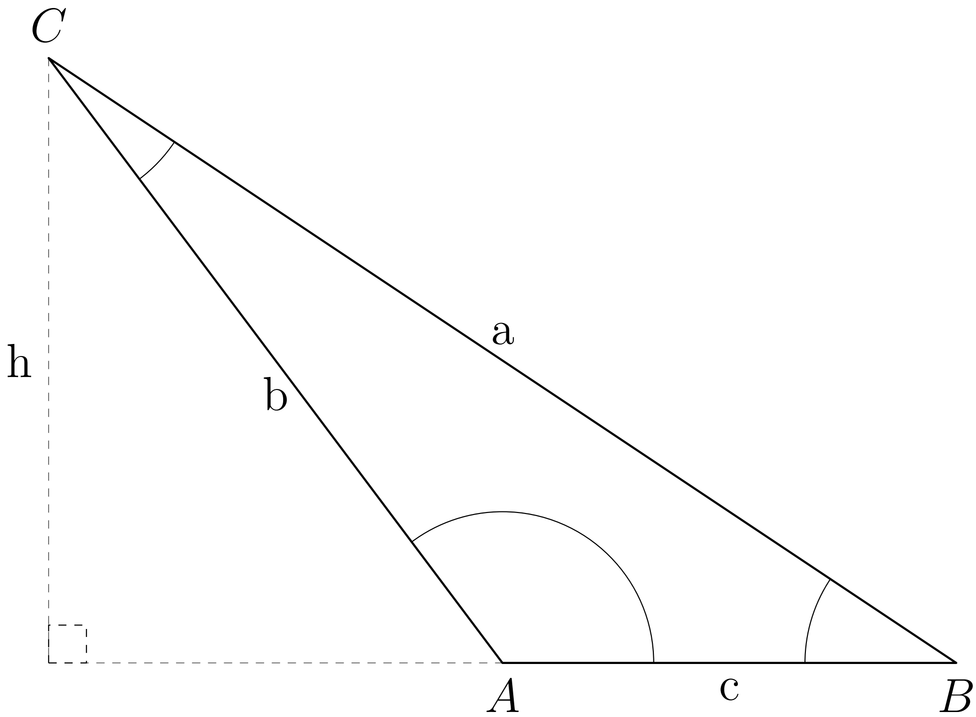
In a triangle like this, $A = \dfrac{ac\sin(B)}{2}$

Proof:

Assume that $A = \frac{ch}{2}$

$$\frac{ac\sin(\angle B)}{2} = \frac{ch}{2}$$
$$\frac{a\sin(\angle B)}{2} = \frac{h}{2}$$
$$a\sin(\angle B) = h$$
$$\because \sin(\angle B) = \frac{h}{a} \implies a\sin(\angle B) = h$$
$$h = h$$
$$LHS = RHS \quad \square$$

Obtuse triangles



For triangles like above, it is true that $A = \frac{cb\sin(\angle A)}{2}$

Proof: Assume that $A = \frac{ch}{2}$

$$\frac{cb\sin(\angle A)}{2} = \frac{ch}{2}$$

$$\frac{b\sin(\angle A)}{2} = \frac{h}{2}$$

$$b\sin(\angle A) = h$$

$$\because (\sin(180\deg - \angle A) = \sin(\angle A)) \land \sin(\angle A) = \frac{h}{b} \implies b\sin(\angle A) = h$$

$$h = h$$

$$LHS = RHS \quad \square$$

Summary

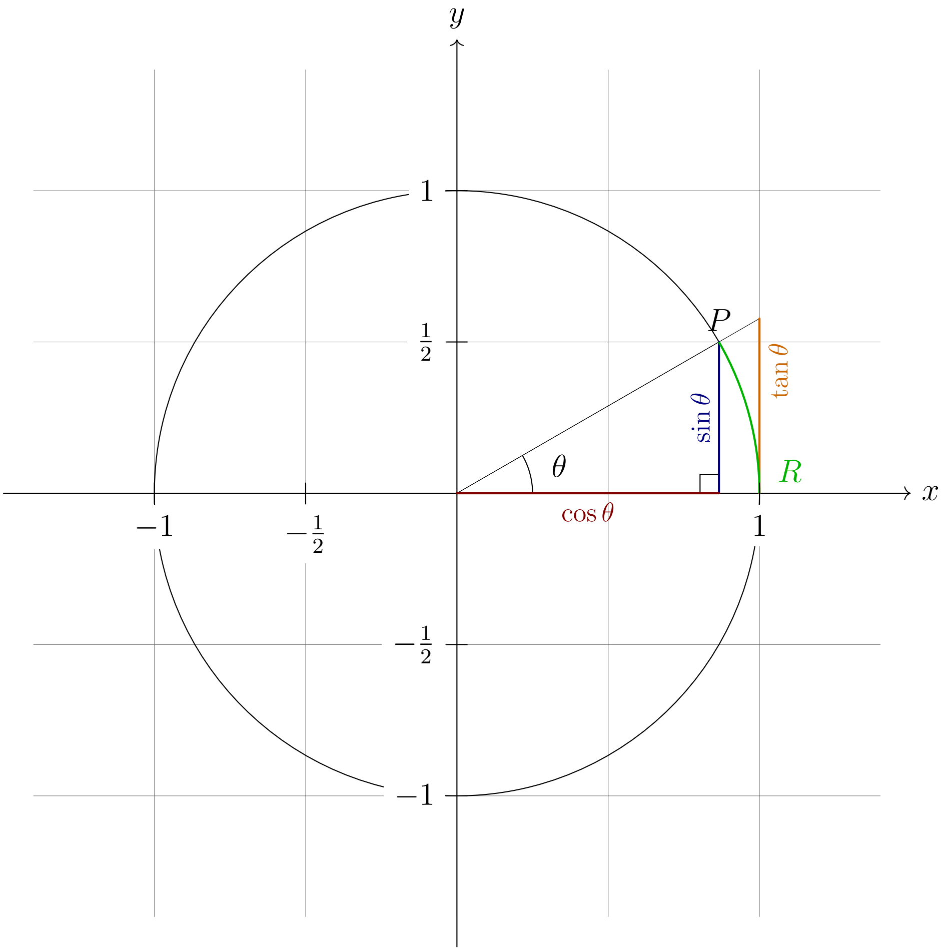
This then implies that *the sine of an interior angle multiplied by it's adjacent sides, all over 2, gives the area of the triangle.*

1.4 The functions ***cot***, ***csc***, and ***sec***

These functions are all reciprocals of their respective trigonometric functions in *1.1* & *1.2*.

2 The Unit Circle

The *Unit Circle* is a circle with a radius of 1.



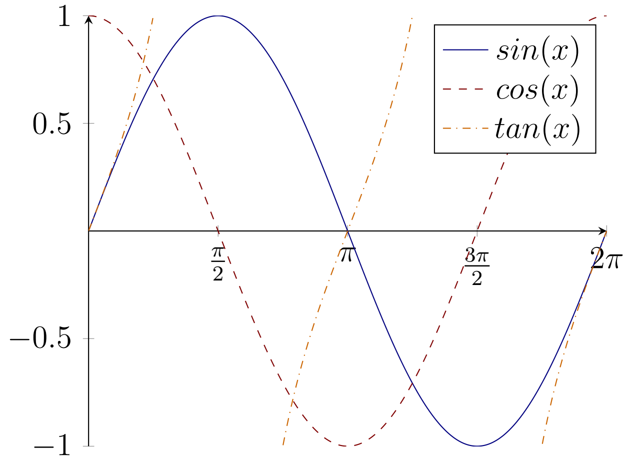
In the Unit Circle, a certain poin P, defined as the corner of a right triangle with a certain angle θ, which is lying on the circumference of the unit circle, has following coordinates:

$$P = (\cos\theta, \sin\theta)$$

For a circle for which $r \neq 1$ is true, it can be more generally represented as:

$$P = (r \ \ \cos\theta, \quad r \ \ \sin\theta)$$

3 The trigonometric Functions on a Graph



Although immediately obscure, the shapes of these functions can be explained by imagining how the lengths of the different functions change as you rotate one revolution around the unit circle.