Trigonometry

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MAT03c: Mathematics 3c

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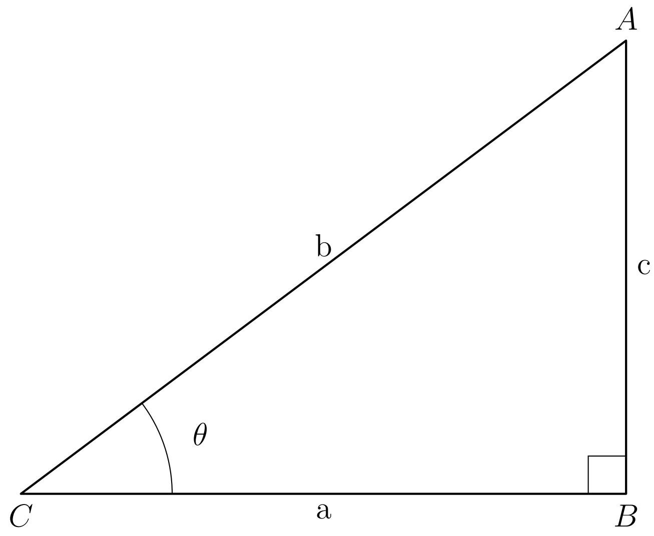


Figure 1: A right angle triangle $\triangle ABC$

1 Trigonometric Functions

1.1 The *sin* & *cos* functions

The sin function is defined as *the ratio between the opposite side and hypotenuse of a right angle triangle, at a certain angle* θ. The cos function, on the other hand is defined as *the ratio between the adjacent side and hypotenuse in a right triangle, defined at a certain angle* θ*.*

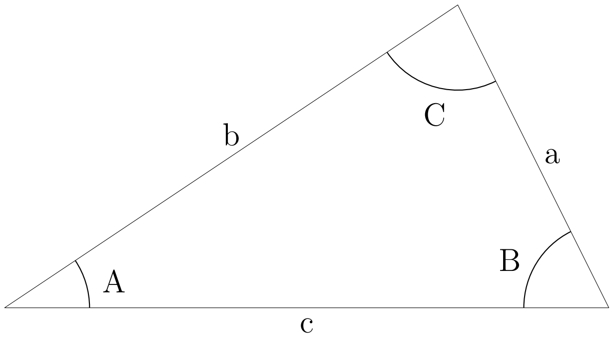
In the above example, $\sin(\theta)$ is defined as follows:

$$\sin(\theta) = \frac{c}{b}$$

And $\cos(\theta)$ as such:

$$\cos(\theta) = \frac{a}{b}$$

1.1.1 The Law of Sines



For this triangle it is true that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

1.2 The ***tan*** function

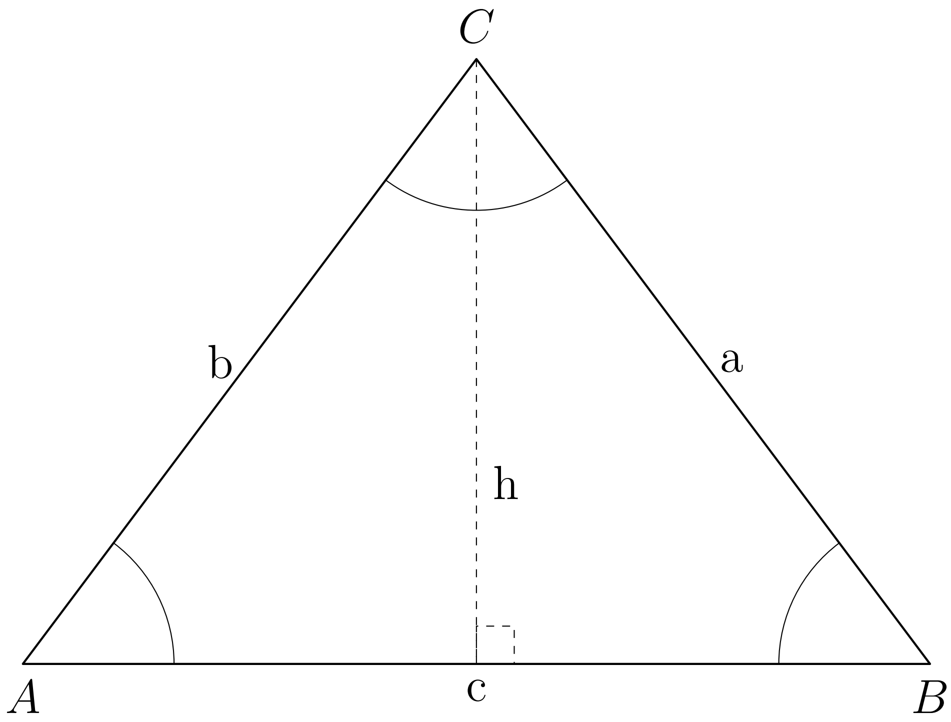
The tan function is defined as *the ratio between the opposite, and adjacent side of a right triangle, defined at a certain angle θ.* See *Fig 1.* Hence, we may define $\tan(\theta)$ as such:

$$\cos(\theta) = \frac{c}{a}$$

1.3 On non-right triangles

1.3.1 Finding Areas

Acute triangles



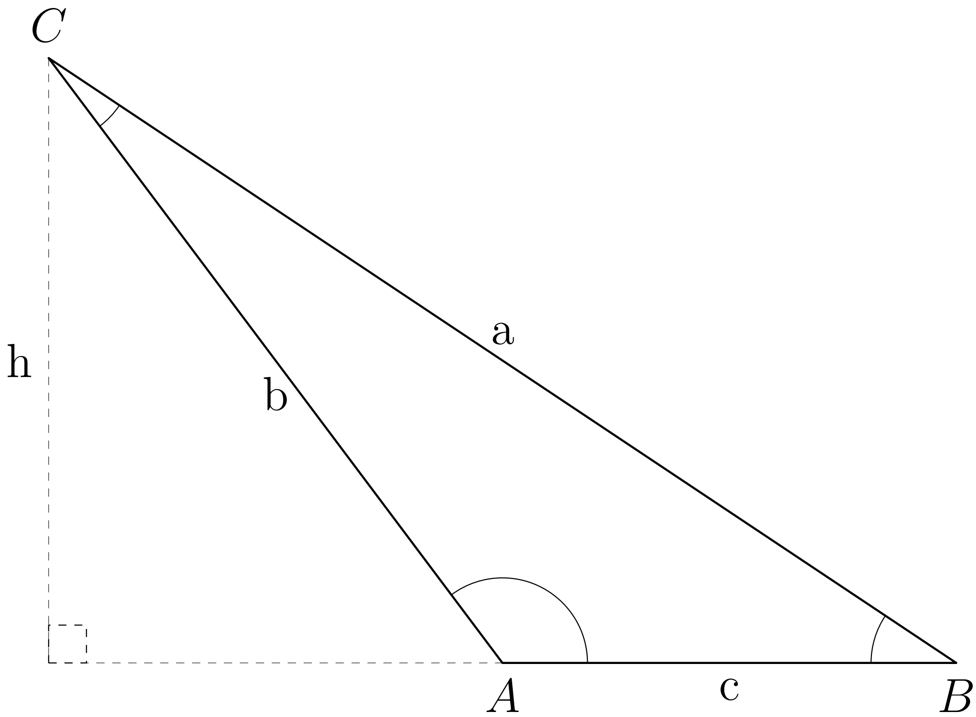
In a triangle like this, $A = \dfrac{ac\sin(B)}{2}$

Proof:

Assume that $A = \frac{ch}{2}$

$$\frac{ac\sin(\angle B)}{2} = \frac{ch}{2}$$
$$\frac{a\sin(\angle B)}{2} = \frac{h}{2}$$
$$a\sin(\angle B) = h$$
$$\because \sin(\angle B) = \frac{h}{a} \implies a\sin(\angle B) = h$$
$$h = h$$
$$LHS = RHS \quad \square$$

Obtuse triangles



For triangles like above, it is true that $A = \frac{cb\sin(\angle A)}{2}$

Proof:

Assume that $A = \frac{ch}{2}$

$$\frac{cb\sin(\angle A)}{2} = \frac{ch}{2}$$

$$\frac{b\sin(\angle A)}{2} = \frac{h}{2}$$

$$b\sin(\angle A) = h$$

$$\because (\sin(180\deg - \angle A) = \sin(\angle A)) \wedge \sin(\angle A) = \frac{h}{b} \implies b\sin(\angle A) = h$$

$$h = h$$

$$LHS = RHS \quad \square$$

Summary

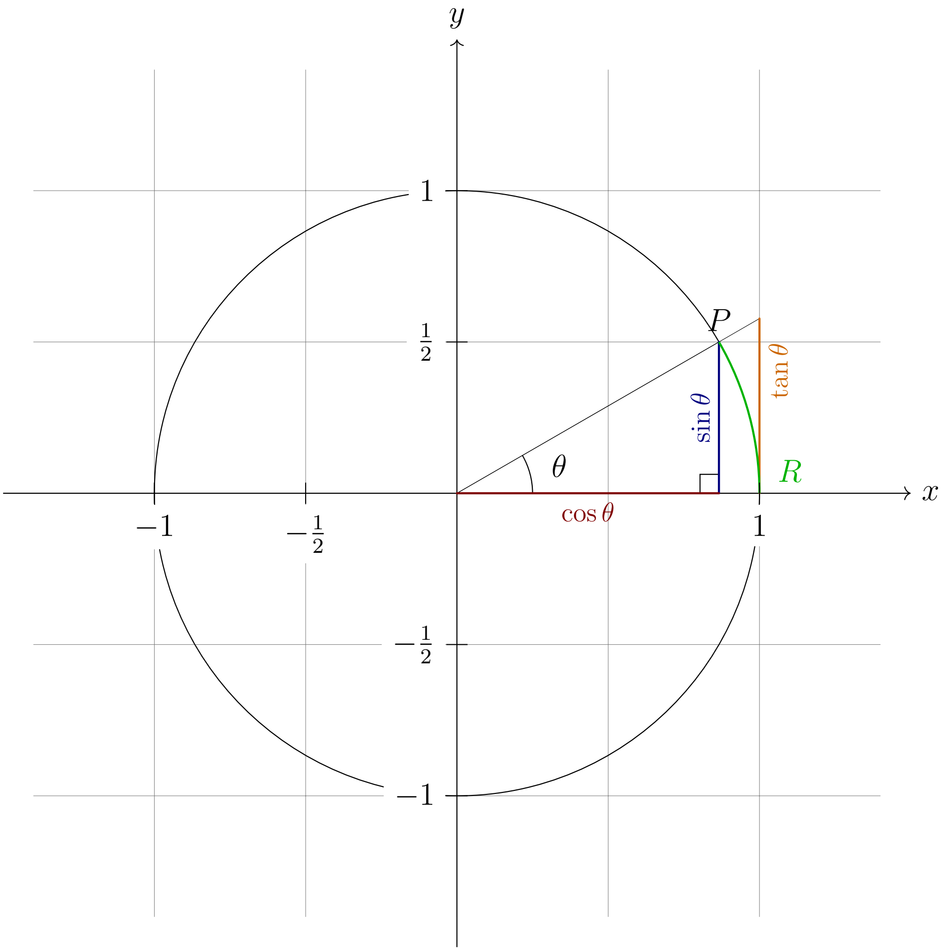
This then implies that *the sine of an interior angle multiplied by it's adjacent sides, all over 2, gives the area of the triangle.*

1.4 The functions *cot*, *csc*, and *sec*

These functions are all reciprocals of their respective trigonometric functions in *1.1* & *1.2*.

2 The Unit Circle

The *Unit Circle* is a circle with a radius of 1.



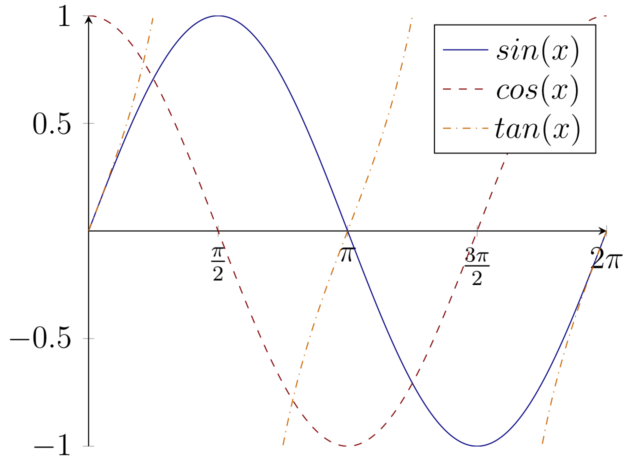
In the Unit Circle, a certain poin P, defined as the corner of a right triangle with a certain angle θ, which is lying on the circumference of the unit circle, has following coordinates:

$$P = (\cos\theta, \sin\theta)$$

For a circle for which $r \neq 1$ is true, it can be more generally represented as:

$$P = (r\ \cos\theta,\ \ r\ \sin\theta)$$

3 The trigonometric Functions on a Graph



Although immediately obscure, the shapes of these functions can be explained by imagining how the lengths of the different functions change as you change θ such that P rotates one revolution around the unit circle.