Summary of the syllabus

1.1 Propositional Logic

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Definition 1: Let p be a proposition. The ***negation*** of p, denoted by $\neg p$ (also denoted by $\bar{p}$), is the statement:

"It is not the case that *p*". The proposition $\neg p$ is read **"not *p*"**.

Definition 2: Let p and q be propositions. The ***conjunction*** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Definition 3: Let p and q be propositions. The ***disjunction*** of p and q, denoted by $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Definition 4: Let p and q be propositions. The ***exclusive*** or of p and q, denoted by ...., is the proposition that is true when exactly one of p and q is true and is false otherwise.

Definition 5: Let p and q be propositions. The ***conditional statement*** $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the ***hypothesis*** and q is called the ***conclusion***.

Definition 6: Let p and q be propositions. The ***biconditional statement*** $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called ***bi-implications***.

Definition 7: A ***bit string*** is a sequence of zero or more bits. The ***length*** of this string is the number of bits in the string.

1.3 Propositional Equivalences

Definition 1: A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a ***tautology***. A compound proposition that is always false is called a ***contradiction***. A compound proposition that is neither a tautology nor a contradiction is called a ***contigency***.

Example: We can construct examples of

Definition 2:

Showing that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

$$\begin{aligned}\neg(p \to q) &\equiv \neg(\neg p \vee q)\\ &\equiv \neg(\neg p \vee q)\\ &\equiv \neg(\neg p) \wedge \neg(q)\\ &\equiv p \wedge \neg q\end{aligned}$$

1.4 Predicates and Quantifiers

1.6 Rules of Inference

1.7 Introduction to Proofs

Note that & is where the equations align.

Example Problem 3

Constructing the *Truth Table* of $(p \to q) \wedge (\neg p \leftrightarrow q)$ in Table 1:

Table 1: Caption here. Leave it blank if you will not refer it.

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Example Problem 4

a) "There is a student in Gryffindor who has taken all elective classes." Solution:

$$\exists x \forall y \forall z (H(x, \text{Gryffindor}) \wedge P(x, y))$$

where

$H(x, z)$ is "x is of z house"
$P(x, y)$ is "x has taken y,"
the domain for x consists of all students in Hogwarts
the domain for y consists of all elective classes,
and the domain for z consists of all Hogwarts houses.

b) Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."
Solution:

1.

$$\forall n(P(n) \to Q(n)),$$

where
$P(n)$ is "n is an odd integer" and
$Q(n)$ is "n^2 is odd."

2. Assume $P(n)$ is true.
3. By definition, an odd integer is $n = 2k + 1$, where k is some integer.
4.

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \end{aligned}$$

5. $\therefore n^2$ is an odd integer. $\square$

c) Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, \{1, 2, 3\}\}$:
Then, $A \in B$ and $A \subseteq B$.

d) Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, \}$, and universe $U = \{1, 2, 3, 4, 5\}$:

$$\begin{aligned} A \cup B &= \{1, 2, 3, 5\}, \\ A \cap B &= \{1, 3\}, \\ A - B &= \{5\}, \\ \bar{A} &= \{2, 4\}, \\ A - A &= \emptyset. \end{aligned}$$