# 3 | Language Competition Dynamics

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## 3.1 | Introduction

In today's interconnected world, languages find themselves without the natural or political borders that many years ago used to separate them. This phenomena has lead to the spread of languages beyond their places of origin giving birth to bilingual communities. In those communities, the coexistence of languages leads to a dynamic in which they compete for dominance in everyday communication.

In this work we will study the language competition dynamics that arises between languages in a bilingual community. In order to do that, we will simulate a competition between two languages and study how the outcome changes when varying different parameters as well as the topology of our model.

## 3.2 | Model Description

To simulate the language competition we will make use of a lattice-like structure with $N = L \times L$ nodes where each node in the lattice is an agent (speaker) of the bilingual community. Each of the nodes are initialized at random with an uniform probability for the different speaking communities.

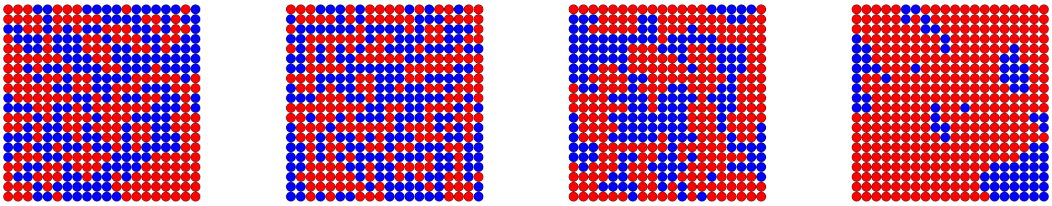
Once the initial state is established we will iterate by a number of $N$ epochs the main algorithm. This will consist in picking a random node $i$ from the lattice and after computing the local densities $\sigma_i$ of each linguistic community in the neighbourhood of agent i we will be able to get the probabilities in which the agent $i$ can change from one linguistic community to another one. Finally, by sampling a random number considering the before mentioned probabilities the linguistic community of the agent will be decided.

We can define two different models for performing this task: The Abrams-Strogatz model and the Bilingual model. Both of them are defined in [14].

The Abrams-Strogatz model supposes that the agents within our community can be in two states. We will call them language A and language B. Their corresponding transition probabilities of them are:

$$p_{i,A->B} = \frac{1}{2}\sigma_i^B \qquad p_{i,B->A} = \frac{1}{2}\sigma_i^A$$

Where $\sigma_i^A$ and $\sigma_i^B$ corresponds to the local densities of language users of each linguistic community in the neighborhood of agent i.

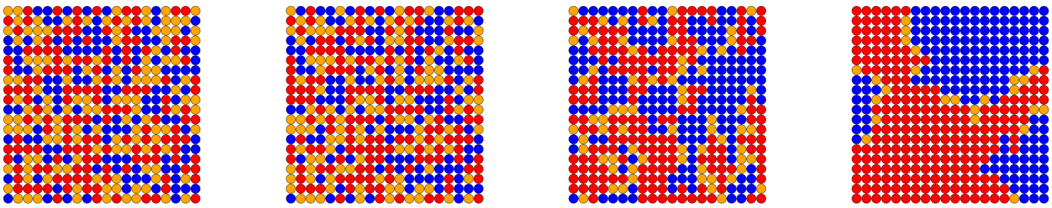


(a) Epoch 0 (b) Epoch 100 (c) Epoch 2500 (d) Epoch 50000

Figure 3.1: Example of the evolution of the language dynamics competition for the Abrams-Strogatz model.

On the other hand, the bilingual model supposes also the existence of bilingual agents within our community. We will call them AB. Now the transition probabilities of our agents will be:

$$p_{i,A->AB} = \frac{1}{2}\sigma_i^B \qquad p_{i,B->AB} = \frac{1}{2}\sigma_i^A$$
$$p_{i,AB->B} = \frac{1}{2}(1-\sigma_i^A) \qquad p_{i,AB->A} = \frac{1}{2}(1-\sigma_i^B)$$



(a) Epoch 0 (b) Epoch 100 (c) Epoch 2500 (d) Epoch 50000

Figure 3.2: Example of the evolution of the language dynamics competition for the Bilingual model.

As observed in 3.1 and 3.2 as the number of epochs increase, spatial domains of each monolingual community are formed and grow in size. Bilingual communities are never formed instead they place themselves in a narrow band between the two monolingual domains.

## 3.3 | Model Analysis

In this section we are going to study both the Abrams-Strogatz and the Bilingual model changing some of the parameters used to characterize the network as well as changing the topology of how our network is created.

In order to study the outcome of these changes we are going to use the mean interface density $\rho$. This is defined as the density of links joining nodes in the network which are in different states. The minimum value $\rho = 0$ corresponds to a stationary configuration in which all the agents belong to the same linguistic community.

### 3.3.1 Number of Nodes N

The first parameter that we can consider changing is the number of nodes N and see if by increasing the lattice size the number of epochs to get to extinction changes.

We can see that the average interface density decays as a power law: $< \rho >\sim t^{-\gamma}$ in both models 3.3 and the decay slows as we increase the number of nodes in our lattice. In both models the $\gamma$ parameter gets lower as we increase the size of our network 3.5

### 3.3.2 Changing the Topology

In order to study the effects that the social structure has in the evolution of the models we make use of a small-world (also called Watts-Strogatz) topology instead of the lattice-like we have been using for now. This one introduces a rewiring probability $p$. Basically how likely is that one of the endpoints of an edge changes.

As we can see in 3.4, a small world topology in the bilingual model produces a fast extinction of one of the monolingual communities. Meanwhile, the effect of changing p is not very big in the Abrams-Strogatz model. We can see that when comparing the $\gamma$ for different values of p in 3.4.

We can also introduce a community structure topology that consists of a combination of random attachment with a search for new contacts in the neighborhood of our random connections we have made. This process is repeated a number of times depending if we want to create more or less community structure. At 3.7 we can see the degree distribution depending on the number of iterations.

An extended explanation of this algorithm and its implementation can be found in [11].

When plotting the average interface density at 3.8 we can see that a community structure topology seems to make the model converge slower which correlates of what should we expect as it acts as resilient factor. In the Bilingual case though the model with more community structure converges faster which may be due to the fact that we are working with a network of N=100 nodes and introducing a lot of connections in the network may instead make the community structure act as a global connected network at the limit.

### 3.3.3 Prestige and Volatility

Prestige and volatility can be used to model a change in the status of the two competing languages and a change in how fast agents take to imitate neighboring agents status respectively. We will model this for the Abrams-Strogatz model. The new transition probabilities will be:

$$p_{i,A->B} = (1-s)(\sigma_i^B)^a \qquad p_{i,B->A} = s(\sigma_i^A)^a$$

Where a represents volatility and s the prestige. At a > 1 an agent is affected by local majorities changes below random imitation while for a < 1 this probability will be above random imitation. Meanwhile, if s > 0.5 there will be a preference for language A and for s < 0.5 will be a preference for language B.

As expected as the prestige gets further from $s = 0.5$ the number of epochs needed to get into the extinction state is lower. In the same way as we increase the volatility the convergence into a monolingual community is also faster 3.5.

## 3.4 | Modelling the Dynamics of Language Death

We can use the probability of transition that takes into account volatility and prestige in the Abrams-Strogatz model with real world data in order to model the dynamics of language death. The evolution dynamics is defined by the following ODE:

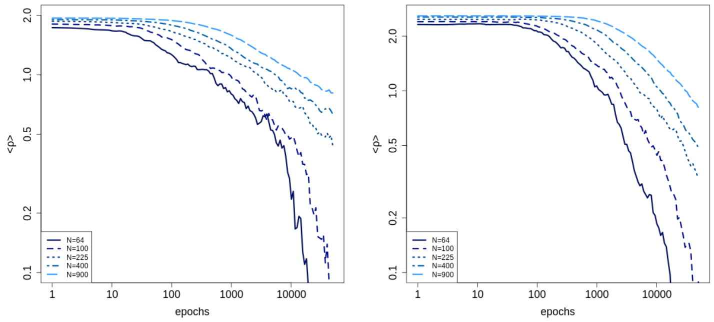
$$\frac{dx}{dy} = \sigma_B p_{B->A} - \sigma_A p_{A->B}$$

Where in this case we consider the densities $\sigma_B$ and $\sigma_A$ as the ones for the whole population. With this in mind we have fitted data that we gathered for different minority languages to this ODE in order to infer the parameters a and s. Being those Welsh in Wales [5], Irish in Ireland [7] and Gaelic Scottish in Sutherland, Scotland [13]. This data can be referenced at 3.1, 3.2 and 3.3 respectively.

The result of fitting the data can be viewed at 3.6. In doing so we are able to infer the s and a parameters for each language, the results are shown at table 3.6.

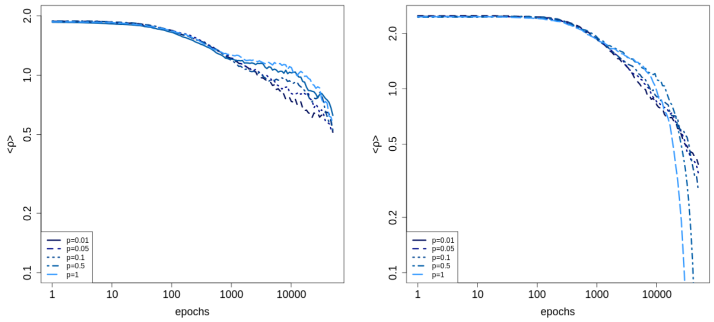
The prestige value for the three languages falls inside the margin of error found at [2] of $1.31 \pm 0.25$ with the exception of Gaelic Scottish that falls outside of it by a little bit.

On the other hand, when calculating the prestige we get approximately the same results as the paper. The computed prestige is 0.453 for Welsh and 0.296 for Gaelic Scottish while the paper gets 0.43 and 0.33 prestige values. The Irish language is not one of the languages studied in [2] so we can't compare its results.



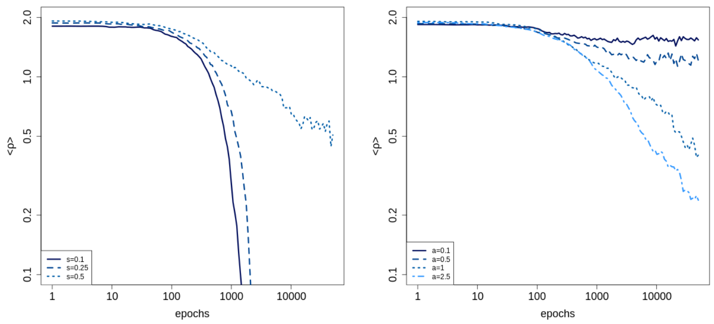

(a) Abrams-Strogatz (b) Bilingual

Figure 3.3: Evolution of the average interface density by different values of N. The results are averaged for 25 iterations.




(a) Abrams-Strogatz (b) Bilingual

Figure 3.4: Evolution of the average interface density for a lattice structure with N=225 and with different rewiring probabilities $p$. The results are averaged for 25 iterations.




(a) Prestige (b) Volatility

Figure 3.5: Evolution of the average interface density for a lattice structure with N=225 with different values of volatility and prestige using the Abrams-Strogatz model. The results are averaged for 8 iterations.

| Year | Welsh Speakers |
|---|---|
| 1891 | 0.545 |
| 1901 | 0.499 |
| 1911 | 0.435 |
| 1921 | 0.371 |
| 1931 | 0.368 |
| 1951 | 0.289 |
| 1961 | 0.260 |
| 1971 | 0.209 |
| 1981 | 0.187 |
| 1991 | 0.186 |
| 2001 | 0.208 |
| 2011 | 0.190 |

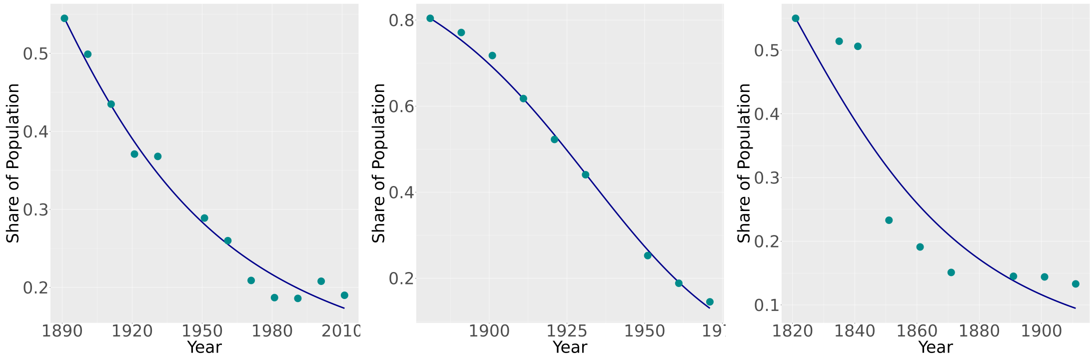
Table 3.1: The table displays the evolution of the number of Welsh speakers in Wales.

| Year | Irish Speakers |
|---|---|
| 1821 | 0.550 |
| 1835 | 0.514 |
| 1841 | 0.506 |
| 1851 | 0.233 |
| 1861 | 0.191 |
| 1871 | 0.151 |
| 1891 | 0.145 |
| 1901 | 0.144 |
| 1911 | 0.133 |

Table 3.2: The table displays the evolution of the number of Irish speakers in Ireland (32 counties).

| Year | Gaelic Scottish Speakers |
|---|---|
| 1881 | 0.804 |
| 1891 | 0.771 |
| 1901 | 0.7175 |
| 1911 | 0.618 |
| 1921 | 0.523 |
| 1931 | 0.441 |
| 1951 | 0.253 |
| 1961 | 0.188 |
| 1971 | 0.145 |

Table 3.3: The table displays the evolution of the number of Gaelic Scottish speakers in Sutherland, Scotland.




(a) Welsh (b) Gaelic Scottish (c) Irish

Figure 3.6: Evolution of the number of speakers of Welsh in Wales, Irish in Ireland and Gaelic Scottish in Sutherland, Scotland overtime.

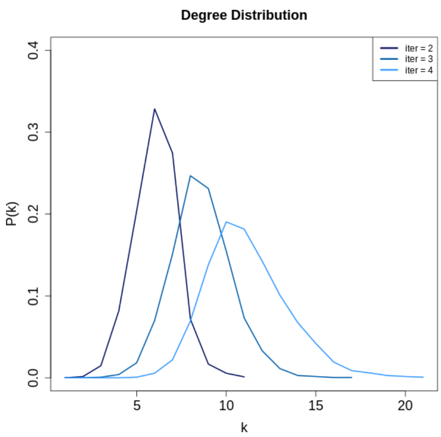
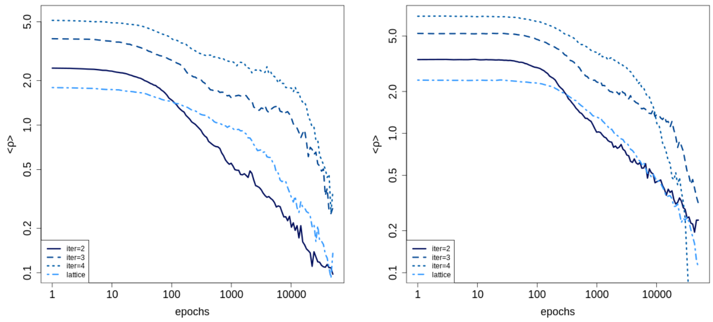



Figure 3.7: Degree distribution depending in the iterations used to generate the community structure topology for a lattice using 100 nodes. The result is averaged over 25 runs.

| Rewiring probability ($p$) | $\gamma$ Parameter | |
|---|---|---|
| | Abrams-Strogatz | Bilingual |
| 0.01 | 0.136 | 0.182 |
| 0.05 | 0.125 | 0.178 |
| 0.1 | 0.118 | 0.175 |
| 0.5 | 0.100 | 0.215 |
| 1 | 0.097 | 0.301 |

Table 3.4: The table displays the value of the decay of the average interface density for different parameters of N.




(a) Abrams-Strogatz (b) Bilingual

Figure 3.8: Average density interface evolution of different community structures for a network with N=100. The result is averaged over 25 runs.

| Number of Nodes ($N$) | $\gamma$ Parameter | |
|---|---|---|
| | Abrams-Strogatz | Bilingual |
| 64 | 0.298 | 0.428 |
| 100 | 0.260 | 0.305 |
| 225 | 0.148 | 0.193 |
| 400 | 0.123 | 0.152 |
| 900 | 0.096 | 0.098 |

Table 3.5: The table displays the value of the decay of the average interface density for different parameters of N.

| Languages | Parameters | |
|---|---|---|
| | Prestige ($s$) | Volatility ($a$) |
| Welsh | 0.453 | 1.245 |
| Irish | 0.898 | 1.274 |
| Gaelic Scottish | 0.296 | 0.950 |

Table 3.6: The table displays the volatility and prestige of different languages after fitting the experimental data to the dynamics equation.