Exam Development

joan alegre canton

March 2019

1 Question 1

We have to solve the following Max problem:

$$\max_{(k1,k2,\ldots,kL)} \quad \sum_{i=1}^{L} s_i k_i^{\gamma}$$

s.t

$$\sum_{i=1}^{L} k_i = K$$

We isolate k1 from the constraint, we plug this into the objective function and we take FOC's with respect ki, having:

$$[k_i] \qquad -\gamma s_i (K - \sum_{i=2} k_i)^{\gamma-1} + \gamma s_i k_i^{\gamma-1} = 0$$

from this foc we can do the following:

$$s_1 k_1^{\gamma-1} = s_i k_i^{\gamma-1}$$

$$s \equiv z_i^{1-\gamma}$$

$$s_1 k_i^{1-\gamma} = s_i k_1^{1-\gamma}$$

$$s_1^{\frac{1}{1-\gamma}} k_i = s_i^{\frac{1}{1-\gamma}} k_1$$

$$z_1 k_i = z_i k_1$$

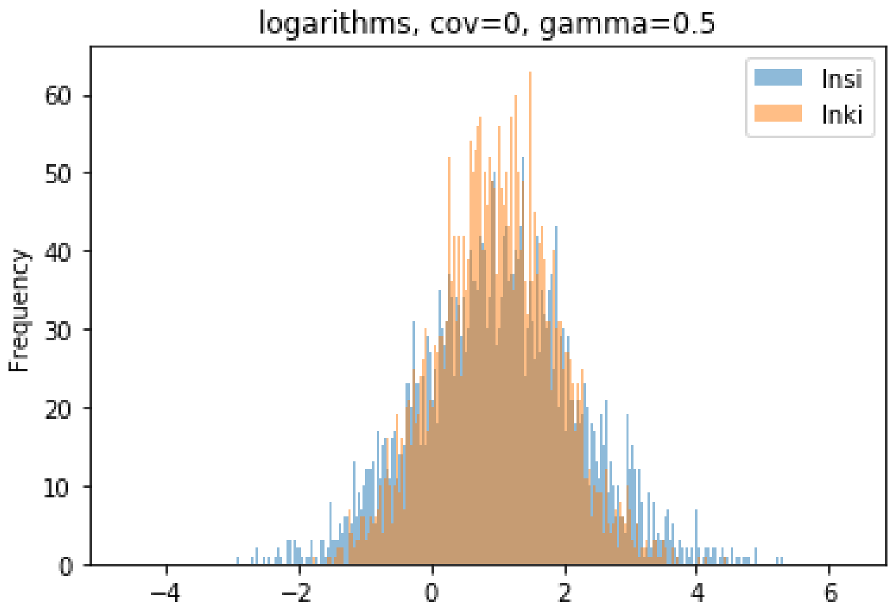
$$z_1 \sum k_i = k_1 \sum z_i$$

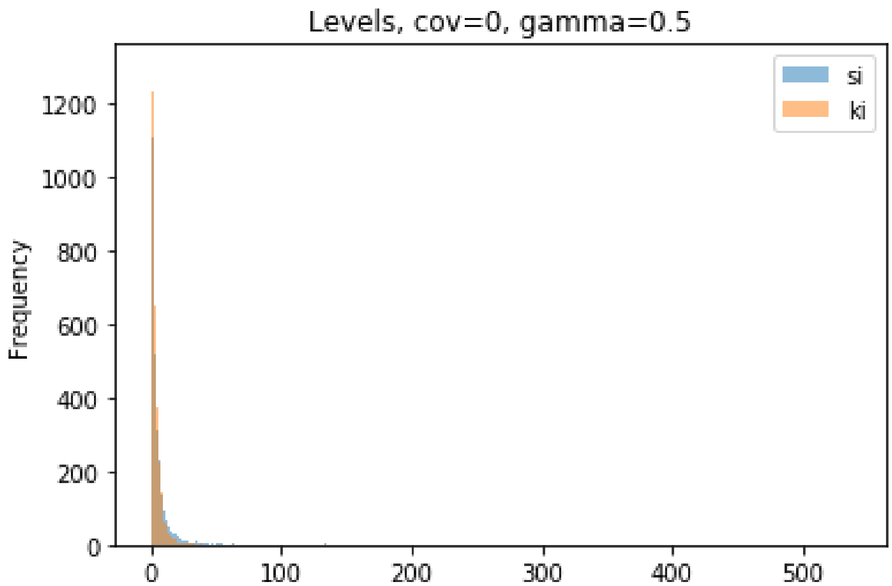
$$k_1 = \frac{z_1}{Z} K$$

In general terms:

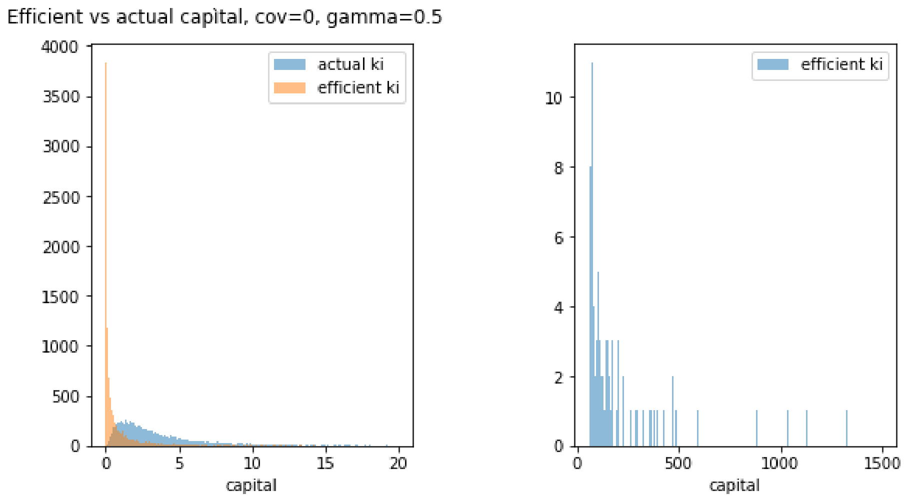
$$k_i = \frac{z_i}{Z} K$$

In the following two plots we can see the distribution of the logarithm of productivity and capital and the distribution in levels of productivity and capital. The only important think is that there are some extreme values of productivity and capital, we cannot see this from this plot, but there are some values of more than 400 of productivity or capital. This is so because of exponential transformation.



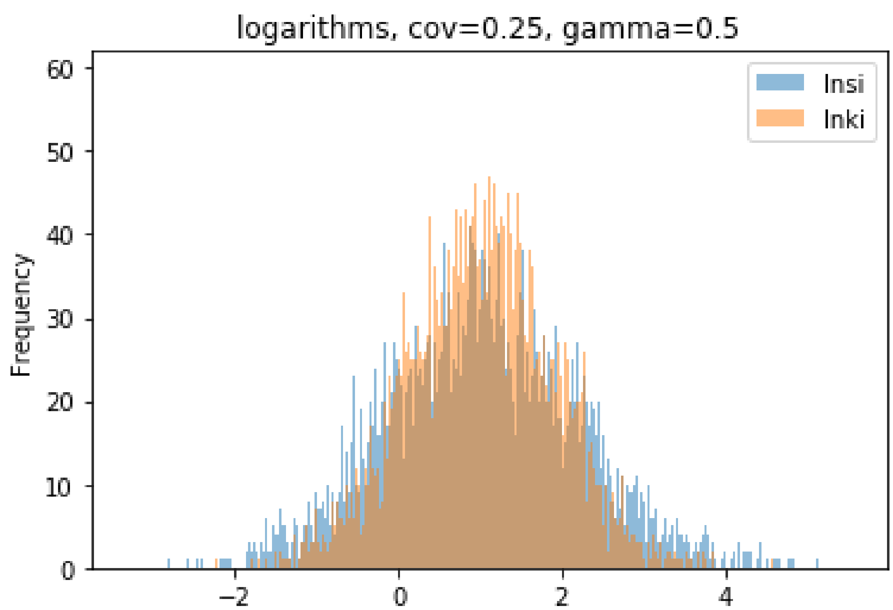


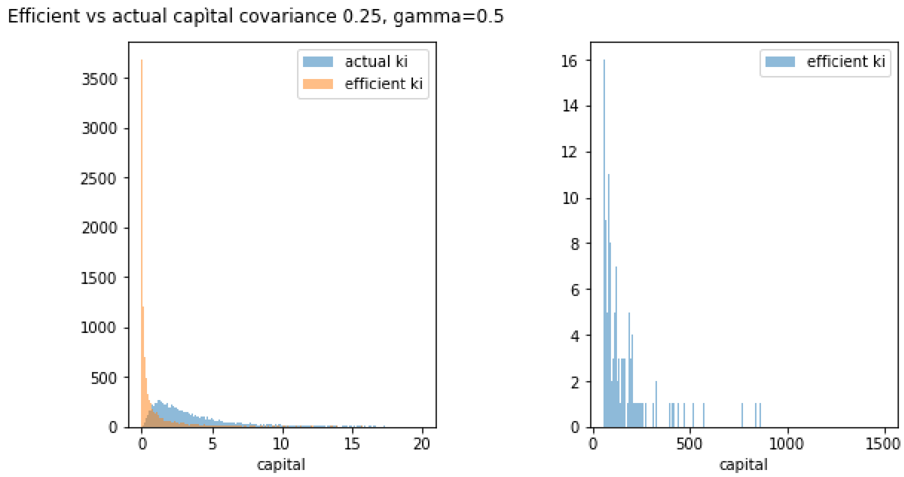
When we apply the efficient equilibrium conditions, and we compute the efficient capital associated to the problem we have the following distribution. In the left, we can see differences on the distribution of actual capital and efficient capital. In the right we can see the distribution of only the efficient capital (do not get confuse with the colors, this blue is the orange on the left plot), this right plot shows that there are ouliers that are monopolizing the productivity weigh, and hence social planner is giving them almost all capital. Notice there are people with more than 1000 of capital, in a world of 40000 aggregate capital an 10000 persons. This is why in the plot of the left there are so many people with low values of capital.



The gain ratio of this case is 2.472, so we could improve in 2.47 times the output of the economy with an efficient redistribution.

Now we do the same for covariances equal 0.25, intuitively if a covariance is higher this means that people with higher productivity has higher quantity of land, and hence gains of a redistribution will be lower than the non-covariances case.

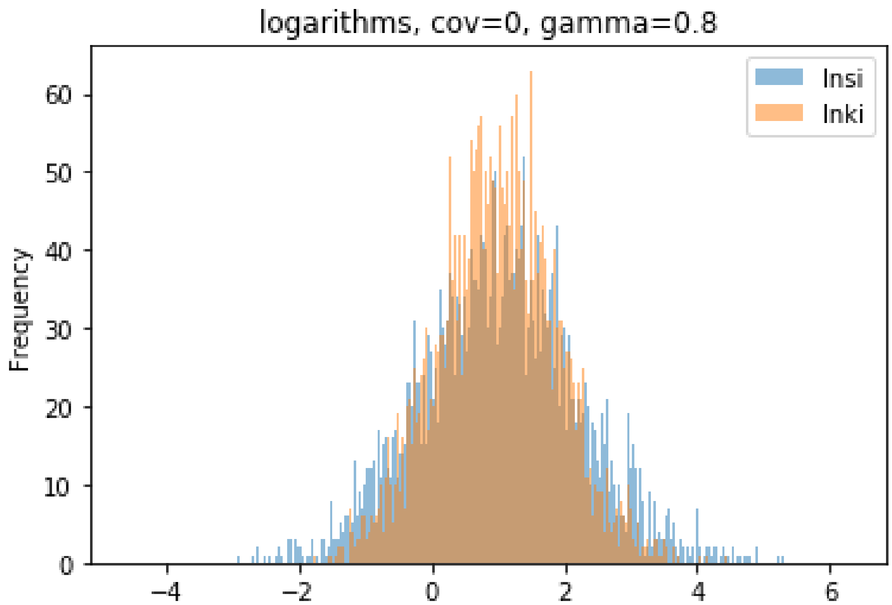


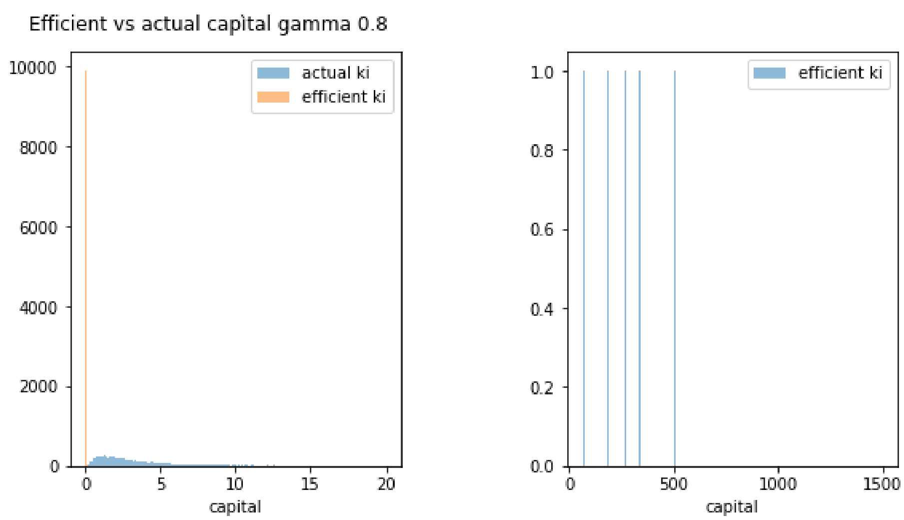


efficient redistribution of capital is actually the same than in the first part of this question (it is not exactly the same because I remake the distribution, but since the DGP is the same and the sample is quite large results are equivalent). Nevertheles, the redistribution gain now is 1.98, which is consistent what we thought, the more correlation we have between capital and productivity the lower is the gain of redistribution.

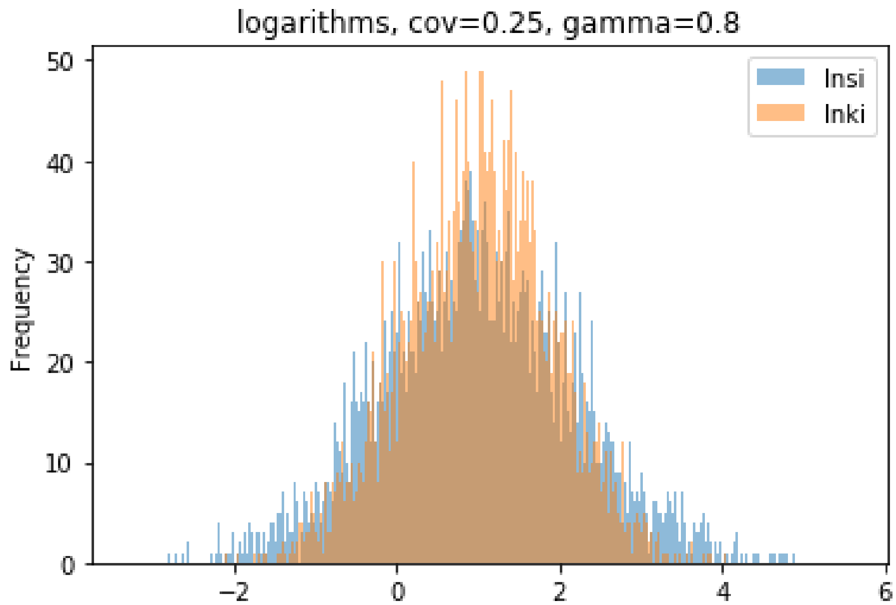
2 Question 2

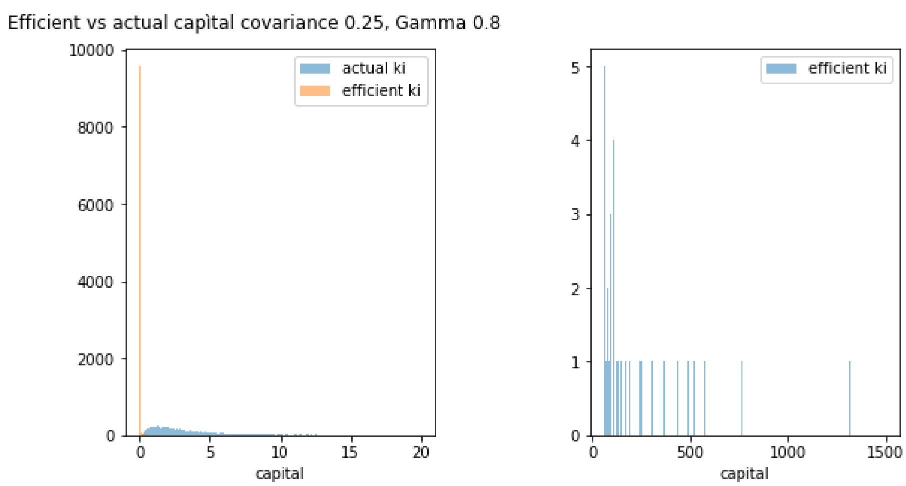
In this question we change gamma, and we redo everything again. Since gamma is limiting the accumulation of capital, a higher gamma will mean that efficient capital will be more accumulated in a few hands. In the extreme of the cases, with a $\gamma = 1$ we would have that the person with higher productivity will use the whole capital of the economy. In this sense, what we should see is that distribution of efficient capital is lower than the previous case, and that gains of redistribution are much more higher in this case than in the one with $\gamma = 0.5$





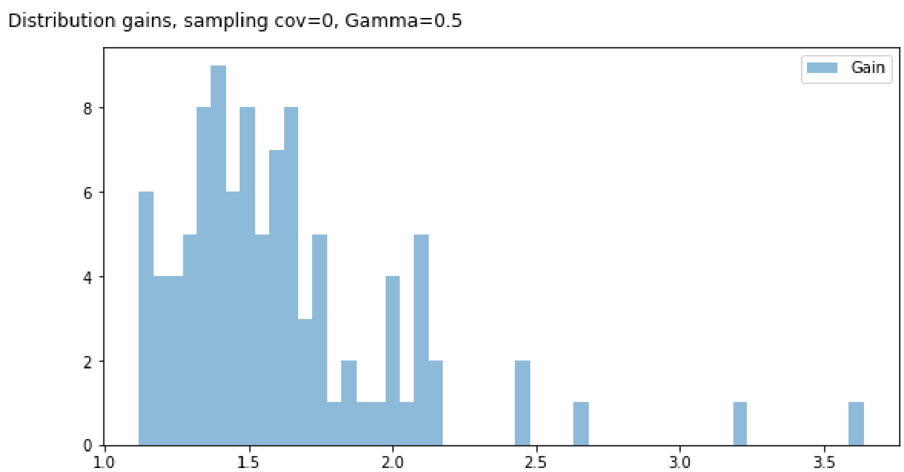
As we expected efficient distribution has a much lower variance than the previous case, most part of the people will have 0 production, and the people with the highest productivity will produce everything. In this case with covariances 0 we have that redistribution gain ratio is 16.26, far much higher than the case of $\gamma = 0.5$, with covariances 0.25 gains should be lower than this one, but still really considerable.





redistribution ratio gain in this case is 4.6, so we can say that difference on covariances when gamma is high makes a lot of difference on the redistribution gain. This later ratio gain is higher than the one in question 1.

3 Question 3



The gain of the administrative data is 2.47 so we can say that sampling with a T=100 is not a good way of infering the true data generating process, and the true redistribution gain. Also notice, that this distribution would be more like a F-snedecor distribution, than a normal distribution. My assumption of why is happening so is that there are too much outliers, that are generating a bias really big when we make samples. As we increase the sample size we will decrease this potential bias, and we will go toward the real redistribution gain. By the way, this outlier problem comes from the exponential transformation of productivity, it gives really extreme values to some observations.

4 Question 4

In this problem we will have that productivity is affected by capital, simplifying this will make that the second derivate of the output with respect of capital is less negative (or even positive for some parameters) making the equivalent of increasing gamma. So what we can expect is that under this framework, with equivalent parameter values than in question 1, efficient capital will be more concentrated in a few hands. Now we have to solve a new sort of problem, close form solutions will be restricted to some set of parameter combinations. In cases where there no exist close form solution we will need to do use numeric methods.

We have to solve the following Max problem:

$$\max_{(k1,k2,\ldots,kL)} \quad \sum_{i=1}^{L} s_i(a_i,k_i)k_i^{\gamma}$$

s.t

$$\sum_{i=1}^{L} k_i = K$$

Doing FOC's:

$$[k_i] \qquad -(\frac{ds(a_1,k_1)}{dk_1}k_1^{\gamma} + \gamma s_i(K - \sum_{i=2} k_i)^{\gamma-1}) + \frac{ds(a_1,k_i)}{dk_1}k_1^{\gamma} + \gamma s_i k_i^{\gamma-1} = 0$$

$$\frac{ds(a_1,k_1)}{dk_1}k_1^{\gamma} + \gamma s_i(K - \sum_{i=2} k_i)^{\gamma-1} = \frac{ds(a_1,k_i)}{dk_1}k_1^{\gamma} + \gamma s_i k_i^{\gamma-1}$$

Notice that if $\frac{ds(a_i,k_i)}{dk_i}$ is 0 then we will be in the same situation than in question 1.
if we do the derivate of the idiosyncratic productivity and wit respect to k_1 it turns out that we end up having this expression: $\frac{ds(a_i,k_i)}{dk_1} = \frac{(1-\alpha)k_i^{-\frac{1}{\sigma}}}{s_i}$.

Let's find the solution to the problem when we have $\sigma = 1$ and $\gamma = 0.5$

$$\frac{(1-\alpha)k_1^{\gamma-1}}{s_1} + \gamma k_1^{\gamma-1}s_1 = \frac{(1-\alpha)k_i^{\gamma-1}}{s_i} + \gamma k_i^{\gamma-1}s_i$$

$$k_1^{\gamma-1}(\frac{(1-\alpha)}{s_1} + \gamma s_1) = k_i^{\gamma-1}(\frac{(1-\alpha)}{s_i} + \gamma s_i)$$

$$k_1(\frac{(1-\alpha)}{s_1} + \gamma s_1)^{\frac{1}{\gamma-1}} = k_i(\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{\gamma-1}}$$

We move the two powered terms to the other side, and we make the sum over all individuals, we end up having:

$$k_1 \sum_i (\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{1-\gamma}} = (\frac{(1-\alpha)}{s_1} + \gamma s_1)^{\frac{1}{1-\gamma}} \sum_i k_i$$

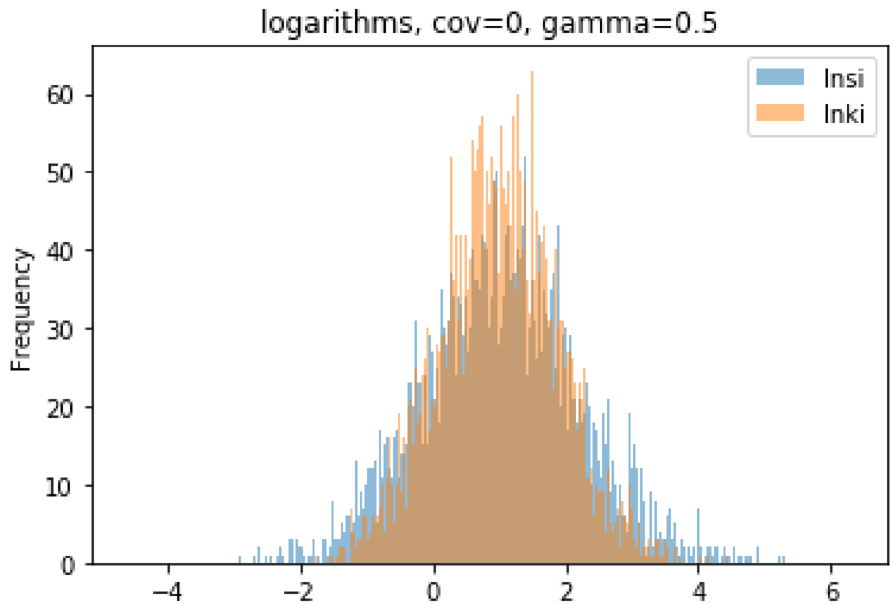
$$k1 = \frac{(\frac{(1-\alpha)}{s_1} + \gamma s_1)^{\frac{1}{1-\gamma}}}{\sum_i (\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{1-\gamma}}} K$$

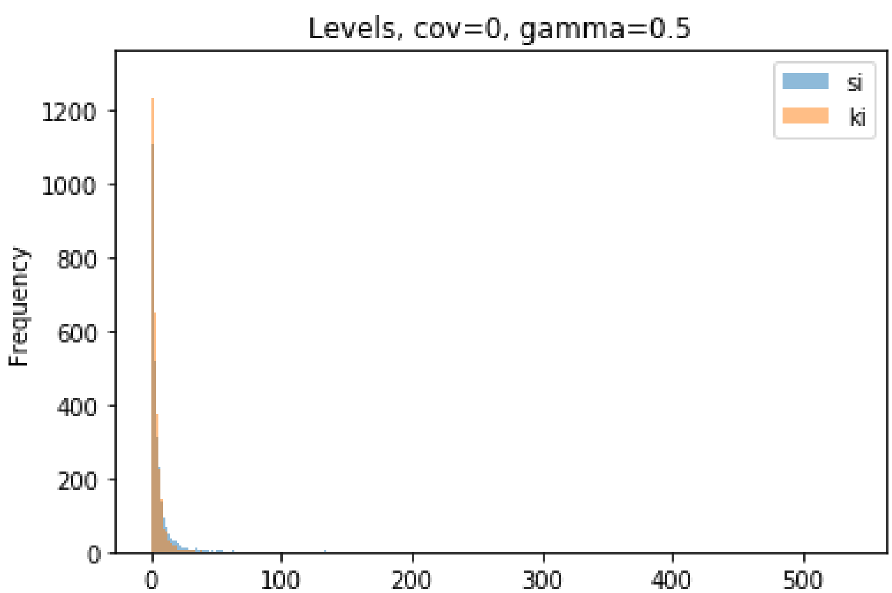
We can generalize to:

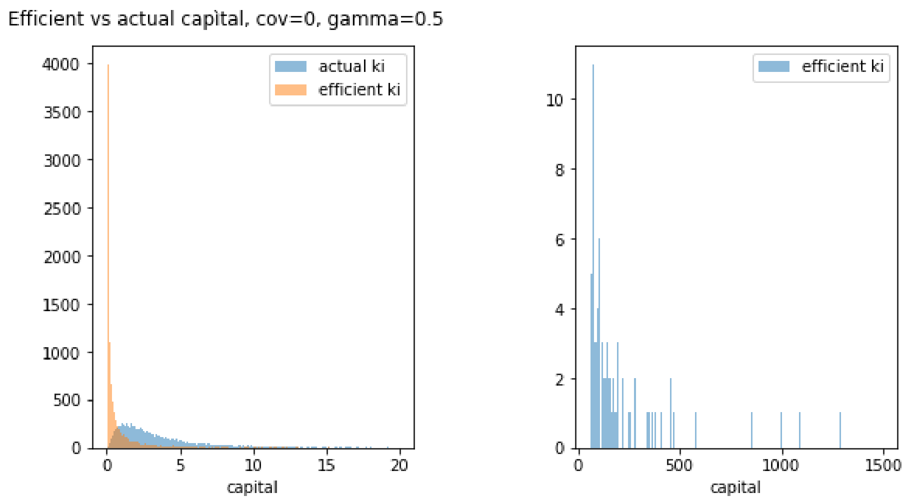
$$ki = \frac{(\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{1-\gamma}}}{\sum_i (\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{1-\gamma}}} K$$

calling $Zi = (\frac{(1-\alpha)}{s_i} + \gamma s_i)^{\frac{1}{1-\gamma}}$ We end up having:

$$ki = \frac{zi}{Z} K$$


logarithms, cov=0, gamma=0.5
lnsi
lnki
Frequency
0
10
20
30
40
50
60
−4
−2
0
2
4
6


Levels, cov=0, gamma=0.5
si
ki
Frequency
0
200
400
600
800
1000
1200
0
100
200
300
400
500



In this exercice we can see fairly the same than in question 1 problem, there are quite a lot outliers, the redistribution ratio gain is 2.4. I was expecting to have a redistribution gain higher than the one in question 1 with covariances 0, and have a more concentrated distribution of efficient capital. But instead, it looks like it is fairly similar, and having a relationship of capital and productivity for $\sigma = 1$ is not affecting too much the result.

In the case of $\sigma = 0.5$ and $\gamma = 0.5$:

$$\frac{(1-\alpha)}{s_1} + \gamma k_1^{0.5} s_1 = \frac{(1-\alpha)}{s_i} + \gamma k_1^{0.5} s_i$$

$$\frac{(1-\alpha)}{s_1} - \frac{(1-\alpha)}{s_i} + \gamma k_1^{0.5} s_1 = \gamma k_1^{0.5} s_i$$

$$k_i = [\frac{(1-\alpha)}{\gamma s_i}(\frac{1}{s_1} - \frac{1}{s_i}) + k_1^{0.5}(\frac{S_1}{s_i})]^2$$

We sum both sides the equation associated to every individual, obtaining:

$$K = \sum[\frac{(1-\alpha)}{\gamma s_i}(\frac{1}{s_1} - \frac{1}{s_i}) + k_1^{0.5}(\frac{S_1}{s_i})]^2$$

if we discompose this squared of a sum, we soon realize that we need to solve an equation of fourth degree, for every individual. This is a tedious task, and it is also a problem since different combinations of idiosyncratic productivity with respect to the others may led to not have a solution in the space of real numbers. I have no time to try this, but to avoid to have the fourth degree equation for every individual, I was thinking in trying to make a polynomial that represents the mapping between productivity of individual i, productivity of the others, and optimal capital for i. And estimate this mapping. To do so I would have solve this degree equation for a representative sample of the 10000 individuals, and I would have estimated this polynomial, having as a result an "Optimal policy of redistribution".

5 Question 5

As I understand the static problem, we are only focus on efficiency since the whole production will be translated to consumption. Nevertheless, in a dynamic problem we cannot do this anymore, since one way of propagation is investing on capital, reducing the possible quantity of consumption today but increasing the future consumption. If we don't care about consumption, and we care only about having larger economies, with lager amounts of capital, the maximization problem is trivial, and the solution will be just the point were the whole quantity of production goes to pay investment. In the case that we care about consumption, in a dynamic problem, we need to assume utilities functions. In this case we will have the following Social Planner problem:

$$\max_{(k_{i,t})\forall t,i} \sum_{\tau=0}^{\infty} \beta^{\tau} \sum_{i=1}^{L} \lambda_{i,t} u(c_{i,t+\tau})$$

s.t

$$\sum_{i=1}^{L} c_{i,t+\tau} = \sum_{i=1}^{L} s_i(a_{i,t+\tau}, k_{i,t+\tau}) k_{i,t+\tau}^{\gamma} - \sum_{i=1}^{L} (k_{i,t+1+\tau} - k_{i,t+\tau}(1-\delta)) \quad \forall \tau$$

And transversality condition.

λ is the weight that the social planner gives to every individual in a determined period. The LHS of the constraint is the Aggregate consumption and the RHS is the aggregate total production menus the aggregate total investment for a given period.

We can solve this problem substituting consumption from the restriction into objective function. We do FOC of capital, we get the Euler equation. We find also intratemporal conditions.

The thing here is that productivity depends on capital. Hence, we could generate an equilibrium where the more we concentrate in a few hands inputs the more output we can obtain,since decreasing yields of capital could be not holding anymore due to the indirect effect of the capital to $s_k(a_{i,t}, k_{i,t})$, in this sense this is equivalent to question 4.