

Joan alegre Problem set 5:

I. A simple wealth model:

In a context of ρ being larger than interest rate according to Euler equation associated to the max problem of the H.H we could say that H.H will consume more in the present than the future in terms of actual consumption.

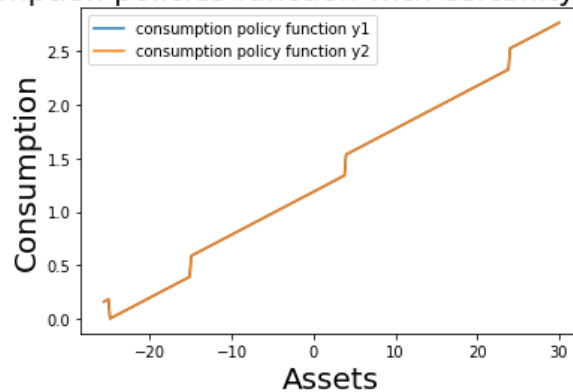
II. Solving the ABHI:6-

II.2: The infinitely-lived household economy (code 56-140 line): Here I provide my code for solving a H.H problem recursively with a Bellman equation.

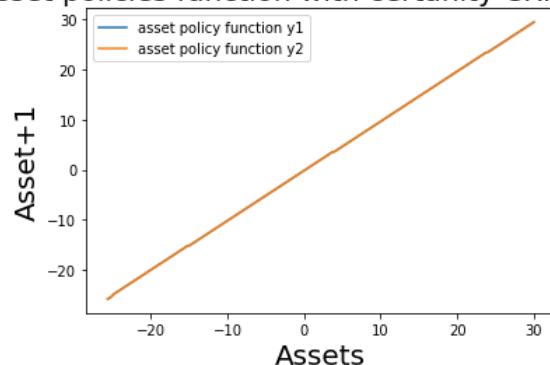
II.3: life-Cicle economy (code 140-162 line): Here I provide code for solving a time limited maximization H.H problem, solving by Backwards with the Belmman equation

II.4.1: Partial equilibrium, Certainty, The following 5 plots are the ones associated to this problem with utility of CRRA:

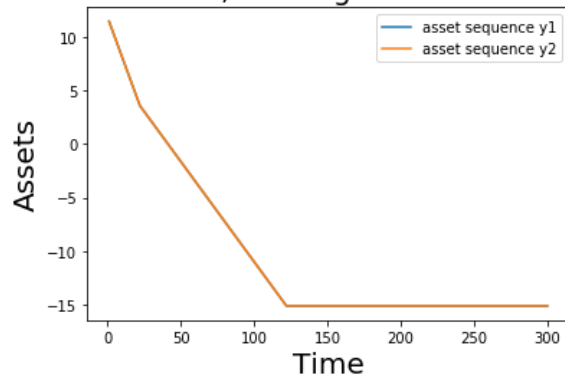
Consumption policies function with certainty CRRA CASE



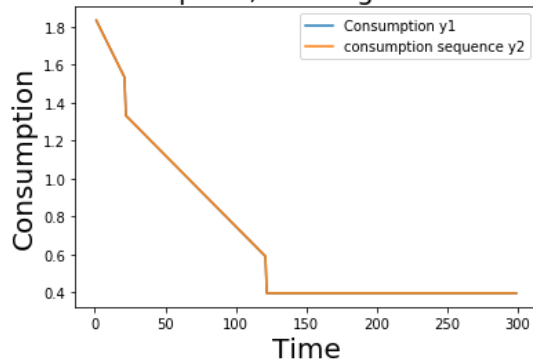
Asset policies function with certainty CRRA CASE



Sequence of assets, starting at $a_0=11.45$ CRRA CASE



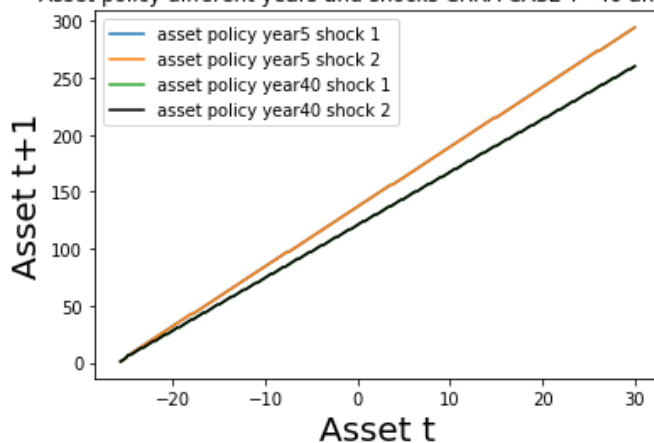
Sequence of consumption, starting at $a_0=11.45$ CRRA CASE



What these 2 last plots show is that H.H prefer to consume on the present large amount even if in the future they will have to fall in debt and consume only 0.4 units forever.

Nevertheless, sequence of assets always negative could look like a Ponzi scheme, but it is not. Why? Because it is not asking debt for repaying the former debt and its interest. It is just borrowing that amount of debt every year to repay the principle, and use part of its dotation income to pay the constant amount of interest rate.

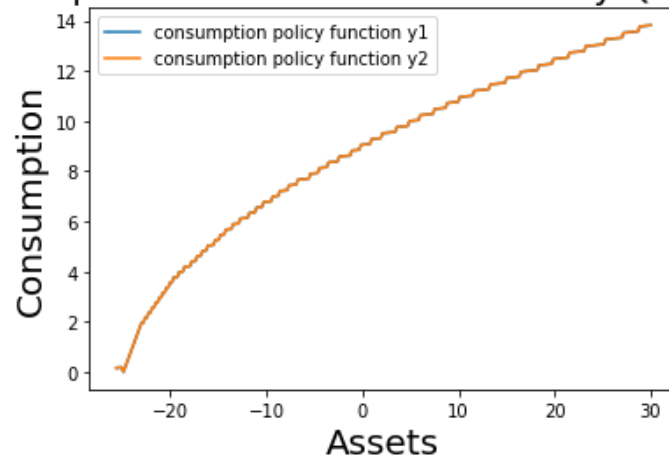
Asset policy different years and shocks CRRA CASE $T=40$ and $T=5$



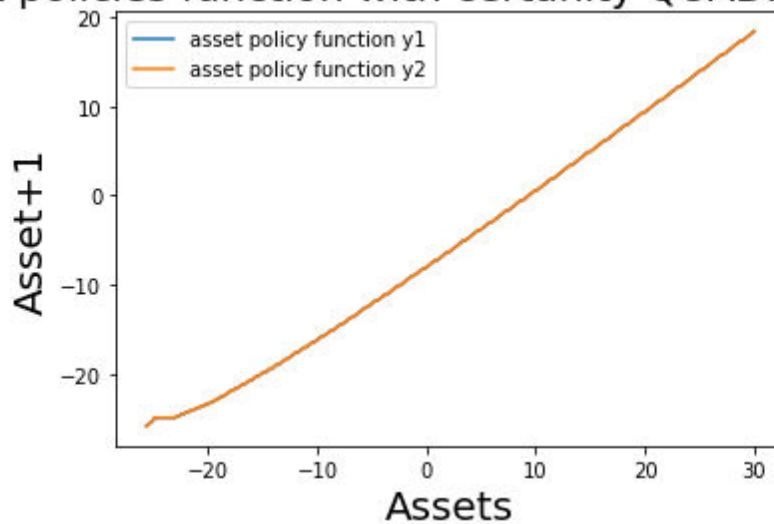
This plot is consistent with one can think will happen. The more near we are of dying the smaller is the amount that we save.

In the following we will see exactly the same than before but for QUADRATIC case:

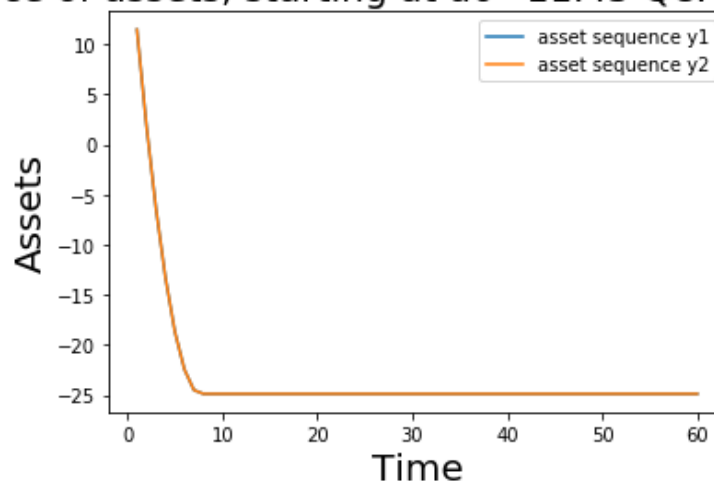
Consumption policies function with certainty QUADRATIC CASE



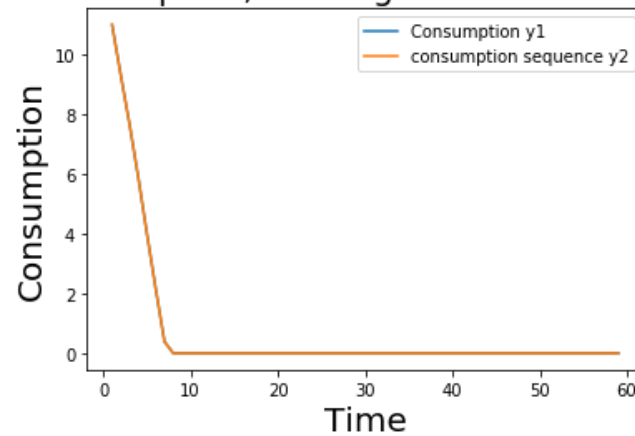
Asset policies function with certainty QUADRATIC CASE



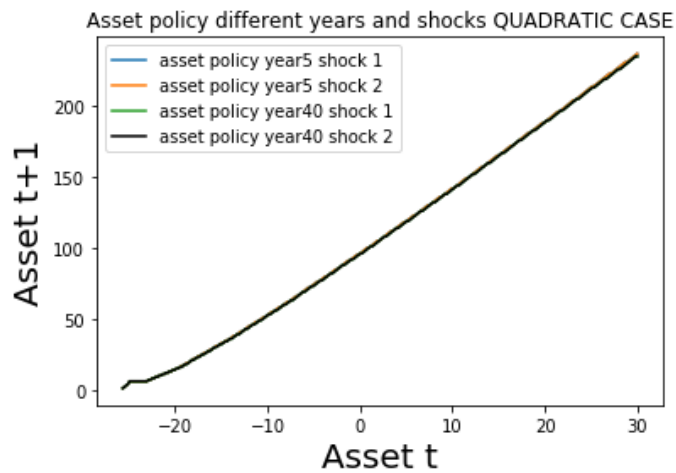
Sequence of assets, starting at $a_0=11.45$ QUADRATIC CASE



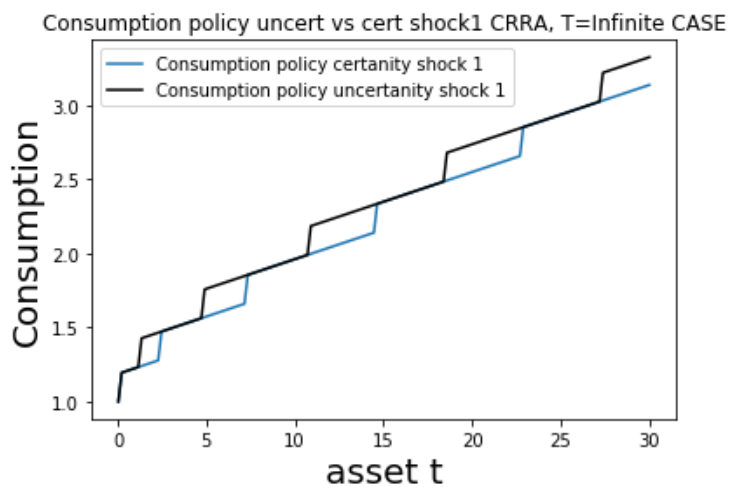
Sequence of consumption, starting at $a_0=11.45$ QUADRATIC CASE

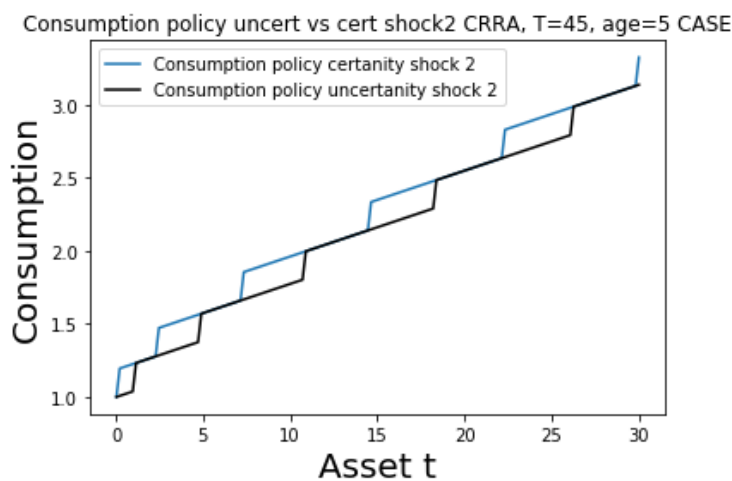
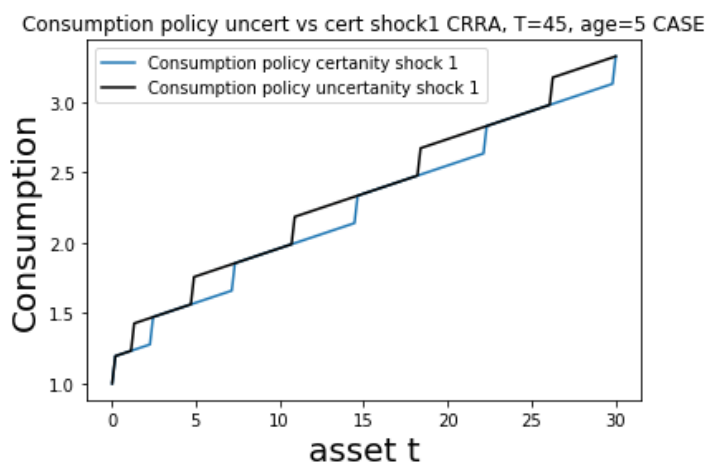
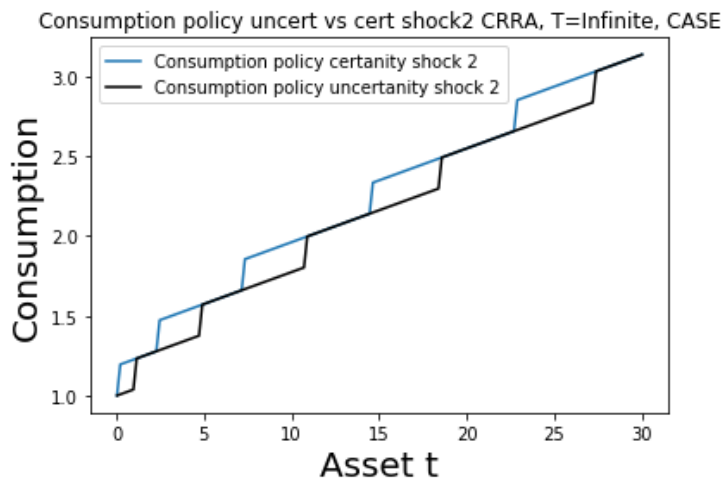


This results are nor really satisfactory, since H.H use its wealth just to consume everything in the short time.



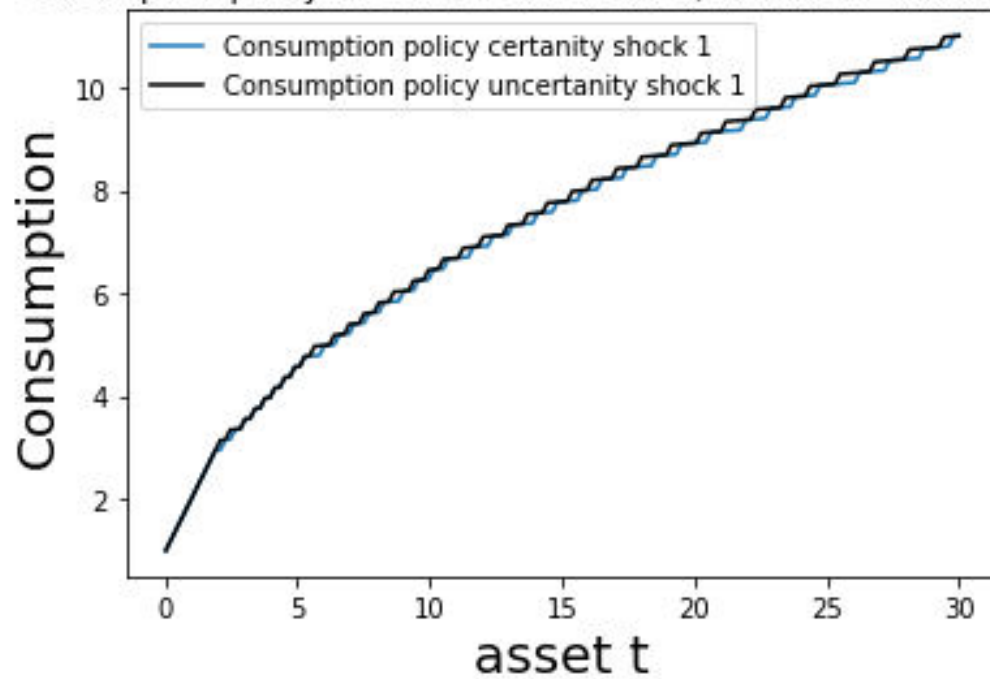
II.4.2: With uncertainty. A) From now on all the H.H problem will have a borrowing limit of 0, $a_{t+1} \geq 0$.



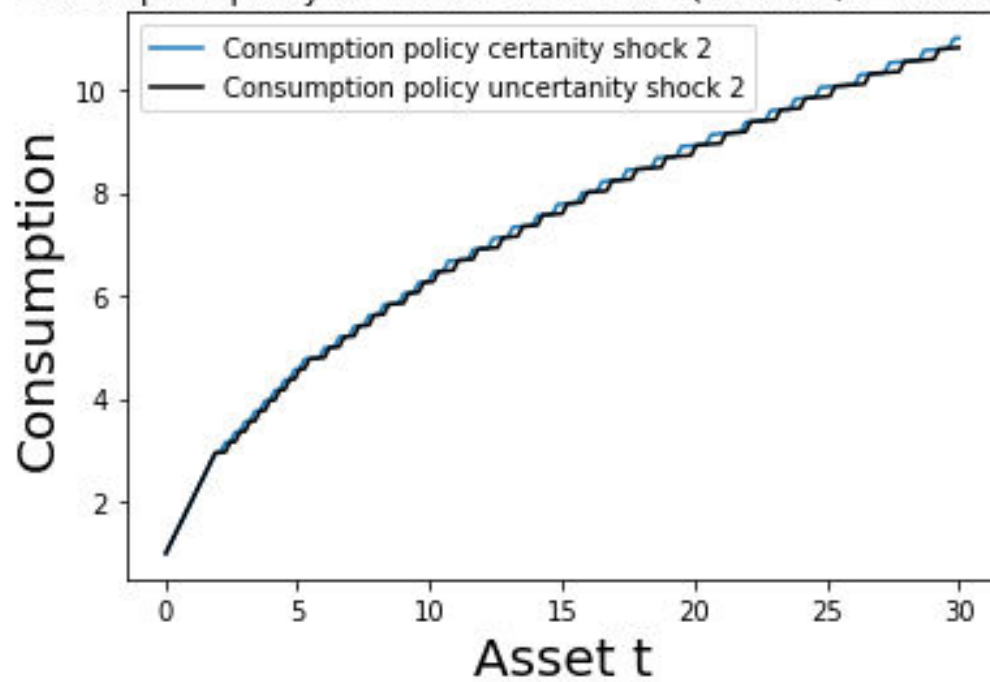


We can see in the case of the second shock (which is the good shock) we have that in the case of certainty we are consuming more than in the case of uncertainty (in both $T = \text{infinite}$ and $T=45$ cases). This is because under the borrowing constraint, and utility function strictly concave we generate a situation of precautionary savings. Since we are investing more than it is optimal, in the time of having a bad shock we will eat the excess of money that we have saved before. Having the case that under uncertainty we are consuming more than under certainty when bad shock.

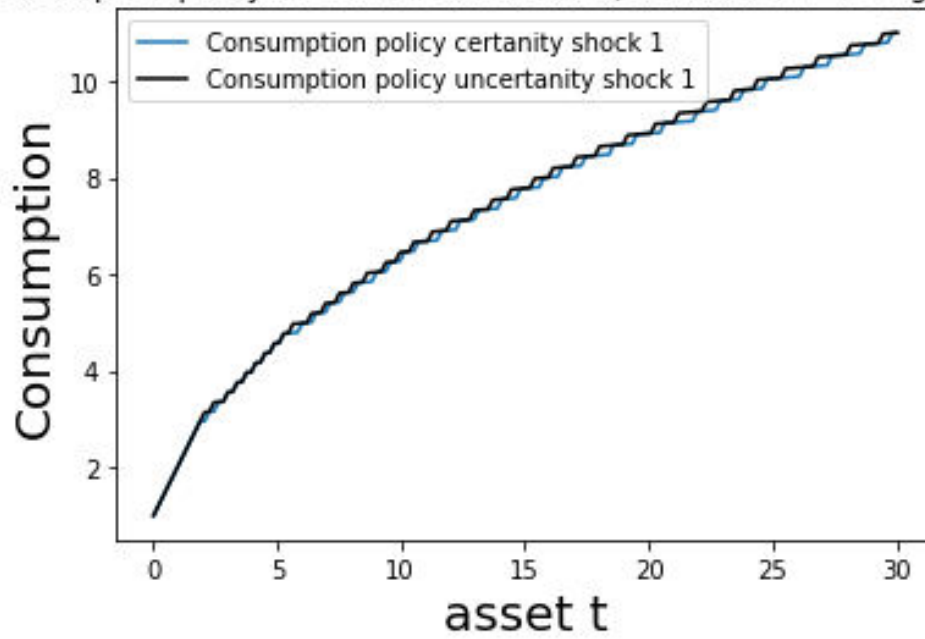
Consumption policy uncert vs cert shock1 Quadratic, T=Infinite, CASE



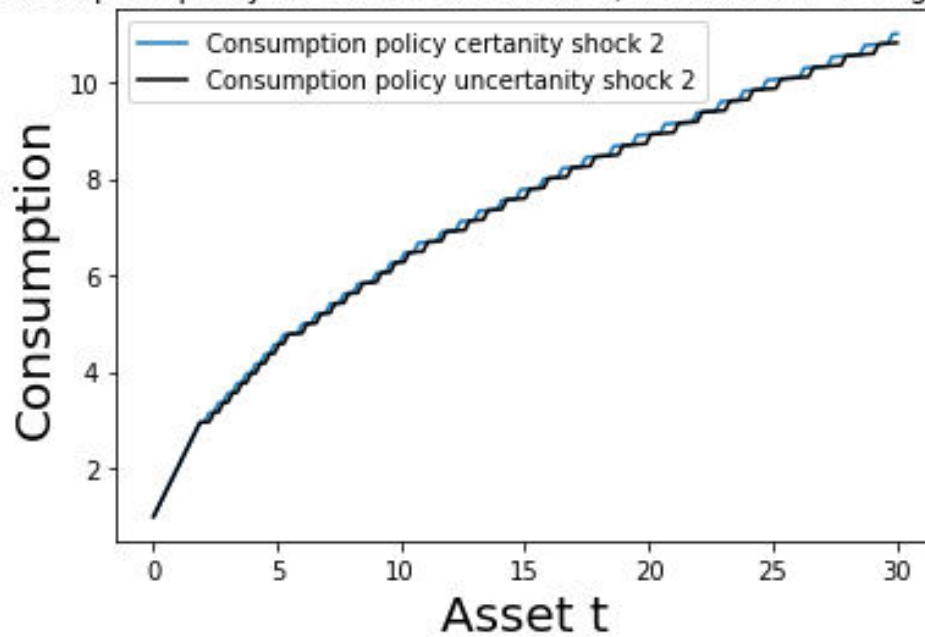
Consumption policy uncert vs cert shock2 Quadratic, T=Infinite, CASE



Consumption policy uncert vs cert shock1 QUADRATIC, T=45, age=5 CASE



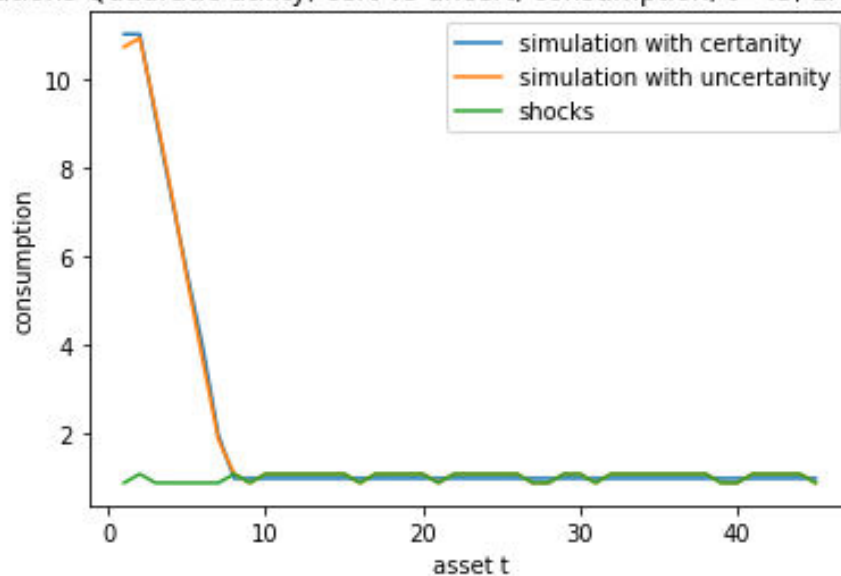
Consumption policy uncert vs cert shock2 QUADRATIC, T=45, age=5 CASE



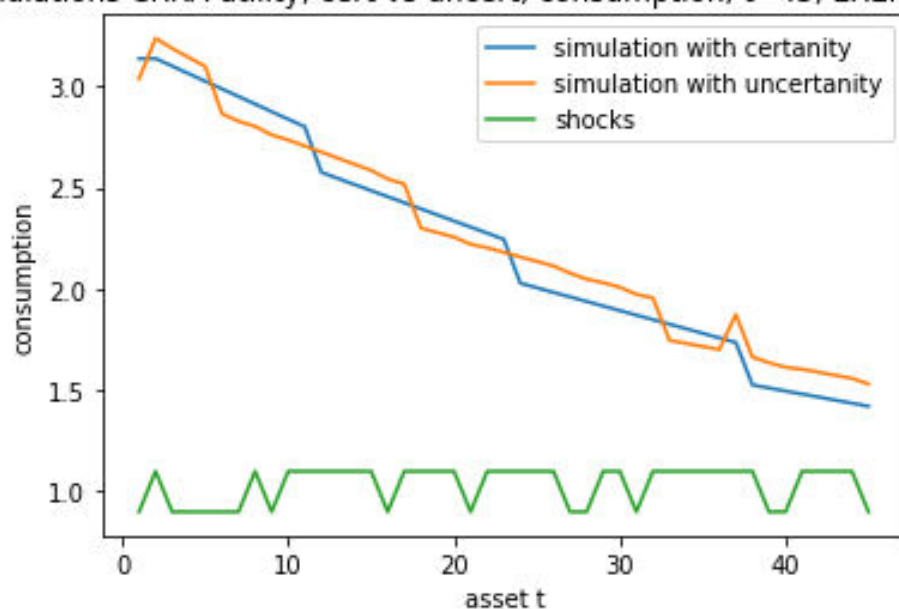
In the case of QUADRATIC preferences we can see exactly the same than the CRRA case, there are precautionary savings.

II.4.2: With uncertainty B)

Simulations Quadratic utility, cert vs uncert, consumption, t=45, EXERCICE II.4.2

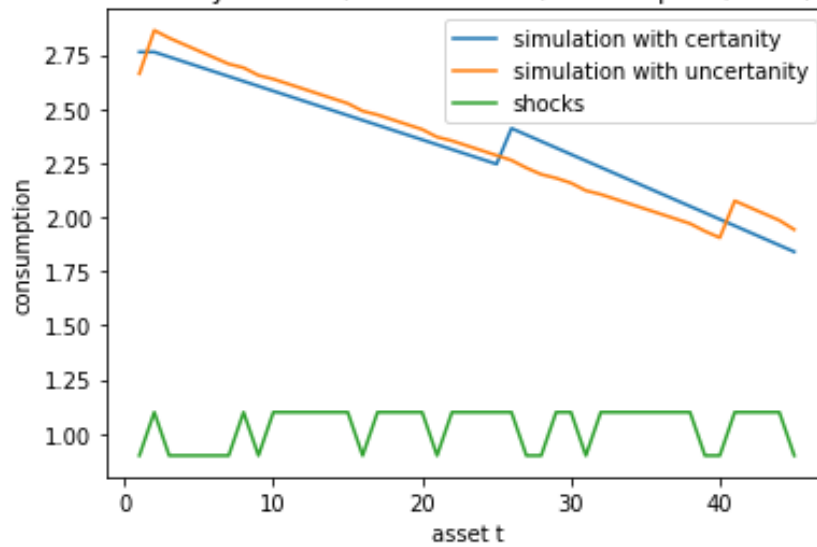


Simulations CRRA utility, cert vs uncert, consumption, t=45, EXERCICE II.4.2

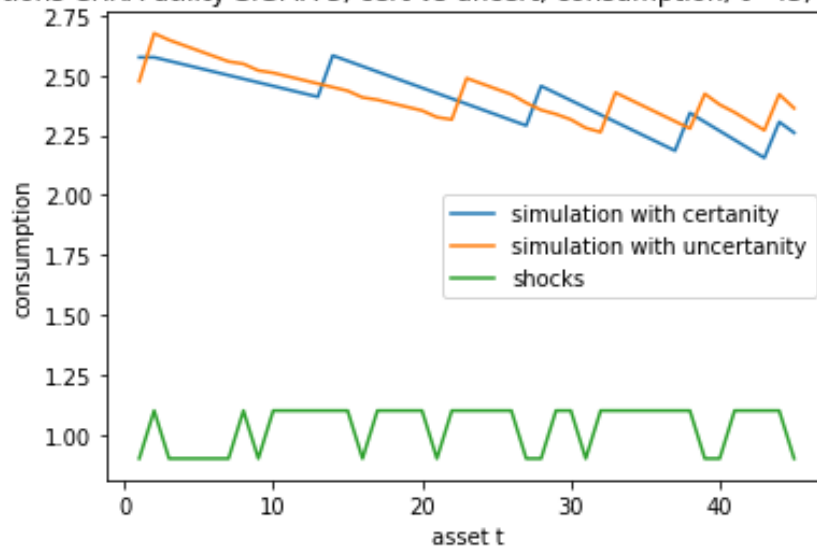


II.4.2.3: CRRA simulation with sigma 2,5,20: In the case of $\sigma=20$ my code could not run (I do believe in human mistakes as well) so, I choose to take $\sigma=6$ (the maximum that my computer wanted to accept, strangers things) rather than $\sigma=20$.

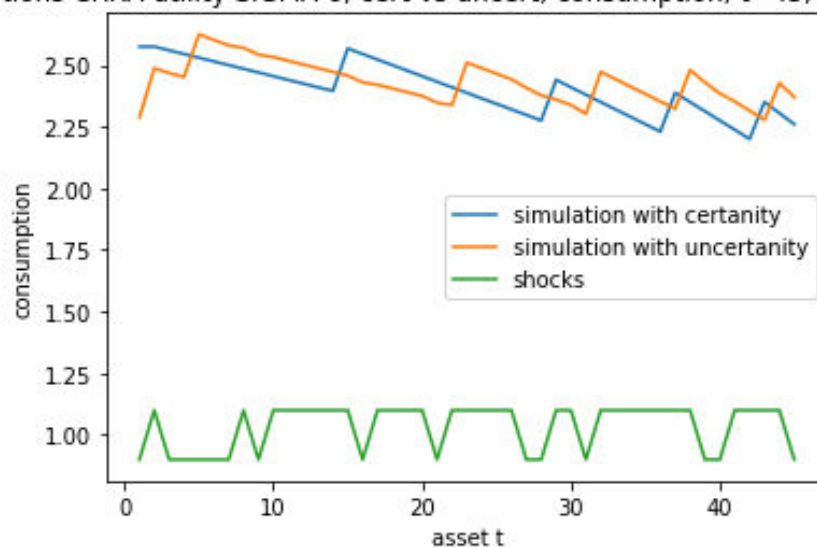
Simulations CRRA utility SIGMA 2, cert vs uncert, consumption, t=45, EXERCICE II.4.3



Simulations CRRA utility SIGMA 5, cert vs uncert, consumption, t=45, EXERCICE II.4.3

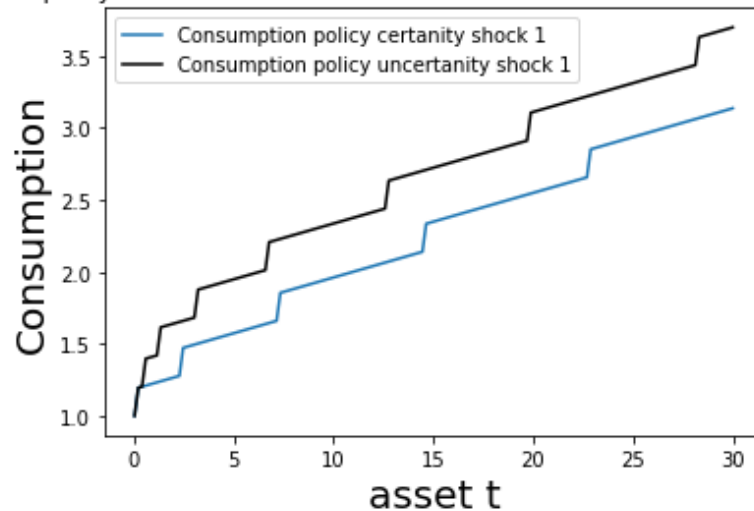


Simulations CRRA utility SIGMA 6, cert vs uncert, consumption, t=45, EXERCICE II.4.3

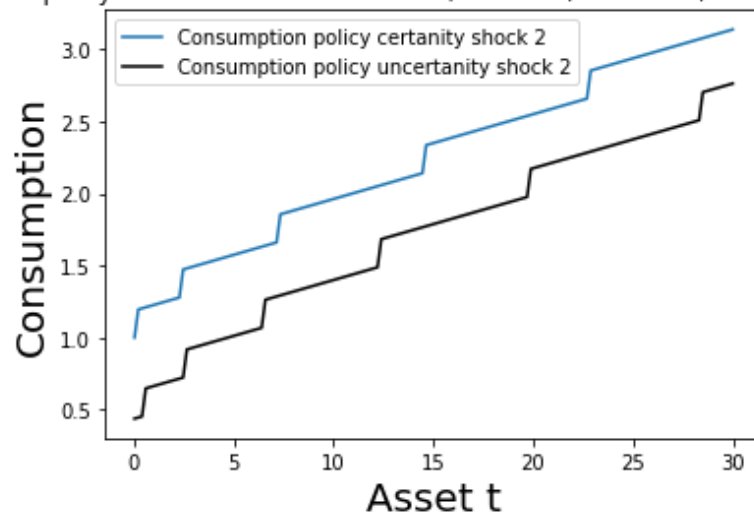


II.4.2.4: Variance of 0.5: I will now plot for CRRA case consumption policies of both uncertainty case and certainty case. I will use CRRA with sigma 1, and variance of 0.5, gamma = 0 as usual. In the case of certainty both policies should be equal, since shocks should be smoothed, First I will plot consumption policy function with certainty and uncertainty in the case of $\text{var}Y = 0.5$, separating different shocks in different plots.

Consumption policy uncert vs cert shock1 CRRA, $\text{Var}Y=0.5$, $T=\text{Infinite}$ CASE, II.4.2.4 EXERCICE



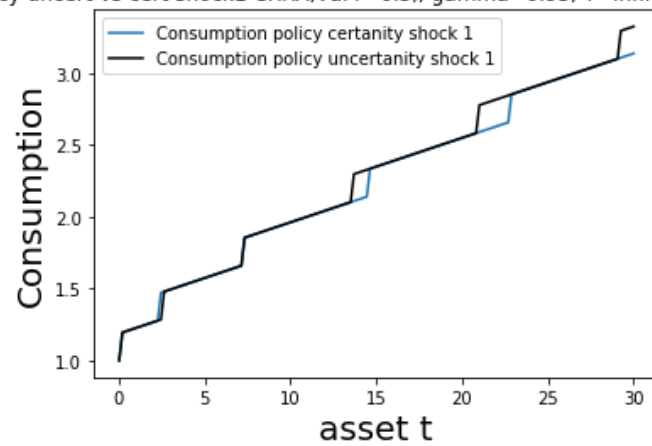
Consumption policy uncert vs cert shock2 CRRA, $\text{Var}Y=0.5$, $T=\text{Infinite}$, CASE II.4.2.4 EXERCICE



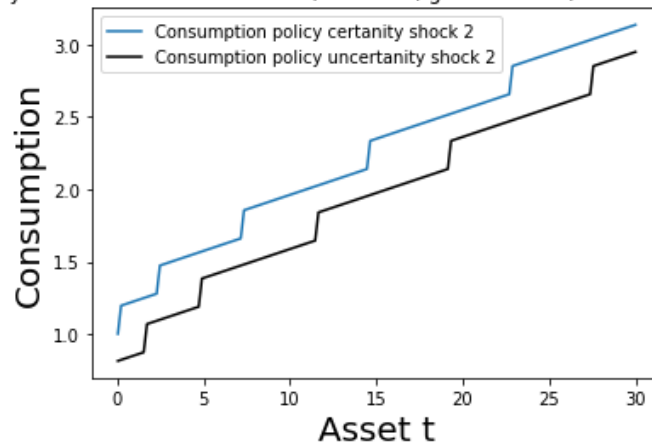
We can see that differences between certainty and uncertainty are higher than the previous cases, we can also see that this is consistent with the idea of precautionary savings since when we have a good shock (the second plot) we have a lower consumption than the certainty one, generating an excess of asset that we will consume on the bad shock generating that in the first plot we consume more than the certainty case.

II.4.2.5: $\text{Var}Y=0.5$ and $\gamma = 0.95$

Consumption policy uncert vs cert shock1 CRRA, $\text{Var}Y=0.5$, $\gamma=0.95$, $T=\text{Infinite}$ CASE, II.4.2.4 EXERCICE



Consumption policy uncert vs cert shock2 CRRA, $\text{Var}Y=0.5$, $\gamma=0.95$, $T=\text{Infinite}$, CASE II.4.2.5 EXERCICE



Results really similar to the other case.

II5.1: The simple ABHI model.

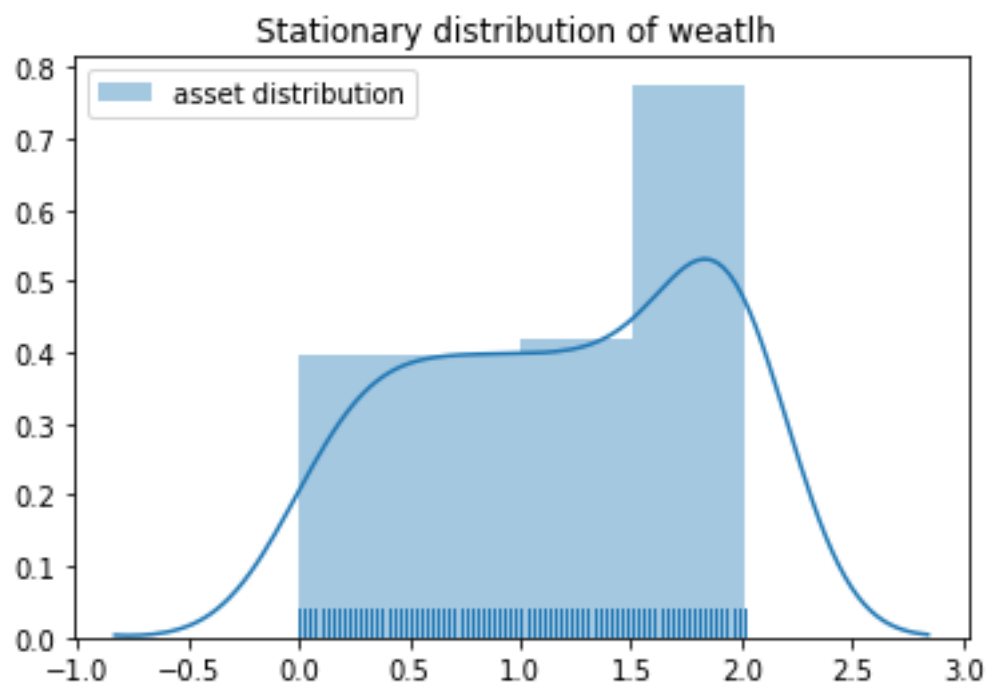
Parameters that I used are, a depreciation of 1%, a rho of 6%, a gamma of 50%.

We solve the problem of the firm with the guess, and assuming that in equilibrium aggregate labor is 1, to make this true I need a distribution of people such that if we integrate y (labor shock) we achieve 1. Since I will use 100 individuals, y that will receive every individual is $1/100$ plus (minus) the variance.

An important detail, I will use borrowing limit equal 0, nobody can fall in debt and CRRA utility function.

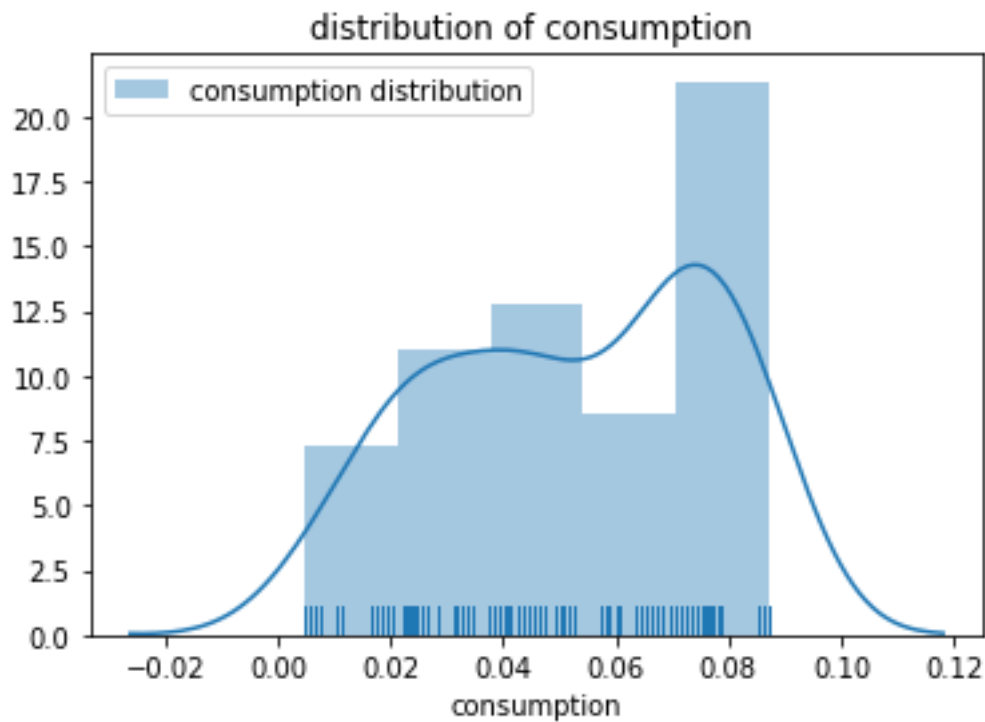
Another issue is that I should design a distribution over assets such that I hold market clearing condition associated to my r guess. Nevertheless, is much easier to guess a distribution of K and find the r and w associated to this. And that is what I will do.

The grid of assets will be a evenly space grid with 200 elements, from 0 to 5. The 100th first elements on the grid will be the initial assets that we will use to find the stationary distribution.

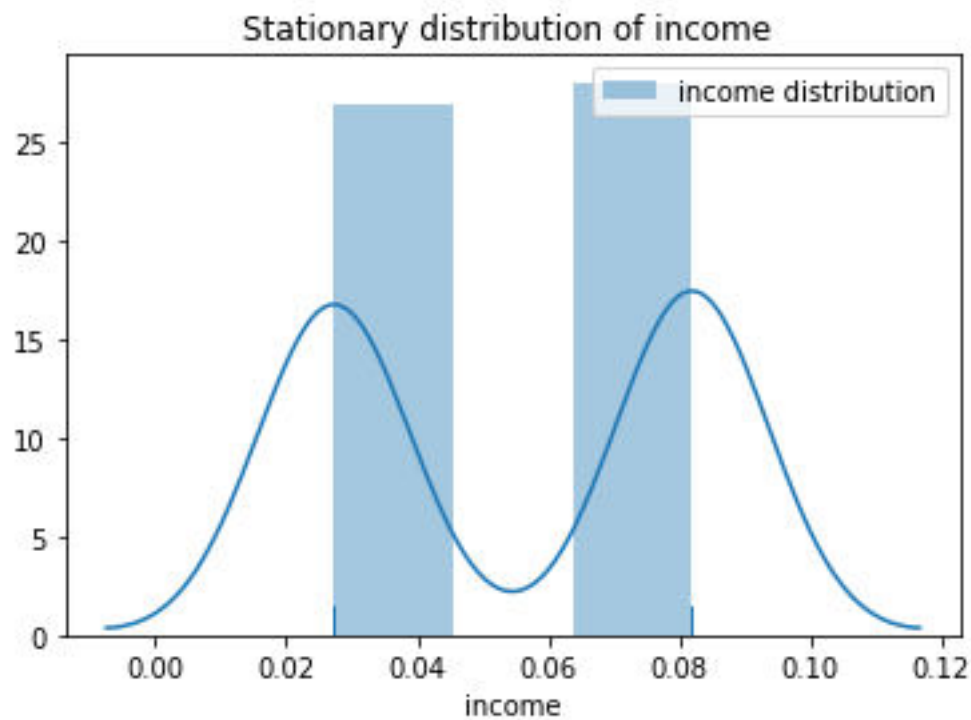


Initial asset really affects the stationary distribution since there are multiple stationary points where policy function is equal to the present asset. If we are really rich (above than 3) then we decrease either we have a good shock or a bad shock, nevertheless when if initial asset is between 2 and 3, good shock do not make the individual reduce its wealth. In the end, everybody will be under 2, where all the wealth perfils are

steady states. This means that if you started being under 2 you will be always under 2 no matter the shock you have.



Consumption is affected by shocks, even if people that is poor will not move from its wealth status, income will affect the consumption, and it happens to be that in this case I simulate a shock vector where most part of the bad shock was taken by the people that would be on the second histogram of assets (people that have between asset of 1 and 1.5), nevertheless what we see is fairly consistent, having that the poorer people consume less than the richest people.



For income distribution we can see that almost half of the people have had a bad shock and half a good shock, this is due to the vectors of the markov chain are really similar, having $[a, 1-a]$, $[b, 1-b]$ and $a = (1-b)$ and $(1-a) = b$.