Llista de problemes per al segon parcial

Entregueu per separat les solucions dels exercicis.

1. **(European debates.)** The association for the promotion of the European Identity is planning a workshop formed by a series of debates over different European topics. They have a *participant's* list formed by the persons that have committed to participate in the workshop provided the association issues them a formal invitation.

   The organizing committee in view of the planned topics and previous experiences has selected for each debate two disjoint lists of people: the *success* list and the *failure* list. The committee is considering those list in an optimistic perspective. The presence of at least one of the persons in the success list or the absence of at least one of the persons in the failure list of a particular debate guarantees that this debate will be successful.

   The organizing committee faces the problem of selecting the subset of people to whom the association will send a formal invitation. The association wish to invite a set of people that guarantees that the number of successful debates (according to the previous rule) is maximized.

   Design a 2-approximation algorithm for the problem. Justify its correctness and efficiency. In the case that the second aim of the organization is to get some sponsor's money per participant, does your algorithm provide always a satisfactory solution regarding this second criteria?

2. **(Min makespan scheduling.)** Consider the following scheduling problem: Given n jobs, m machines and, for each $1 \leq i \leq m$ and each $1 \leq j \leq n$, the amount of time t_{ij} required for the i-th machine to process the j-th job, find the schedule for all n jobs on these m machines that minimizes the makespan, i.e., the maximum processing time over all machines. You can assume that once a job starts in a machine it must run in this machine until it finishes.

   A solution for the problem can be represented by 0-1 variables, x_{ij}, $1 \leq i \leq m$ and $1 \leq j \leq n$, indicating whether the i-th machine will process the j-th job.

   (i) Using those variables provide an integer programming formulation for the problem and its corresponding LP relaxation.

   Le us call *fractional optimal schedule* an optimal solution of the LP and opt the makespan of an optimal solution of the IP.

   Observe that if $t_{ij} > \mathsf{opt}$, then job j will not be assigned to machine i in an optimal solution. This suggest that we can set an upper bound T and set $x_{ij} = 0$, whenever $t_{ij} > T$. Of course, we need to seek a suitable bound T so that a good assignment of tasks to machines is still possible.

   (ii) Provide an LP formulation to check if, for a given bound T, a fractional optimal schedule in which $x_{ij} = 0$ when $t_{ij} > T$ exists.

   (iii) Show how to find in polynomial time the value T^*, the minimum among all the integers T's for which the previous condition holds.

3. **(Tightly linked group.)** The students of the MEI PTDMA course have to design an algorithm to disclose a tightly linked group in a social network. To pass the course it is enough to design an App that identifies a smallest *distorsion set* of nodes in the social network. A distorsion set is a subset of participants whose removal leaves a *tightly linked* vertex subset. For this exercise, you can assume that the social network is an undirected connected graph and that a vertex subset X is *tightly linked* if and only if, for every pair of distinct vertices in X, either they are connected by an edge or they are both connected by an edge to a third vertex in X.

   Prove that the decision version of the problem when parameterized by the size of the distortion set belongs to FPT.

4. **(Kernel for vertex cover.)**

   Considereu el problema de la coberta de vèrtexs (min vertex cover). Saps construir en temps polinomial una solució òptima x a la versió relaxada del problema de programació entera que el descriu. En aquesta sol·lució, $x_u \in [0,1]$. Considereu els conjunts

$$S_1 = \{v \in V \mid x_v > \frac{1}{2}\}$$
$$S_{\frac{1}{2}} = \{v \in V \mid x_v = \frac{1}{2}\}$$
$$S_0 = \{v \in V \mid x_v < \frac{1}{2}\}$$

   Demostreu que sota la parametrització natural, $(G[S_{\frac{1}{2}}], k - |S_1|)$ és un kernel. Dona una mida del kernel com a funció del paràmetre.

5. **(2-colorable?)** Consider the following graph streaming algorithm

```
procedure PartTree(int n, stream s)
    F = ∅, x = 1
    while not s.end() do
        (u, v) = s.read()
        if x then
            if F ∪ {(u, v)} does not contain a cycle then
                F = F ∪ {(u, v)}
            else
                if F ∪ {(u, v)} contains an odd cycle then
                    x = 0
    On query, report x
```

   - Analyze the performance of PartTree.
   - Show that the answer to a query is 1 iff and only if the already seen graph is 2-colorable.
   - Modify the algorithm so that in addition, when the answer is 1, it provides a valid 2-coloring of G. Justify the efficiency and correctness of your proposal.