A Nonlocal Variational Approach for P+XS Image Fusion

CSASC'13

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June 10, 2013

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Contents

Introduction

The pansharpening problem

Multispectral image

+

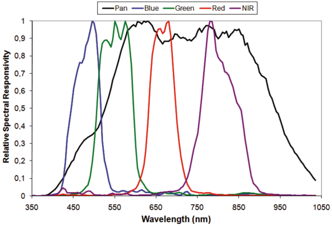
Panchromatic image

$\longrightarrow$

Pansharpened image

Introduction

The pansharpening problem



- Why not MS sensors with high spatial resolution?
  - The incoming radiation energy to the sensor
  - Bandwidth transmission from satellite to Earth.
- **Applications** $\rightarrow$ remote sensing (detection and classification), astronomy, military tasks, soil measure content, improving geometric correction, enhancing features...

Introduction

The pansharpening problem

MS image

Detail extension of MS to PAN resolution by bicubic interpolation

Truth image detail

Introduction

The pansharpening problem

Existing techniques

- IHS $\Longrightarrow$ RGB$\rightarrow$ IHS, replace I by PAN, get back to RGB.
- PCA $\Longrightarrow$ Replace the first principal component by the PAN and reverse PCA transform.
- Brovey transform $\Longrightarrow$ Normalize the MS image and multiply it by the PAN.
- Wavelet methods $\Longrightarrow$ Replace the high frequency detail coefficients by those of the PAN
- **Variational models** $\Longrightarrow$ compute fused image by minimizing an energy functional.

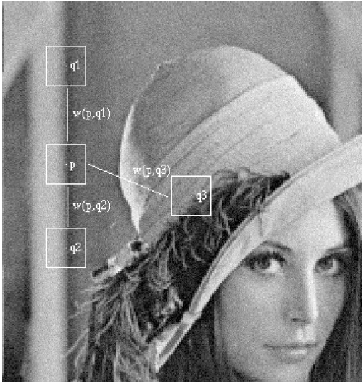
Truth image

IHS fused image

Introduction

Nonlocal-means filter

Introduce the geometry of PAN into MS using **neighbourhood filters**[1].



Non-local means filter for image denoising:

$$NL[f](x) = \frac{1}{C(x)} \int_\Omega e^{-\frac{d_\rho(f(x),f(y))}{h^2}} f(y)dy,$$

$$C(x) = \int_\Omega e^{-\frac{d_\rho(f(x),f(y)}{h^2}} dy$$

$$d_\rho(f(x), f(y)) = \int_\Omega G_\rho(t)|f(x+t) - f(y+t)|^2 dt.$$

- G_ρ Gaussian kernel, h filtering parameter.
- Regularity assumption → self-similarity.
- **Nonlocal geometry**.

[1] A. Buades, B. Coll, J.-M. Morel. *A Review of Image Denoising Algorithms, with a New One*. Mult. Modeling and Sim., 4(2):490-530, 2005.

Contents

The nonlocal functional

Notations:

- $\Omega \subset \mathbb{R}^N$ open and bounded domain.
- $S \subseteq \Omega$ **sampling grid** $\leftrightarrow$ low-resolution pixels.
- PAN image: $P : \Omega \to \mathbb{R}$.
- MS image: $\vec{u}^S = (u_1^S, \ldots, u_M^S)$, $u_m^S : S \to \mathbb{R}$, M spectral bands.
- Pansharpened image: $\vec{u} = (u_1, \ldots, u_M)$, $u_m : \Omega \to \mathbb{R}$.

The nonlocal functional

Goal $\rightarrow$ To obtain $\vec{u}$ from P and $\vec{u}^S$ as the minimizer of a functional.

P+XS $\rightarrow$ Ballester *et al.*[2] proposed to minimize

$$J(\vec{u}) = \sum_{m=1}^{M} \gamma_m \int_\Omega |\theta^\perp \cdot \nabla u_m|^2 dx \quad + \lambda \int_\Omega \left(\sum_{m=1}^{M} \alpha_m u_m(x) - P(x) \right)^2 dx +$$

$$\Downarrow \qquad + \mu \sum_{m=1}^{M} \int_\Omega \Pi_S \left(k_m * u_m(x) - u_m^\Omega(x) \right)^2 dx.$$

Nonlocal reguarization term

[2] C. Ballester, V. Caselles, L. Igual, J. Verdera. *A Variational Model for P+XS Image Fusion*. Int. Journal of Comp. Vision, 69(1):43-58, 2006.

The nonlocal functional

Panchromatic matching term

- **Assumption**: PAN and spectral bands are aligned.
- **Constraint**:

$$P(x) = \underbrace{\sum_{m=1}^{M} \alpha_m u_m(x)}_{\text{grayscale of } \vec{u}}, \quad \forall x \in \Omega,$$

where $\alpha_m \geq 0$ and $\sum_m \alpha_m = 1$.

- **Minimization term**:

$$\boxed{\int_\Omega \left(\sum_{m=1}^{M} \alpha_m u_m(x) - P(x) \right)^2 dx}$$

The nonlocal functional

Spectral correlation preserving term

- **Assumption**: Low-resolution pixels are formed from the high-resolution ones by low-pass filtering followed by subsampling.
- **Constraint**:

$$u_m^S(i,j) = k_m * u_m(si,sj), \quad \forall (i,j) \in S.$$

- **Minimization term**:

$$\boxed{\sum_{m=1}^{M} \int_\Omega \Pi_S \cdot \Big(k_m * u_m(x) - u_m^\Omega(x)\Big)^2 dx}$$

  - $k_m \rightarrow$ kernel of a convolution operator mapping $L^2(\Omega)$ into $\mathcal{C}(\bar{\Omega})$.
  - $u_m^\Omega \rightarrow$ arbitrary continuous extension of u_m^S from S to Ω.
  - $\Pi_S = \sum_{(i,j)\in S} \delta_{(i,j)} \rightarrow$ **Dirac's comb** defined by grid S.

The nonlocal functional

Nonlocal geometry enforcing term

Gilboa et al. [3] $\rightarrow$ quadratic nonlocal functional

$$\int_\Omega \int_\Omega (u(x) - u(y))^2 w_{u_0}(x,y)\,dx\,dy$$

Our propose $\Longrightarrow$ weights computed on Pan

$$\omega(x,y) = \frac{1}{C(x)} e^{-\frac{d_\rho(P(x),P(y))}{h^2}}, \quad C(x) = \text{normalizing factor}$$

Minimization term:

$$\boxed{\sum_{m=1}^{M} \int_\Omega \int_\Omega \big(u_m(x) - u_m(y)\big)^2 \omega(x,y)\,dx\,dy.}$$

[3] G. Gilboa, S. Osher, *Nonlocal image regularization and supervised segmentation* SIAM Multiscale Modelling and Simulation 6(2), 2007

Contents

Analysis of the functional

Existence and uniqueness of minimizer

- **Nonlocal boundary** $\rightarrow$ $\Gamma \subset \mathbb{R}^N \setminus \Omega$ surrounding Ω s.t. $|\Gamma| \neq 0$ and $\bar{\Omega} \cap \Gamma = \partial\Omega$.
- **Nonlocal domain** $\rightarrow$ $\tilde{\Omega} = \Omega \cup \Gamma$.
- **The solution space: Weighted $\mathbf{L^2}$ space**

$$L^2_\omega(\tilde{\Omega}\times\tilde{\Omega}) = \left\{ f : \tilde{\Omega} \times \tilde{\Omega} \rightarrow \mathbb{R} \;\middle|\; f \text{ measurable, } \int_{\tilde{\Omega}} \int_{\tilde{\Omega}} |f(x,y)|^2 \omega(x,y) dx dy < +\infty \right\}$$

- For $z : \tilde{\Omega} \rightarrow \mathbb{R}$ define $\hat{z}(x,y) = z(x) - z(y)$
- Functions with a **zero trace**:

$$\mathcal{Z} = \left\{ z : \tilde{\Omega} \rightarrow \mathbb{R} \middle| \hat{z} \in L^2_\omega(\tilde{\Omega} \times \tilde{\Omega}),\ z|_\Gamma = 0 \right\},$$

- **Class of admissible functions**: If $\vec{g} = (g_1, \ldots, g_M)$, with $g_m \in L^2(\Gamma)$,

$$\mathcal{A} = \left\{ \vec{u} = (u_1, \ldots, u_M) \;\middle|\; \begin{array}{c} u_m : \tilde{\Omega} \rightarrow \mathbb{R},\ \ u_m \in L^2(\Omega),\ \ \forall 1 \leq m \leq M \\ u_m = g_m + z_m \ \text{ for some } \ z_m \in \mathcal{Z} \end{array} \right\}.$$

Analysis of the functional

Existence and uniqueness of minimizer

$$
\begin{aligned}
J(\vec{u}) &= \frac{\gamma}{2}\sum_{m=1}^{M}\int_{\Omega}\int_{\Omega}\big(u_m(x)-u_m(y)\big)^2\omega(x,y)\,dx\,dy + \frac{\lambda}{2}\int_{\Omega}\left(\sum_{m=1}^{M}\alpha_m u_m(x) - P(x)\right)^2 \\
&+ \frac{\mu}{2}\sum_{m=1}^{M}\int_{\Omega}\Pi_S\cdot\big(k_m * u_m(x) - u_m^{\Omega}(x)\big)^2 dx
\end{aligned}
$$

where $\gamma, \lambda, \mu > 0$, $\vec{u} \in \mathcal{A}$.

Lemma

$J(\vec{u})$ is strictly convex for any $\vec{u} \in \mathcal{A}$.

Theorem

If $\vec{g} = (g_1, \ldots, g_M)$, with $g_m \in L^2(\Gamma)$, then $\exists!\, \vec{u}^* \in \mathcal{A}$ s.t.

$$J(\vec{u}^*) = \inf_{\vec{u}\in\mathcal{A}} J(\vec{u}).$$

Go to proof

Analysis of the functional

Optimality condition

$\vec{u} = (u_1, \ldots, u_M) \in \mathcal{A}$ minimizer of J.

- **Euler-Lagrange equation** for each u_k, $1 \leq k \leq M$:

$$\gamma \int_{\tilde{\Omega}} \big(u_k(x) - u_k(y)\big)\big(\omega(x,y) + \omega(y,x)\big) dy$$
$$+ \lambda \alpha_k \left(\sum_{m=1}^{M} \alpha_m u_m(x) - P(x) \right)$$
$$+ \mu k_k^{\mathsf{T}} * \left[\Pi_S \cdot \left(k_k * u_k(x) - u_k^{\Omega}(x) \right) \right] = 0, \quad \forall x \in \Omega,$$

  where $k_k^{\mathsf{T}}(x) = k_k(-x)$.

- **Nonlocal boundary condition**:

$$u_k = g_k, \quad \forall x \in \Gamma.$$

Contents

The proposed fusion algorithm

Discrete formulation

- PAN given on $I = \{0, 1, \ldots, N-1\} \times \{0, 1, \ldots, N-1\}$.
- Pixel $\rightarrow p = (p_i, p_j) \in I$.
- MS given on sampling grid $S \subseteq I$ of size $\frac{N}{s} \times \frac{N}{s}$.
- **Discrete functional**:

$$
\begin{aligned}
J(\vec{u}) &= \frac{\gamma}{2} \sum_{m=1}^{M} \sum_{p,q \in I} (u_m(p) - u_m(q))^2 \omega(p,q) \\
&+ \frac{\lambda}{2} \sum_{p \in I} \left(\sum_{m=1}^{M} \alpha_m u_m(p) - P(p) \right)^2 \\
&+ \frac{\mu}{2} \sum_{m=1}^{M} \sum_{p \in S} (k_m * u_m(p) - u_m^{\Omega}(p))^2.
\end{aligned}
$$

The proposed fusion algorithm

Discrete formulation

Gradient descent method → compute solution with **explicit scheme**:

$$\begin{aligned} u_m^{(n+1)}(p) &= u_m^{(n)}(p) - \Delta t \gamma \sum_{p,q \in I} \left(u_m^{(n)}(p) - u_m^{(n)}(q)\right)\left(\omega(p,q) + \omega(q,p)\right) \\ &- \Delta t \lambda \alpha_m \left(\sum_{k=1}^{M} \alpha_k u_m^{(n)}(p) - P(p) \right) \\ &- \Delta t \mu k_m^t * \left[\Pi_S \cdot \left(k_m * u_m^{(n)}(p) - u_m^{\Omega}(p) \right) \right], \quad \forall p \in I, \ \forall 1 \le m \le M. \end{aligned}$$

- $n \geq 0$ → iteration number.
- $\Delta t > 0$ → time step in the descent direction.
- k_m → Gaussian kernels of s.d. $\sigma = 1.2$ if $s = 2$ and $\sigma = 2.2$ if $s = 4$.
- Π_S is a $N \times N$ matrix s.t.

$$\Pi_S(p) = \begin{cases} 1 & \text{if } p \in S, \\ 0 & \text{if } p \notin S, \end{cases} \qquad \forall p \in I.$$

The proposed fusion algorithm

Computing the nonlocal weights

- Search for similar pixels restricted to pixels at a certain distance.
- $L \rightarrow$ fixed spatial support parameter.
- $\mathcal{N}_0 \rightarrow l \times l$ window centered at $(0,0)$.
- **Discrete weights**:

$$\omega(p,q) = \begin{cases} \frac{1}{C(p)} e^{-\frac{1}{h^2}\sum_{t\in\mathcal{N}_0}\|P(p+t)-P(q+t)\|^2} & \text{if } \|p-q\| \leq L, \\ 0 & \text{otherwise.} \end{cases}$$

- **Normalizing factor**:

$$C(p) = \sum_{q:\|p-q\|\leq L} e^{-\frac{1}{h^2}\sum_{t\in\mathcal{N}_0}\|P(p+t)-P(q+t)\|^2}$$

- $w(p,p)$ set to the maximum weight.

The proposed fusion algorithm

Fusion algorithm

$\vec{u}^{(0)} \leftarrow$ Initialization by IHS technique.

for all pixel $p \in I$ **do**
  Compute similarity of all 3×3 patches in search window I_p.
  Compute the weights $\omega(p,q)$ for all $q \in I_p$.
  $\omega(p,p) \leftarrow$ maximum weight.
end for

while not convergence **do**
  $\vec{u}^{(n)} \leftarrow$ solution of Euler-Lagrange equation.
end while

return $\vec{u}^{(\infty)} = (u_1^{\infty}, \dots u_M^{\infty})$.

Contents

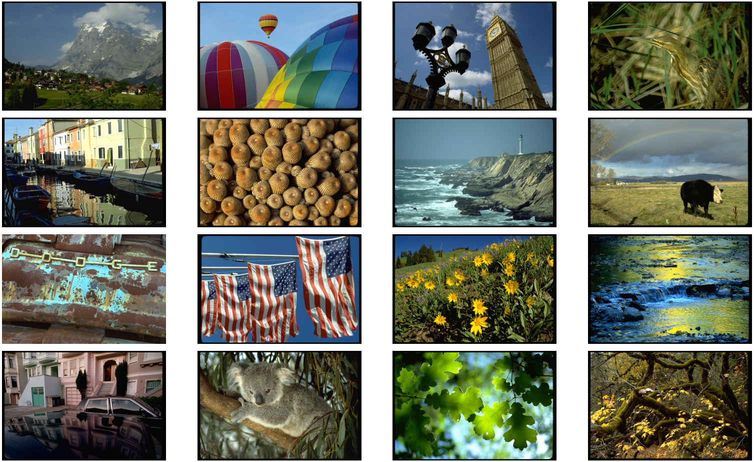
Experimental results

Description of parameters

- $M = 3 \rightarrow$ Red, Blue and Green channels.
- $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$.
- $u_m^\Omega \rightarrow$ extension of u_m^S by replication of 4-factor.
- NL-filter parameters $\rightarrow h = 2.5$, 7×7 support window, 3×3 comparison window.
- $k_1, k_2, k_3 \rightarrow$ Gaussian kernels of standard deviation 2.2,.
- Parameters of the functional $\rightarrow \gamma = 1.0$, $\lambda = 7.5$ and $\mu = 7.5 \cdot 4^2$.
- Compromise between **spatial resolution** and **spectral resolution**.

Experimental results

Database



Experimental results

Numerical comparison

	IHS	Brovey	Wavelets	P+XS	Ours
Alps	2.14	1.84	2.62	1.95	**1.47**
Balloons	2.75	2.71	4.01	2.68	**1.57**
Big Ben	2.11	2.04	2.90	2.10	**1.49**
Bittern	2.03	1.85	2.77	1.96	**1.26**
Burano	3.66	3.40	4.45	3.41	**2.61**
Cactus	3.02	2.80	3.72	3.01	**2.32**
Coast	1.67	1.63	2.26	1.70	**1.35**
Cow	1.68	1.62	2.37	1.74	**1.28**
Dodge	3.72	3.70	4.91	3.95	**2.52**
Flags	4.57	4.43	6.62	4.41	**2.65**
Flower Hill	7.29	5.93	8.37	5.95	**4.30**
Gold Water	6.49	6.31	7.91	6.93	**4.79**
Houses	1.46	1.46	2.07	1.49	**1.18**
Koala	2.08	2.05	2.79	2.17	**1.44**
Leaves	3.61	3.06	4.65	3.04	**2.20**
Woods	5.09	4.54	6.10	4.91	**3.25**
Average	3.34	3.09	4.28	3.21	**2.23**

Experimental results

Visual comparison

- Low RMSE $\leftrightarrow$ rejection by human visual inspection.
- Colours from low-resolution images are not necessarily preserved:
  - Wavelets-based dismissed due to huge color artifacts.
  - Ours outperforms IHS, Brovey, Wavelets and P+XS in spectral quality.
  - Spectral distortion more obvious in Red and Blue.
- Similar spatial quality of all techniques.
- Ours reduces the inherent noise of images.

Truth image

IHS fused image

Brovey fused image

Wavelets fused image

P+XS fused image

Ours fused image

Truth image

IHS fused image

Brovey fused image

Wavelets fused image

P+XS fused image

Ours fused image

Truth blue band

IHS blue band

Brovey blue band

Wavelets blue band

P+XS blue band

Ours blue band

Truth image

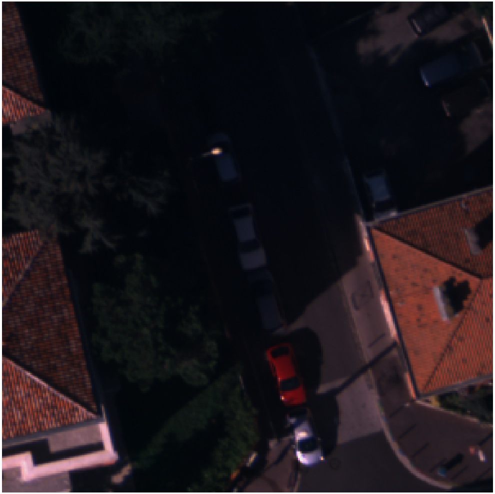
IHS fused image

Brovey fused image

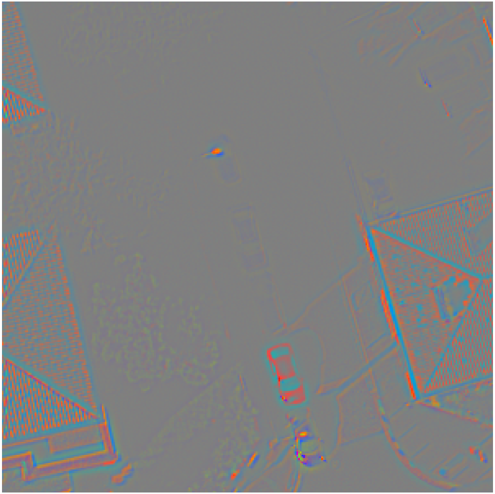
Wavelets fused image

P+XS fused image

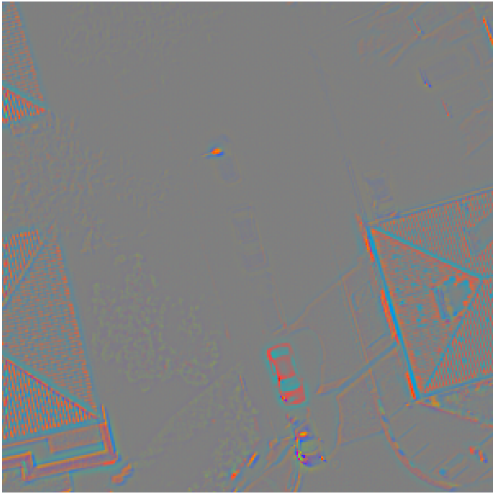
Ours fused image



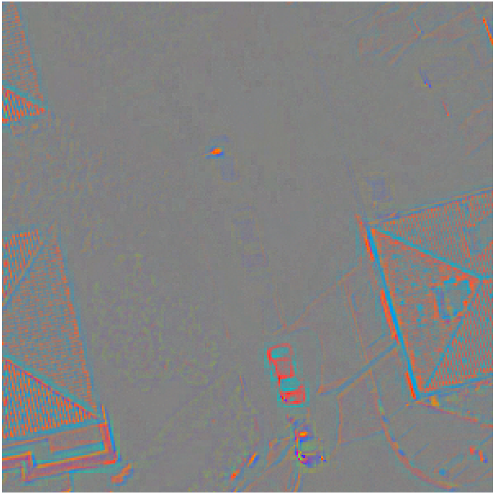
Truth Image



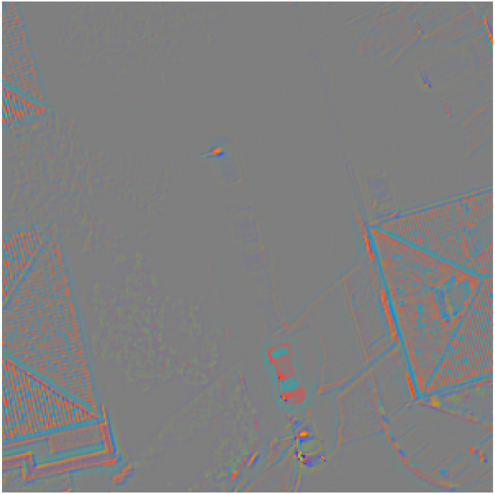
Difference with IHS fused image



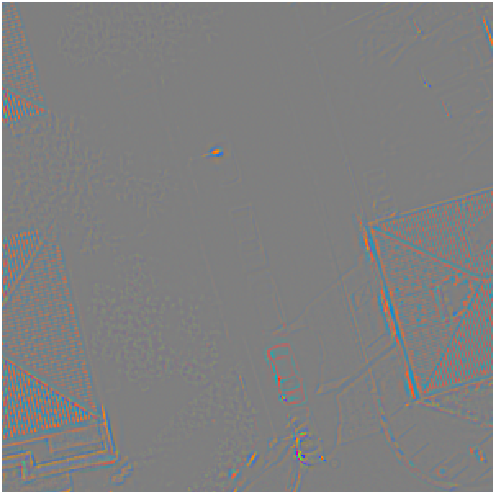
Difference with Brovey fused image



Difference with Wavelets fused image



Difference with P+XS fused image



Difference with ours fused image

Contents

Conclusions

- Satellites provide a high-resolution PAN image and a low-resolution MS image.
- Pansharpening $\rightarrow$ increase spatial resolution of MS preserving spectral quality.
- Our variational method $\rightarrow$ minimize an energy functional composed of
  - a panchromatic matching term,
  - a spectral correlation preserving term,
  - and a nonlocal geometry enforcing term.
- PAN geometry is introduced into fused image through a nonlocal filter.
- We have proved the existence and uniqueness of minimizer.
- We have developed a pansharpening algorithm from associated E-L equation.
- Experimental results:
  - Our algorithm outperforms compared techniques in RMSE.
  - Similar spatial quality of IHS, Brovey, P+XS and ours.
  - Our algorithm outperforms compared techniques in spectral quality.

Acknowledgements

These authors were supported by the Ministerio de Ciencia e Innovación under grant TIN2011-27539. During this work, the third author had a fellowship of the Conselleria d'Educació, Cultura i Universitats of the Govern de les Illes Balears for the realization of his Ph.D. thesis, which has been selected under an operational program co-financed by the European Social Fund.

Thank you for your attention

- **Boundary condition** $\rightarrow g \in L^2(\Gamma)$.
- Denote by $g + \mathcal{Z}$ functions $v : \tilde{\Omega} \rightarrow \mathbb{R}$ s.t. $v = g + z$ for some $z \in \mathcal{Z}$.

Lemma (Nonlocal Poincaré's inequality)

Let $g \in L^2(\Gamma)$. If $v \in g + \mathcal{Z}$, then $\exists \beta > 0$ and $C_g > 0$ ($C_g = 0$ iff $g \equiv 0$) s.t.

$$\beta \|v\|_{L^2(\Omega)} \leq \|\nabla_\omega v\|_{L^2(\tilde{\Omega}\times\tilde{\Omega})} + Cg,$$

or, equivalently,

$$\beta \|v\|_{L^2(\Omega)} \leq \|\hat{v}\|_{2,\omega} + C_g.$$

- **Consequence** $\rightarrow \ \mathcal{Z} \subset L^2(\Omega)$.
- **Class of admissible functions**:

$$\mathcal{A} = \left\{ \vec{u} = (u_1, \ldots, u_M) \ \middle| \ \begin{array}{c} u_m : \tilde{\Omega} \rightarrow \mathbb{R}, \ \ u_m \in L^2(\Omega), \ \ \forall 1 \leq m \leq M \\ u_m = g_m + z_m \ \ \text{for some} \ \ z_m \in \mathcal{Z} \end{array} \right\}.$$

Sketch of the Proof.

- $J(\vec{u}) \geq 0,\ \forall \vec{u} \in \mathcal{A} \ \Rightarrow\ \exists \inf_{\vec{u} \in \mathcal{A}} J(\vec{u}) = L.$
- $L < +\infty$ since $J(\vec{u}) < +\infty$ for $\vec{u} \in \mathcal{A}$ s.t. $\vec{u}|_\Omega = \vec{0}$ and $\vec{u}|_\Gamma = \vec{g}$.
- Consider a minimizing sequence $\{\vec{u}^k\}_{k \geq 0}$ in $\mathcal{A}$.
- Nonlocal Poincaré's inequality $\Rightarrow$ $\{\vec{u}^k\}_{k \geq 0}$ bounded.
- Weak Compactness Th. $\Rightarrow$ $\exists$ subsequence $\{\vec{u}^{k_j}\}_{j \geq 0}$ and $\vec{u}^* \in \mathcal{A}$ s.t. $\vec{u}^{k_j} \rightharpoonup \vec{u}^*$.
- Mazur's Lemma $\Rightarrow$ $\exists \{\vec{v}^s\}_{s \geq 0}$ convex combinations s.t. $\vec{v}^s \to u^*$.
- Convexity and Fatou's Lemma $\Rightarrow$ $J(\vec{u}^*) \leq L$.
- Strict convexity of J $\Rightarrow$ Uniqueness of minimizer.

$\square$

return