

Problem 1

Within *mixtureMean.RData*, there are two test cases (A with large number of components K ; and B with small K) consisting of a vector of μ values, a list of weights and a list of IDs that map the weights to the corresponding components in the mean vector.

(a)

One line code using *sapply()* that will calculate the weighted mean $\sum_{k=1}^{m_i} w_{i,k} \mu_{ID_{i,k}}$.

```
#### Problem Set 3 1. mixtureMean
library(rbenchmark)
rm(list = ls(all = TRUE)) # remove all objects
load("mixtureMean.RData") # import data
```

```
# (a) original data storage
mixmeanA <- sapply(1:length(IDsA), function(i) {
  return(sum(muA[IDsA[[i]]] * wgtsA[[i]]))
}) # sapply test case A
mixmeanB <- sapply(1:length(IDsB), function(i) {
  return(sum(muB[IDsB[[i]]] * wgtsB[[i]]))
}) # sapply test case B
```

(b)

I set up the objects under case A as two matrices, one storing the μ values used for calculation and one storing the corresponding weights for those mean values. The size of these two matrices are the same, which is $n \times \max(m_i)$, i.e. the number of observations times the maximum number of components per observation.

```
# (b) data setup for A: K=1000 table out as matrix storing the
# mu[ids]/wgts in cols for each observation row
idnum <- length(IDsA)
idlen <- sapply(1:idnum, function(i) {
  return(length(IDsA[[i]]))
})
maxmi <- max(idlen)
muidA <- matrix(as.numeric(NA), nr = idnum, nc = maxmi)
for (i in 1:idnum) {
  muidA[i, 1:idlen[i]] <- IDsA[[i]]
}
wtidA <- matrix(as.numeric(NA), nr = idnum, nc = maxmi)
for (i in 1:idnum) {
  wtidA[i, 1:idlen[i]] <- wgtsA[[i]]
}
mixmeanA2 <- rowSums(muA[muidA] * wtidA, na.rm = TRUE)
```

(c)

The set up of data objects under case B is even simpler. As $K = 10$ which is small, I just use a $n \times K$ matrix to store all the weights for the observations of the mean vector, leaving the untouched components with weight 0.

```

# (c) data setup for B: K=10 small K can allow us to store all the IDs as
# truth table for each u
idnum <- length(IDsB)
munum <- length(muB) # K is small
wtidB <- matrix(0, nr = munum, nc = idnum)
for (i in 1:idnum) {
  tmpwt <- rep(0, munum)
  tmpwt[IDsB[[i]]] <- wgtsB[[i]]
  wtidB[, i] <- tmpwt
}
mixmeanB2 <- colSums(muB * wtidB)

```

(d)

The comparison for the two test cases are shown below using benchmarking functions.

```

# (d) efficiency comparison
# (d) efficiency comparison
benchmark(A1 = {mixmeanA <- sapply(1:length(IDsA),
                                   function(i){ return( sum(muA[IDsA[[i]]*wgtsA[[i]]) ) } )},
          A2 = {mixmeanA2 <- rowSums(muA[muidA]*wtidA, na.rm = TRUE)},
          replications = 5)

##   test replications elapsed relative user.self sys.self user.child
## 1   A1              5    5.29      10.8      5.27      0.00         NA
## 2   A2              5     0.49       1.0      0.45      0.02         NA
##   sys.child
## 1         NA
## 2         NA

all.equal(mixmeanA, mixmeanA2)

## [1] TRUE

benchmark(B1 = {mixmeanB <- sapply(1:length(IDsB),
                                   function(i){ return( sum(muB[IDsB[[i]]*wgtsB[[i]]) ) } )},
          B2 = {mixmeanB2 <- colSums(muB*wtidB)},
          replications = 5)

##   test replications elapsed relative user.self sys.self user.child
## 1   B1              5    5.36       67      5.35      0.02         NA
## 2   B2              5     0.08       1      0.04      0.03         NA
##   sys.child
## 1         NA
## 2         NA

all.equal(mixmeanB, mixmeanB2)

## [1] TRUE

```

We can see that there is about 10X (b) and 100X (c) speed-up compared to the original case, which shows the advantages of re-arranging the data objects.

Problem 2

The CSC (compressed sparse column) format for storing sparse matrices has three components:

values: a vector of non-zero entries, stored with column major ordering

rowIndices: a vector of row indices, one for each non-zero entry

colPointers: a vector of entry index at which each column starts (length of $ncol + 1$)

```
#### Problem Set 3 2. CSC matrix
library(Matrix) # sparse matrix package
library(compiler)
library(rbenchmark)
rm(list = ls(all = TRUE)) # remove all objects
source("cscFromC.R")
```

(a)

I re-write the C-style *makeCSC()* function as below.

```
### (a) R version of makeCSC
makeCSCr <- function(matT) {
  matCSC = list()
  dimInfo <- which(matT != 0, arr.ind = TRUE) # array indices info (row, col)
  matCSC$values <- matT[dimInfo] #non-zero matrix entries
  matCSC$rowIndices <- dimInfo[, 1] # row indices
  matCSC$colPointers <- c(1, cumsum(tabulate(dimInfo[, 2], nbins = ncol(matT)))) +
    1) # cumsum the num of entries in each col
  return(matCSC)
}

m <- matrix(c(1, 0, 0, 7, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4), nr = 4)
makeCSCr(m)

## $values
## [1] 1 7 2 4
##
## $rowIndices
## [1] 1 4 2 4
##
## $colPointers
## [1] 1 3 4 4 5
```

(b)

I used the profiling tool to assess the computation bottleneck of my function, which is shown below. I generated the test matrices using the *makeTestMatrix()* function provided in *cscFromC.R*.

```
m <- makeTestMatrix(2500)
Rprof("makeCSCr.prof", interval = 0.01)
system.time(mr <- makeCSCr(m))

##      user  system elapsed
##      0.69    0.01    0.75
```

```
Rprof(NULL)
summaryRprof("makeCSCr.prof")$by.self

##          self.time self.pct total.time total.pct
## !=           0.52   55.32         0.52   55.32
## which         0.21   22.34         0.73   77.66
## gc            0.20   21.28         0.20   21.28
## makeCSCr      0.01    1.06         0.74   78.72
```

So, an improved version is shown below with built-in packages.

```
### (b) # more efficient after Rprof() and improvement in coding
makeCSCr2 <- function(matT) {
  matCSC = list()
  M <- as(matT, "dgCMatrix")
  matCSC$values <- M@x #non-zero matrix entries
  matCSC$rowIndices <- M@i + 1 # row indices
  matCSC$colPointers <- M@p + 1 # cumsum the num of entries in each col
  return(matCSC)
}
m <- matrix(c(1, 0, 0, 7, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 4), nr = 4)
makeCSCr2(m)

## $values
## [1] 1 7 2 4
##
## $rowIndices
## [1] 1 4 2 4
##
## $colPointers
## [1] 1 3 4 4 5
```

(c)

The speed test for C-style, original R-style, package-based R-style and byte-compiled R is shown below. I have also made a comparison function for checking the accuracy of the functions written in different styles.

```
compMat <- function(m1, m2) {
  # compare the CSC representation of matrix
  flag = all.equal(m1$values, m2$values)
  flag = flag && all.equal(m1$rowIndices, m2$rowIndices)
  flag = flag && all.equal(m1$colPointers, m2$colPointers)
  return(flag)
}
makeCSCrCMP <- cmpfun(makeCSCr) # byte compiled
##### Test #####
m <- makeTestMatrix(6400)

system.time(mc <- makeCSC(m))

##    user  system elapsed
##    3.76    0.03     3.79
```

```

system.time(mr <- makeCSCr(m))

##      user  system elapsed
##    1.55    0.17    1.71

compMat(mc, mr)

## [1] TRUE

##### benchmark #####
benchmark(makeCSC(m), makeCSCr(m), makeCSCr2(m), makeCSCrCMP(m), replications = 2,
  columns = c("test", "replications", "elapsed", "relative", "user.self",
    "sys.self"))

##           test replications elapsed relative user.self sys.self
## 1  makeCSC(m)           2    7.55    3.020      7.53    0.02
## 2  makeCSCr(m)          2    3.08    1.232      2.70    0.35
## 3  makeCSCr2(m)         2    2.50    1.000      1.85    0.63
## 4 makeCSCrCMP(m)        2    2.53    1.012      2.03    0.50

```

We can see that the package-based R-style code runs the fastest, that byte-compiled version runs slightly faster than the original one and that C-style code runs the slowest.

(d)

In my R-style *makeCSCr()* function, there is no additional copy of the full matrix or other matrices of that same size. The result is shown below with the *gc()* function.

```

### (d) memory usage
gc() #initial

##           used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  1390450  74.3   2251281 120.3   2251281 120.3
## Vcells 26919727 205.4   77314428 589.9 130664802 996.9

m <- makeTestMatrix(2500)
gc() # with m

##           used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  1390470  74.3   2251281 120.3   2251281 120.3
## Vcells 30044761 229.3   77314428 589.9 130664802 996.9

system.time(mr <- makeCSCr(m))

##      user  system elapsed
##    0.14    0.00    0.14

gc() # after mr

##           used (Mb) gc trigger (Mb) max used (Mb)
## Ncells  1390492  74.3   2251281 120.3   2251281 120.3
## Vcells 30059608 229.4   77314428 589.9 130664802 996.9

```

Problem 3

We would like to set the lower triangle of a matrix to equal the transpose of the upper triangle.

```
#### Problem Set 3 3. lower.triangle(mat) = transpose(upper.triangle(mat))
rm(list = ls(all = TRUE)) # remove all objects
m <- matrix(1:25, nr = 5) # simple test matrix
```

(a)

The code shown below does not work. As R uses column-major ordering to store matrix entries, when extracted using *upper.tri()*, the ordering is messed up as shown below inline with the diagnosis code.

```
m <- matrix(1:25, nr = 5) # simple test matrix
m #showcase

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   1   6  11  16  21
## [2,]   2   7  12  17  22
## [3,]   3   8  13  18  23
## [4,]   4   9  14  19  24
## [5,]   5  10  15  20  25

#### (a)
m[lower.tri(m)] = t(m[upper.tri(m)]) # this code does not work
m #showcase

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   1   6  11  16  21
## [2,]   6   7  12  17  22
## [3,]  11  17  13  18  23
## [4,]  12  18  22  19  24
## [5,]  16  21  23  24  25

# should be row-major conversion here for substitution
m[upper.tri(m)] # convert to col-major vector

## [1]  6 11 12 16 17 18 21 22 23 24
```

(b)

The working code is shown below with inline explanation of how it works. The main idea is to extract the entries from the transposed matrix to preserve the row-major ordering of the original matrix.

```
#### (b)
m <- matrix(1:25, nr = 5) # simple test matrix
m #debug

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   1   6  11  16  21
## [2,]   2   7  12  17  22
## [3,]   3   8  13  18  23
## [4,]   4   9  14  19  24
## [5,]   5  10  15  20  25
```

```

m[upper.tri(m)] #debug, col-major

## [1] 6 11 12 16 17 18 21 22 23 24

### after examination, we have to get row-major upper triangle
t(m) # transpose of original m

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1    2    3    4    5
## [2,] 6    7    8    9   10
## [3,] 11   12   13   14   15
## [4,] 16   17   18   19   20
## [5,] 21   22   23   24   25

t(m)[lower.tri(m)] # col-major of t(m) = row-major of m

## [1] 6 11 16 21 12 17 22 18 23 24

### the working one-line code to do the job
m #showcase

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1    6   11   16   21
## [2,] 2    7   12   17   22
## [3,] 3    8   13   18   23
## [4,] 4    9   14   19   24
## [5,] 5   10   15   20   25

m[lower.tri(m)] <- t(m)[lower.tri(m)] # this code works
m #showcase

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1    6   11   16   21
## [2,] 6    7   12   17   22
## [3,] 11   12   13   18   23
## [4,] 16   17   18   19   24
## [5,] 21   22   23   24   25

```

(c)

The `lower.tri<-` replacement function is shown below. The optional `byrow` argument is similar to that in `matrix()`, which let the function to fill the matrix with row-major ordering when TRUE.

```

### (c)
`lower.tri<-` <- function(x, byrow = FALSE, value){
  #replacement function for lower.tri

  x <- as.matrix(x)

  if ( length(x[lower.tri(x)]) != length(value) )
    stop("Vector length mismatch!")

  if (!byrow) x[lower.tri(x)] <- value # col-major
  else { # row-major

```

```

    tmpx <- t(x) # no replacement function for t()
    tmpx[upper.tri(tmpx)] <- value
    x <- t(tmpx)
  }
  return(x)
}

#####test#####
m <- matrix(1:25, nr = 5) # simple test matrix
m

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    6   11   16   21
## [2,]    2    7   12   17   22
## [3,]    3    8   13   18   23
## [4,]    4    9   14   19   24
## [5,]    5   10   15   20   25

lower.tri(m) <- 1:10 ; m

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    6   11   16   21
## [2,]    1    7   12   17   22
## [3,]    2    5   13   18   23
## [4,]    3    6    8   19   24
## [5,]    4    7    9   10   25

lower.tri(m, byrow = TRUE) <- 1:10 ; m

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    6   11   16   21
## [2,]    1    7   12   17   22
## [3,]    2    3   13   18   23
## [4,]    4    5    6   19   24
## [5,]    7    8    9   10   25

lower.tri(m) <- c(1,2,3)

## Error: Vector length mismatch!

```


Problem 4

For the object-oriented coding to generate a Markov chain with a fixed number of states, I choose to use the widely accepted and fast speed S3 class. A simple test case is generated with matrix attributes required for a Markov chain object.

```
#### Problem Set 3 4. Markov Chain (OOP)
library(methods)
rm(list = ls(all = TRUE)) # remove all objects
rnd <- rnorm(50000) # rnd sample
rnd <- rnd[rnd > 0] # prob > 0

p = 5 # num of states
x0 <- rep(0, p) # get length p vector
x0[1] <- 1 # initialize with starting from state 1
xm <- matrix(sample(rnd, p * p), nr = p) # get p-by-p matrix of rnds
xm <- xm/rowSums(xm) # normalize rows

drawState <- function(p, pr) {
  if (length(pr) != p)
    stop("Vector length mismatch!")
  states <- diag(nrow = p, ncol = p) # state vectors
  id <- sample(1:5, 1, prob = pr)
  return(as.vector(states[id, ])) # get random states according to prob=pr
}
```

For an object in Markov chain class, it has the initial state vector *init* and two matrices i.e. transition matrix *P* and chain matrix *chain* which stores the process. In details, the transition matrix *P* of dimension $p \times p$ gives the transition probability $P[i, j] = Pr(X_n = j | X_{n-1} = i)$.

(a)

The constructor that creates an *Markov* class object utilizes a method *runSteps()* to run the chain for a given number of steps requested by user. The validity is checked inline with the code shown below.

```
### (a) create the Markov S3 class:
### initial state, transition matrix, num of steps, current state and [if needed, chain];

### creating a "runSteps" method for the MarkovS3 class construction
runSteps <- function(object, ...) UseMethod("runSteps") # generic method

runSteps.MarkovS3 <- function(obj) {
  if (obj$n == 0) {
    obj$chain <- matrix(obj$init, nr = 1)
    return(obj) # no more steps, simple return the initial state
  } else {
    p = length(obj$init)
    obj$chain <- matrix(as.numeric(NA), nr = obj$n + 1, nc = p)
    obj$chain[1, ] <- obj$init
    for (i in 1:obj$n)
      obj$chain[i + 1, ] <- drawState(p, as.vector(obj$chain[i, ] %*% obj$P))
  }
  return(obj)
}
```

```

### constructor for 'MarkovS3' class
MarkovS3 <- function(init = NA, P = NA, n = 0, chain = NA){
  # check validity
  p = length(init) # num of states
  nr = dim(P)[1]; nc = dim(P)[2]; # dim of transition matrix

  if (n < 0)
    stop("Need positive integer steps!")
  if (p < 2)
    stop("Need at least two states for the Markov chain!")
  if (nr != nc)
    stop("The transistion matrix has to be square!")
  if (p != nr)
    stop("Mismatch between state vector length and transition matrix dimension!")
  if (length(init[init < 0]) > 0)
    stop("All state probabilities should be positive!")
  if (length(P[P < 0]) > 0)
    stop("All transition probabilities should be positive!")
  if (sum(init) != 1)
    stop("State probabilities should sum to one!")
  if (!all.equal(rowSums(P), rep(1, p)))
    stop("Transition probabilities for each state should sum to one!")

  # initialize the Markov chain
  obj <- list(init = init, P = P, n = n, chain = NA)
  class(obj) <- 'MarkovS3'

  # construct the Markov chain
  obj <- runSteps(obj)
  return(obj)
}

### simple tests
mkc0 <- MarkovS3(x0, xm)
mkc1 <- MarkovS3(x0, xm, n = 1)
mkc1

## $init
## [1] 1 0 0 0 0
##
## $P
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0.07202 0.06330 0.2873 0.10163 0.4757
## [2,] 0.13766 0.40616 0.2335 0.04011 0.1826
## [3,] 0.25238 0.19789 0.1208 0.22651 0.2024
## [4,] 0.05318 0.02573 0.3286 0.43496 0.1575
## [5,] 0.12563 0.08561 0.2353 0.35161 0.2018
##
## $n
## [1] 1
##
## $chain
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1 0 0 0 0

```

```
## [2,]    0    1    0    0    0
##
## attr("class")
## [1] "MarkovS3"

mkc2 <- MarkovS3(x0, xm, n = 1000)
```

(b)

Three operators ('+', '-', '[') for the Markov chain class is developed as shown below.

```
#### (b) Markov chain S3 class-specific operators
`+` .MarkovS3` <- function(obj, incr) {
  # check validity
  if (incr < 0) stop("Need positive integer steps!")
  if (incr == 0) return (obj)

  p = length(obj$init)
  addchain <- matrix(as.numeric(NA), nr = incr + 1, nc = p)
  addchain[1, ] <- obj$chain[obj$n + 1, ] # current state
  for (i in 1:incr)
    addchain[i + 1, ] <- drawState(p, as.vector(addchain[i, ] %*% obj$P))

  obj$n <- obj$n + as.integer(incr) # add steps
  obj$chain <- rbind(obj$chain, addchain[2:(incr+1), ])
  return(obj)
}

`-` .MarkovS3` <- function(obj, decr) {
  # check validity
  if (decr < 0) stop("Need positive integer steps!")
  if (obj$n < decr) stop("Not enough steps of states to remove!")
  if (decr == 0) return (obj)

  obj$n <- obj$n - as.integer(decr) # remove steps
  obj$chain <- obj$chain[1:(obj$n + 1), ]
  if (obj$n == 0)
    obj$chain <- matrix(obj$chain, nr = 1)
  return(obj)
}

`[` .MarkovS3` <- function(obj, idVec) {
  # check validity
  if (length(idVec) == 0) stop("No indices for extracting chain states!")
  if (length(idVec[idVec < 1 | idVec > obj$n+1])) stop("Indices out of bound!")

  return(obj$chain[idVec, ]) # init state is 1st
}

# show only the chain
mkc1$chain

##      [,1] [,2] [,3] [,4] [,5]
```

```
## [1,] 1 0 0 0 0
## [2,] 0 1 0 0 0

(mkc1 + 1)$chain

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1 0 0 0 0
## [2,] 0 1 0 0 0
## [3,] 0 0 1 0 0

(mkc1 - 1)$chain

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1 0 0 0 0

mkc2[10:15]

##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0 0 1 0 0
## [2,] 1 0 0 0 0
## [3,] 0 0 0 0 1
## [4,] 0 0 1 0 0
## [5,] 0 0 0 0 1
## [6,] 0 1 0 0 0
```

(c)

Three methods, i.e. *plot()*, *summary()*, *print()* or *show()* that produce nice, formatted output are developed for Markov chain class shown below.

```
##### (c) Markov chain S3 class-specific functions
plot.MarkovS3 <- function(obj, ...) {
  if (obj$n == 0)
    cat("The current state is the same as the initial state\n", obj$chain[1, ], "\n")
  if (obj$n > 0){
    data <- which(obj$chain > 0, arr.ind = TRUE)
    plot(data[,1], data[,2], xlab="Steps", ylab="States Prob", ...)
    hist(data[,2], c(0:5+0.5), probability = TRUE,
         main = "Histogram of states", xlab = "Markov chain states")
  }
}

summary.MarkovS3 <- function(obj, ...) {
  p <- length(obj$init)
  allPre <- colSums(obj$chain[-(obj$n+1), ]) # previous states counts
  Tsub <- obj$chain[-1, ] - obj$chain[-(obj$n+1), ] # transition records (1:from,-1:to)
  Tadd <- obj$chain[-1, ] + obj$chain[-(obj$n+1), ]
  Tadd[which(Tadd == 1)] <- 0 # retaining the state
  Pemp <- diag(colSums(Tadd)/2/allPre, nrow = p, ncol = p) #retaining states counts, diag
  for (i in 1:p)
    for (j in 1:p)
      if (i != j)
        Pemp[i,j] <- length(which(Tsub[,i] == 1 & Tsub[,j] == -1))/allPre[i]
```

```

cat("The initial state of the Markov chain is\n"); print(obj$init)
cat("The transition matrix of this chain is\n"); print(obj$P)
cat("The current step is", obj$n, ", and the current state is\n");
print(obj$chain[obj$n+1, ])
cat("The empirical transition probabilities are \n"); print(Pemp)
cat("The empirical state probabilities are \n"); print(colMeans(obj$chain))
}

print.MarkovS3 <- function(obj, ...) {
  cat("Initial state\n"); print(obj$init)
  cat("Transition matrix\n"); print(obj$P)
  cat("Current step\n"); print(obj$n)
  cat("Current state\n"); print(obj$chain[obj$n+1, ])
}

summary(mkc2)

## The initial state of the Markov chain is
## [1] 1 0 0 0 0
## The transition matrix of this chain is
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0.07202 0.06330 0.2873 0.10163 0.4757
## [2,] 0.13766 0.40616 0.2335 0.04011 0.1826
## [3,] 0.25238 0.19789 0.1208 0.22651 0.2024
## [4,] 0.05318 0.02573 0.3286 0.43496 0.1575
## [5,] 0.12563 0.08561 0.2353 0.35161 0.2018
## The current step is 1000 , and the current state is
## [1] 0 0 0 1 0
## The empirical transition probabilities are
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0.05109 0.13869 0.4380 0.05839 0.3066
## [2,] 0.06400 0.33600 0.3920 0.04000 0.1680
## [3,] 0.17241 0.11638 0.1164 0.37931 0.2155
## [4,] 0.04563 0.02281 0.1939 0.44867 0.2928
## [5,] 0.28807 0.12757 0.1852 0.18107 0.2181
## The empirical state probabilities are
## [1] 0.1369 0.1249 0.2318 0.2637 0.2428

print(mkc1)

## Initial state
## [1] 1 0 0 0 0
## Transition matrix
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 0.07202 0.06330 0.2873 0.10163 0.4757
## [2,] 0.13766 0.40616 0.2335 0.04011 0.1826
## [3,] 0.25238 0.19789 0.1208 0.22651 0.2024
## [4,] 0.05318 0.02573 0.3286 0.43496 0.1575
## [5,] 0.12563 0.08561 0.2353 0.35161 0.2018
## Current step
## [1] 1
## Current state
## [1] 0 1 0 0 0

```

```

plot(mkc0)

## The current state is the same as the initial state
## 1 0 0 0 0

plot(mkc2)

```



