Chapter 1: Production function estimation

Joan Llull

Structural Econometrics for Labor Economics
and Industrial Organization

IDEA PhD Program

Introduction

Introduction

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.
- Factor **intensity** and **skill-biased technical change**.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.
- Factor **intensity** and **skill-biased technical change**.
- **Learning-by-doing**.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.
- Factor **intensity** and **skill-biased technical change**.
- **Learning-by-doing**.
- Technology adoption and endogenous **innovation**.

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.
- Factor **intensity** and **skill-biased technical change**.
- **Learning-by-doing**.
- Technology adoption and endogenous **innovation**.
- ...

Introduction

Production functions are important elements in Economics, especially IO and also have important applications in labor.

They play a key role in determining:

- Aggregate **productivity** and its **dispersion**.
- **Miss-allocation** of resources.
- **Marginal costs**, **marginal productivity**, and input **prices**.
- Factor **intensity** and **skill-biased technical change**.
- **Learning-by-doing**.
- Technology adoption and endogenous **innovation**.
- ...

Two **approaches**: firm level and aggregate.

Firm-level estimation

A simple Cobb-Douglas production framework

A simple Cobb-Douglas production framework

Let y_{it} denote **output** of firm i at time t, and k_{it} and l_{it} the two inputs used in production, namely **capital** and **labor**:

A simple Cobb-Douglas production framework

Let y_{it} denote **output** of firm i at time t, and k_{it} and l_{it} the two inputs used in production, namely **capital** and **labor**:

$$y_{it} = \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta},$$

where ζ_{it} is firm's **total factor productivity** (TFP).

A simple Cobb-Douglas production framework

Let y_{it} denote **output** of firm i at time t, and k_{it} and l_{it} the two inputs used in production, namely **capital** and **labor**:

$$y_{it} = \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta},$$

where ζ_{it} is firm's **total factor productivity** (TFP).

Taking **logs**:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \nu_{it} + \varepsilon_{it},$$

where $\nu_{it} \equiv \ln \zeta_{it}$, and ε_{it} is **measurement error**.

A simple Cobb-Douglas production framework

Let y_{it} denote **output** of firm i at time t, and k_{it} and l_{it} the two inputs used in production, namely **capital** and **labor**:

$$y_{it} = \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta},$$

where ζ_{it} is firm's **total factor productivity** (TFP).

Taking **logs**:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \nu_{it} + \varepsilon_{it},$$

where $\nu_{it} \equiv \ln \zeta_{it}$, and ε_{it} is **measurement error**.

Even a linear regression can be a **structural model**!

OLS biases

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

$$\max_{k_{it},l_{it}} \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta} - r_{it} k_{it} - w_{it} l_{it}, \Rightarrow$$

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

$$\max_{k_{it},l_{it}} \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta} - r_{it}k_{it} - w_{it}l_{it}, \Rightarrow$$

$$\Rightarrow \begin{cases} w_{it} = \beta\zeta_{it}k_{it}^{\alpha}l_{it}^{\beta-1} \\ r_{it} = \alpha\zeta_{it}k_{it}^{\alpha-1}l_{it}^{\beta} \end{cases}$$

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

$$\max_{k_{it},l_{it}} \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta} - r_{it}k_{it} - w_{it}l_{it}, \Rightarrow$$

$$\Rightarrow \begin{cases} w_{it} = \beta\zeta_{it}k_{it}^{\alpha}l_{it}^{\beta-1} \\ r_{it} = \alpha\zeta_{it}k_{it}^{\alpha-1}l_{it}^{\beta} \end{cases} \qquad \Rightarrow \qquad \begin{cases} l_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\beta}{w_{it}}\left(\frac{w_{it}\alpha}{r_{it}\beta}\right)^{\alpha} \\ k_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\alpha}{r_{it}}\left(\frac{r_{it}\beta}{w_{it}\alpha}\right)^{\beta} \end{cases}.$$

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

$$\max_{k_{it},l_{it}} \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta} - r_{it}k_{it} - w_{it}l_{it}, \Rightarrow$$

$$\Rightarrow \begin{cases} w_{it} = \beta\zeta_{it}k_{it}^{\alpha}l_{it}^{\beta-1} \\ r_{it} = \alpha\zeta_{it}k_{it}^{\alpha-1}l_{it}^{\beta} \end{cases} \qquad \Rightarrow \qquad \begin{cases} l_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\beta}{w_{it}}\left(\frac{w_{it}\alpha}{r_{it}\beta}\right)^{\alpha} \\ k_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\alpha}{r_{it}}\left(\frac{r_{it}\beta}{w_{it}\alpha}\right)^{\beta} \end{cases}.$$

Measurement error in the inputs $\Rightarrow$ standard measurement error bias.

OLS biases

OLS has one problem: the **simultaneity bias**. In particular:

$$\max_{k_{it},l_{it}} \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta} - r_{it}k_{it} - w_{it}l_{it}, \Rightarrow$$

$$\Rightarrow \begin{cases} w_{it} = \beta\zeta_{it}k_{it}^{\alpha}l_{it}^{\beta-1} \\ r_{it} = \alpha\zeta_{it}k_{it}^{\alpha-1}l_{it}^{\beta} \end{cases} \qquad \Rightarrow \qquad \begin{cases} l_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\beta}{w_{it}}\left(\frac{w_{it}\alpha}{r_{it}\beta}\right)^{\alpha} \\ k_{it}^{1-\alpha-\beta} = \zeta_{it}\frac{\alpha}{r_{it}}\left(\frac{r_{it}\beta}{w_{it}\alpha}\right)^{\beta} \end{cases}.$$

Measurement error in the inputs $\Rightarrow$ standard measurement error bias.

Selection bias driven by endogenous exits of firms ($\mathbb{E}[\nu_{it}|k_{it}, l_{it}, d_{it} = 1] \neq 0$).

Instrumental Variables

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Exogeneity: competitive setting with different markets: variation in prices that is **exogenous** to firm's productivity.

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Exogeneity: competitive setting with different markets: variation in prices that is **exogenous** to firm's productivity.

Drawbacks:

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Exogeneity: competitive setting with different markets: variation in prices that is **exogenous** to firm's productivity.

Drawbacks:

- Input prices may not be **observable**.

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Exogeneity: competitive setting with different markets: variation in prices that is **exogenous** to firm's productivity.

Drawbacks:

- Input prices may not be **observable**.
- Firms often operate in **non-competitive** settings: input prices may be affected by firm's productivity.

Instrumental Variables

One of the classic solutions: **input prices as instruments**.

The demands are functions of prices and productivity (**relevance**).

Exogeneity: competitive setting with different markets: variation in prices that is **exogenous** to firm's productivity.

Drawbacks:

- Input prices may not be **observable**.
- Firms often operate in **non-competitive** settings: input prices may be affected by firm's productivity.
- Variation in prices **rejects itself the constant parameter model**: $\beta = \frac{w_{it} l_{it}}{y_{it}}$ is not constant in the data.

Fixed effects

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.

2. υ_{it} is i.i.d. over time.

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.
2. υ_{it} is i.i.d. over time.
3. within-firm over time variation in input demands.

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.
2. υ_{it} is i.i.d. over time.
3. within-firm over time variation in input demands.

Drawbacks:

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.
2. υ_{it} is i.i.d. over time.
3. within-firm over time variation in input demands.

Drawbacks:

- 2) and 3) are plausible in **agricultural firms** in developing countries, unlikely to hold for manufacturing in developed countries.

Fixed effects

Alternative approach: $\nu_{it} = \eta_i + \delta_t + \upsilon_{it}$, where υ_{it} is unknown by the firm at the time of setting up demands.

Fixed effects (within groups) consistent if:

1. υ_{it} is uncorrelated with inputs demands.
2. υ_{it} is i.i.d. over time.
3. within-firm over time variation in input demands.

Drawbacks:

- 2) and 3) are plausible in **agricultural firms** in developing countries, unlikely to hold for manufacturing in developed countries.
- **Measurement error bias** exacerbated by the within groups transformation, especially when there is little variation in inputs.

Dynamic panel data

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic demands $\Rightarrow$ as long as υ_{it} is i.i.d over time, k_{it-j}, l_{it-j}, and Y_{it-j} for $j \geq 2$ valid **instruments** for:

$$\Delta \ln y_{it} = \alpha \Delta \ln k_{it} + \beta \Delta \ln l_{it} + \Delta \delta_t + \Delta \upsilon_{it}.$$

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic demands $\Rightarrow$ as long as υ_{it} is i.i.d over time, k_{it-j}, l_{it-j}, and Y_{it-j} for $j \geq 2$ valid **instruments** for:

$$\Delta \ln y_{it} = \alpha \Delta \ln k_{it} + \beta \Delta \ln l_{it} + \Delta \delta_t + \Delta \upsilon_{it}.$$

Drawbacks:

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic demands $\Rightarrow$ as long as υ_{it} is i.i.d over time, k_{it-j}, l_{it-j}, and Y_{it-j} for $j \geq 2$ valid **instruments** for:

$$\Delta \ln y_{it} = \alpha \Delta \ln k_{it} + \beta \Delta \ln l_{it} + \Delta \delta_t + \Delta \upsilon_{it}.$$

Drawbacks:

- I.i.d. assumption is **testable** (Arellano-Bond). Often rejected!

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic demands $\Rightarrow$ as long as υ_{it} is i.i.d over time, k_{it-j}, l_{it-j}, and Y_{it-j} for $j \geq 2$ valid **instruments** for:

$$\Delta \ln y_{it} = \alpha \Delta \ln k_{it} + \beta \Delta \ln l_{it} + \Delta \delta_t + \Delta \upsilon_{it}.$$

Drawbacks:

- I.i.d. assumption is **testable** (Arellano-Bond). Often rejected!
- Instruments often **weak** (strong persistence in the demands).

Dynamic panel data

Strict exogeneity can be relaxed assuming **dynamic demands**:

$$k_{it} = F_K(k_{it-1}, l_{it-1}, \nu_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it-1}, l_{it-1}, \nu_{it}).$$

Multiple reasons hiring and firing costs for labor, irreversibility of some capital investments, installation costs, time-to-build capital,...

Dynamic demands $\Rightarrow$ as long as υ_{it} is i.i.d over time, k_{it-j}, l_{it-j}, and Y_{it-j} for $j \geq 2$ valid **instruments** for:

$$\Delta \ln y_{it} = \alpha \Delta \ln k_{it} + \beta \Delta \ln l_{it} + \Delta \delta_t + \Delta \upsilon_{it}.$$

Drawbacks:

- I.i.d. assumption is **testable** (Arellano-Bond). Often rejected!
- Instruments often **weak** (strong persistence in the demands).
- First differences **remove cross-sectional variation** & worsen **measurement error**.

Blundell and Bond (2000)

Blundell and Bond (2000)

Blundell and Bond (2000): $\nu_{it} = \rho\nu_{it-1} + \eta_{it} + \delta_t + \upsilon_{it}$.

Blundell and Bond (2000)

Blundell and Bond (2000): $\nu_{it} = \rho\nu_{it-1} + \eta_{it} + \delta_t + \upsilon_{it}$.

Under this assumption (ignoring measurement error):

$$\begin{aligned}\ln y_{it} = \rho \ln y_{it-1} &+ \alpha(\ln k_{it} - \rho \ln k_{it-1}) \\ &+ \beta(\ln l_{it} - \rho \ln l_{it-1}) + \eta_{it} + \delta_t + \upsilon_{it}.\end{aligned}$$

Blundell and Bond (2000)

Blundell and Bond (2000): $\nu_{it} = \rho\nu_{it-1} + \eta_{it} + \delta_t + \upsilon_{it}$.

Under this assumption (ignoring measurement error):

$$\begin{aligned}\ln y_{it} = \rho \ln y_{it-1} &+ \alpha(\ln k_{it} - \rho \ln k_{it-1}) \\ &+ \beta(\ln l_{it} - \rho \ln l_{it-1}) + \eta_{it} + \delta_t + \upsilon_{it}.\end{aligned}$$

Estimate using **Blundell and Bond** (1998), based on **Arellano and Bond** (1991) and **Arellano and Bover** (1995).

Control function approaches

Control function approaches

Olley and Pakes (1996) and **Levinshon and Petrin (2003)** propose “control function approaches”.

Control function approaches

Olley and Pakes (1996) and **Levinshon and Petrin (2003)** propose "control function approaches".

Instead of finding instruments for l_{it} and k_{it} add **observables** that can "control" for unobserved total factor productivity.

Control function approaches

Olley and Pakes (1996) and **Levinshon and Petrin (2003)** propose "control function approaches".

Instead of finding instruments for l_{it} and k_{it} add **observables** that can "control" for unobserved total factor productivity.

These control variables come from a **model of firm behavior**.

Olley and Pakes (1996): setup

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

- $F_K(\cdot)$ is **invertible** in ν_{it}.

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

- $F_K(\cdot)$ is **invertible** in ν_{it}.
- no **cross-sectional variation** in prices, $\boldsymbol{r}_{it} = \boldsymbol{r}_t$ for all i.

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

- $F_K(\cdot)$ is **invertible** in ν_{it}.
- no **cross-sectional variation** in prices, $\boldsymbol{r}_{it} = \boldsymbol{r}_t$ for all i.
- ν_{it} follows a first order **Markov process**.

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

- $F_K(\cdot)$ is **invertible** in ν_{it}.
- no **cross-sectional variation** in prices, $\boldsymbol{r}_{it} = \boldsymbol{r}_t$ for all i.
- ν_{it} follows a first order **Markov process**.
- **Time-to-build**: investment i_{it} is chosen in period t, but it is not productive until $t+1$, when $k_{it+1} = (1-\delta)k_{it} + i_{it}$.

Olley and Pakes (1996): setup

Consider the following **modification of the demands** above:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r}_{it}),$$

where i_{it} denotes investment at time t, and $\boldsymbol{r}_{it}$ is the vector of factor prices, in this case, $\boldsymbol{r}_{it} = (r_{it}, w_{it})'$.

Assumptions:

- $F_K(\cdot)$ is **invertible** in ν_{it}.
- no **cross-sectional variation** in prices, $\boldsymbol{r}_{it} = \boldsymbol{r}_t$ for all i.
- ν_{it} follows a first order **Markov process**.
- **Time-to-build**: investment i_{it} is chosen in period t, but it is not productive until $t+1$, when $k_{it+1} = (1-\delta)k_{it} + i_{it}$.

This approach can be adjusted to also deal with the **endogenous exit** selection bias (see Aguirregabiria, 2019).

Olley and Pakes (1996): method

Olley and Pakes (1996): method

Two-stage estimation.

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$
$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Recursive algorithm:

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Recursive algorithm: i) guess $\alpha^{(0)}$,

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Recursive algorithm: i) guess $\alpha^{(0)}$, ii) compute $\hat{\phi}_{it-1} - \alpha^{(0)} \ln k_{it-1}$,

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Recursive algorithm: i) guess $\alpha^{(0)}$, ii) compute $\hat{\phi}_{it-1} - \alpha^{(0)} \ln k_{it-1}$, iii) update $\alpha^{(1)}$ as the coefficient of $\ln k_{it}$ in the semi-parametric regression

Olley and Pakes (1996): method

Two-stage estimation.

First stage: (based on invertibility and no cross-sectional variation):

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \varepsilon_{it},$$

where $\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t)$.

Estimation: semi-parametric methods like kernel regressions or polynomial series approximation.

Drawback: requires that there is enough variation in l_{it} to identify β after controlling for l_{it-1}, k_{it}, and i_{it}.

Second stage: (based on Markovian nature of ν_{it} and time-to-build):

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t) = \nu_{it} = \mathbb{E}[\nu_{it}|\nu_{it-1}] + \xi_{it} \equiv h(\nu_{it-1}) + \xi_{it}, \Rightarrow$$

$$\Rightarrow \hat{\phi}_{it} = \alpha \ln k_{it} + h(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}) + \xi_{it},$$

where $\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}$.

Recursive algorithm: i) guess $\alpha^{(0)}$, ii) compute $\hat{\phi}_{it-1} - \alpha^{(0)} \ln k_{it-1}$, iii) update $\alpha^{(1)}$ as the coefficient of $\ln k_{it}$ in the semi-parametric regression and iv) iterate until convergence.

Levinshon and Petrin (2003): setup

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

- **Investment can be responsive** to more persistent TFP shocks.

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

- **Investment can be responsive** to more persistent TFP shocks.
- **Zero investment** very present in many data-sets (at $i_{it} = 0$, corner solution, there is no invertibility between i_{it} and ν_{it}).

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

- **Investment can be responsive** to more persistent TFP shocks.
- **Zero investment** very present in many data-sets (at $i_{it} = 0$, corner solution, there is no invertibility between i_{it} and ν_{it}).

Levinshon and Petrin (2003) consider alternative setup:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \gamma \ln M_{it} + \nu_{it} + \varepsilon_{it},$$

where M_{it} denotes intermediate inputs (materials).

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

- **Investment can be responsive** to more persistent TFP shocks.
- **Zero investment** very present in many data-sets (at $i_{it} = 0$, corner solution, there is no invertibility between i_{it} and ν_{it}).

Levinshon and Petrin (2003) consider alternative setup:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \gamma \ln M_{it} + \nu_{it} + \varepsilon_{it},$$

where M_{it} denotes intermediate inputs (materials).

Investment equation replaced with **demand for materials**:

$$M_{it} = F_M(l_{it-1}, k_{it}, \nu_{it}, \boldsymbol{r}_{it}),$$

assumed, again, to be **invertible** in ν_{it}.

Levinshon and Petrin (2003): setup

Some drawbacks of Olley-Pakes:

- **Investment can be responsive** to more persistent TFP shocks.
- **Zero investment** very present in many data-sets (at $i_{it} = 0$, corner solution, there is no invertibility between i_{it} and ν_{it}).

Levinshon and Petrin (2003) consider alternative setup:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \gamma \ln M_{it} + \nu_{it} + \varepsilon_{it},$$

where M_{it} denotes intermediate inputs (materials).

Investment equation replaced with **demand for materials**:

$$M_{it} = F_M(l_{it-1}, k_{it}, \nu_{it}, \boldsymbol{r}_{it}),$$

assumed, again, to be **invertible** in ν_{it}.

Other assumptions (no cross-sectional variation, Markovian structure, and time-to-build) still assumed to hold.

Levinshon and Petrin (2003): method

Levinshon and Petrin (2003): method

First step is analogous to that in Olley and Pakes, except that investment is replaced by the demand for materials:

$$\ln y_{it} = \beta \ln l_{it} + \varphi_t(l_{it-1}, k_{it}, M_{it}) + \varepsilon_{it},$$

where $\varphi_t(l_{it}, k_{it}, M_{it}) \equiv \alpha \ln k_{it} + \gamma \ln M_{it} + F_M^{-1}(l_{it-1}, k_{it}, M_{it}, \boldsymbol{r}_{it})$.

Levinshon and Petrin (2003): method

First step is analogous to that in Olley and Pakes, except that investment is replaced by the demand for materials:

$$\ln y_{it} = \beta \ln l_{it} + \varphi_t(l_{it-1}, k_{it}, M_{it}) + \varepsilon_{it},$$

where $\varphi_t(l_{it}, k_{it}, M_{it}) \equiv \alpha \ln k_{it} + \gamma \ln M_{it} + F_M^{-1}(l_{it-1}, k_{it}, M_{it}, \boldsymbol{r}_{it})$.

Second step also analogous:

$$\hat{\varphi}_{it} = \alpha \ln k_{it} + \gamma \ln M_{it} + h(\hat{\varphi}_{it-1} - \alpha \ln k_{it-1} - \gamma \ln M_{it-1}) + \xi_{it}.$$

Levinshon and Petrin (2003): method

First step is analogous to that in Olley and Pakes, except that investment is replaced by the demand for materials:

$$\ln y_{it} = \beta \ln l_{it} + \varphi_t(l_{it-1}, k_{it}, M_{it}) + \varepsilon_{it},$$

where $\varphi_t(l_{it}, k_{it}, M_{it}) \equiv \alpha \ln k_{it} + \gamma \ln M_{it} + F_M^{-1}(l_{it-1}, k_{it}, M_{it}, \boldsymbol{r}_{it})$.

Second step also analogous:

$$\hat{\varphi}_{it} = \alpha \ln k_{it} + \gamma \ln M_{it} + h(\hat{\varphi}_{it-1} - \alpha \ln k_{it-1} - \gamma \ln M_{it-1}) + \xi_{it}.$$

Important difference: $\mathbb{E}[\xi_{it} \ln M_{it}] \neq 0$ (it is zero for capital because of the time-to-build assumption).

Levinshon and Petrin (2003): method

First step is analogous to that in Olley and Pakes, except that investment is replaced by the demand for materials:

$$\ln y_{it} = \beta \ln l_{it} + \varphi_t(l_{it-1}, k_{it}, M_{it}) + \varepsilon_{it},$$

where $\varphi_t(l_{it}, k_{it}, M_{it}) \equiv \alpha \ln k_{it} + \gamma \ln M_{it} + F_M^{-1}(l_{it-1}, k_{it}, M_{it}, \boldsymbol{r}_{it})$.

Second step also analogous:

$$\hat{\varphi}_{it} = \alpha \ln k_{it} + \gamma \ln M_{it} + h(\hat{\varphi}_{it-1} - \alpha \ln k_{it-1} - \gamma \ln M_{it-1}) + \xi_{it}.$$

Important difference: $\mathbb{E}[\xi_{it} \ln M_{it}] \neq 0$ (it is zero for capital because of the time-to-build assumption).

Solution: instrument using lags of M_{it} as in Blundell-Bond.

Ackerberg, Caves, and Fazer (2015) critique

Ackerberg, Caves, and Fazer (2015) critique

Given the assumptions of invertibility and no cross-sectional variation in prices, we can **rewrite the labor demand** as:

$$l_{it} = F_L(l_{it-1}, k_{it}, F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t), \boldsymbol{r}_t) \equiv G_t(l_{it-1}, k_{it}, i_{it}).$$

Ackerberg, Caves, and Fazer (2015) critique

Given the assumptions of invertibility and no cross-sectional variation in prices, we can **rewrite the labor demand** as:

$$l_{it} = F_L(l_{it-1}, k_{it}, F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t), \boldsymbol{r}_t) \equiv G_t(l_{it-1}, k_{it}, i_{it}).$$

Therefore, once l_{it-1}, k_{it}, and i_{it} are non-parametrically controlled for in the first stage, **no variation** left to identify β.

Ackerberg, Caves, and Fazer (2015) critique

Given the assumptions of invertibility and no cross-sectional variation in prices, we can **rewrite the labor demand** as:

$$l_{it} = F_L(l_{it-1}, k_{it}, F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r}_t), \boldsymbol{r}_t) \equiv G_t(l_{it-1}, k_{it}, i_{it}).$$

Therefore, once l_{it-1}, k_{it}, and i_{it} are non-parametrically controlled for in the first stage, **no variation** left to identify β.

$\Rightarrow$ model incorrectly specified or β is **identified spuriously**.

Ackerberg, Caves, and Fazer (2015) solution

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption (“exclusion restriction”):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Consistent with the following **economic assumptions**:

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Consistent with the following **economic assumptions**:

- **Capital markets are perfectly competitive** $\Rightarrow$ price of capital constant across firms, i.e. $r_{it} = r_t$ for all i (independence of w_{it} and r_{it} hard to sustain otherwise).

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Consistent with the following **economic assumptions**:

- **Capital markets are perfectly competitive** $\Rightarrow$ price of capital constant across firms, i.e. $r_{it} = r_t$ for all i (independence of w_{it} and r_{it} hard to sustain otherwise).
- **Internal labor markets** $\Rightarrow$ price of labor cross-sectional variability.

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Consistent with the following **economic assumptions**:

- **Capital markets are perfectly competitive** $\Rightarrow$ price of capital constant across firms, i.e. $r_{it} = r_t$ for all i (independence of w_{it} and r_{it} hard to sustain otherwise).
- **Internal labor markets** $\Rightarrow$ price of labor cross-sectional variability.
- **Cost of labor occurs after investment decisions** $\Rightarrow$ w_{it} does not enter the investment function.

Ackerberg, Caves, and Fazer (2015) solution

Additional assumption ("exclusion restriction"):

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, r_{it}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, w_{it}),$$

where w_{it} and r_{it} satisfy that, conditional on t, i_{it}, k_{it}, and l_{it-1}:

- w_{it} has cross-sectional variation, i.e. $\text{Var}(w_{it}|t, i_{it}, k_{it}, l_{it-1}) > 0$.
- w_{it} and r_{it} are independently distributed.

Consistent with the following **economic assumptions**:

- **Capital markets are perfectly competitive** $\Rightarrow$ price of capital constant across firms, i.e. $r_{it} = r_t$ for all i (independence of w_{it} and r_{it} hard to sustain otherwise).
- **Internal labor markets** $\Rightarrow$ price of labor cross-sectional variability.
- **Cost of labor occurs after investment decisions** $\Rightarrow$ w_{it} does not enter the investment function.
- Idiosyncratic labor costs **not serially correlated** $\Rightarrow$ lagged labor cost shocks not state variables for investment.

Aggregate production functions

Aggregate production functions

Aggregate production functions

Focus often on:

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).
- Construction of **factor shares** and **depreciation rates** often seen more as an accounting/measurement than estimation.

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).
- Construction of **factor shares** and **depreciation rates** often seen more as an accounting/measurement than estimation.

General equilibrium approaches: estimate the production function within the model.

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).
- Construction of **factor shares** and **depreciation rates** often seen more as an accounting/measurement than estimation.

General equilibrium approaches: estimate the production function within the model.

This chapter: **partial equilibrium**.

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).
- Construction of **factor shares** and **depreciation rates** often seen more as an accounting/measurement than estimation.

General equilibrium approaches: estimate the production function within the model.

This chapter: **partial equilibrium**.

Two approaches: aggregate time-series (and cross-inputs) variation vs cross-sectional (e.g. spatial) variation.

Aggregate production functions

Focus often on:

- **Elasticities of substitution** across inputs.
- Evolution of technology over time (**technical change**).
- Construction of **factor shares** and **depreciation rates** often seen more as an accounting/measurement than estimation.

General equilibrium approaches: estimate the production function within the model.

This chapter: **partial equilibrium**.

Two approaches: aggregate time-series (and cross-inputs) variation vs cross-sectional (e.g. spatial) variation.

Only a **few examples**.

Nested CES

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Example: Card and Lemieux (2001), Borjas (2003), and Ottaviano and Peri (2012) for education i and experience j groups:

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Example: Card and Lemieux (2001), Borjas (2003), and Ottaviano and Peri (2012) for education i and experience j groups:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Example: Card and Lemieux (2001), Borjas (2003), and Ottaviano and Peri (2012) for education i and experience j groups:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad L_t \equiv \left[\sum_i \theta_{it} L_{it}^{\rho} \right]^{\frac{1}{\rho}},$$

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Example: Card and Lemieux (2001), Borjas (2003), and Ottaviano and Peri (2012) for education i and experience j groups:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad L_t \equiv \left[\sum_i \theta_{it} L_{it}^{\rho}\right]^{\frac{1}{\rho}},$$

$$L_{it} \equiv \left[\sum_j \gamma_{ij} L_{ijt}^{\eta}\right]^{\frac{1}{\eta}},$$

Nested CES

Convenient way to estimate **elasticities of substitution** across inputs (or allow for imperfect substitutability across them).

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- Elasticity of substitution **inside one nest** estimated without info on inputs or parameters in the **nests that lie above**.

Example: Card and Lemieux (2001), Borjas (2003), and Ottaviano and Peri (2012) for education i and experience j groups:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad L_t \equiv \left[\sum_i \theta_{it} L_{it}^{\rho}\right]^{\frac{1}{\rho}},$$

$$L_{it} \equiv \left[\sum_j \gamma_{ij} L_{ijt}^{\eta}\right]^{\frac{1}{\eta}}, \qquad L_{ijt} \equiv [\lambda L_{ijNt}^{\phi} + (1-\lambda) L_{ijMt}^{\phi}]^{\frac{1}{\phi}}.$$

Nested CES: estimation

Nested CES: estimation

Few periods ⇒ α **assumed** to be 0.3.

Nested CES: estimation

Few periods $\Rightarrow \alpha$ **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Nested CES: estimation

Few periods ⇒ α **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

Nested CES: estimation

Few periods $\Rightarrow$ α **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Nested CES: estimation

Few periods $\Rightarrow \alpha$ **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

Nested CES: estimation

Few periods $\Rightarrow \alpha$ **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

$$\ln w_{ijt} = \ln \left(\frac{\partial Y_t}{\partial L_t} \times \frac{\partial L_t}{\partial L_{it}} \times \frac{\partial L_{it}}{\partial L_{ijt}} \right) = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1) \ln L_{ijt}.$$

Nested CES: estimation

Few periods $\Rightarrow$ α **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

$$\ln w_{ijt} = \ln \left(\frac{\partial Y_t}{\partial L_t} \times \frac{\partial L_t}{\partial L_{it}} \times \frac{\partial L_{it}}{\partial L_{ijt}} \right) = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1) \ln L_{ijt}.$$

Impose $\sum_j \gamma_{ij} = 1$ to get γ_{ij}, construct L_{it} and **estimate**:

Nested CES: estimation

Few periods $\Rightarrow \alpha$ **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

$$\ln w_{ijt} = \ln \left(\frac{\partial Y_t}{\partial L_t} \times \frac{\partial L_t}{\partial L_{it}} \times \frac{\partial L_{it}}{\partial L_{ijt}} \right) = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1) \ln L_{ijt}.$$

Impose $\sum_j \gamma_{ij} = 1$ to get γ_{ij}, construct L_{it} and **estimate**:

$$\ln w_{it} = \kappa_t + \ln \theta_{it} + (\rho - 1) \ln L_{it}.$$

Nested CES: estimation

Few periods $\Rightarrow$ α **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

$$\ln w_{ijt} = \ln \left(\frac{\partial Y_t}{\partial L_t} \times \frac{\partial L_t}{\partial L_{it}} \times \frac{\partial L_{it}}{\partial L_{ijt}} \right) = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1) \ln L_{ijt}.$$

Impose $\sum_j \gamma_{ij} = 1$ to get γ_{ij}, construct L_{it} and **estimate**:

$$\ln w_{it} = \kappa_t + \ln \theta_{it} + (\rho - 1) \ln L_{it}.$$

Borjas (2003): θ_{it} specified as education group-specific **time trends**.

Nested CES: estimation

Few periods $\Rightarrow \alpha$ **assumed** to be 0.3.

Sequential estimation based on **first order conditions**:

Lowest level:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln \frac{1-\lambda}{\lambda} + (\phi - 1) \ln \frac{L_{ijMt}}{L_{ijNt}}.$$

Construct **next level aggregates** L_{ijt} using λ and ϕ and **estimate**:

$$\ln w_{ijt} = \ln \left(\frac{\partial Y_t}{\partial L_t} \times \frac{\partial L_t}{\partial L_{it}} \times \frac{\partial L_{it}}{\partial L_{ijt}} \right) = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1) \ln L_{ijt}.$$

Impose $\sum_j \gamma_{ij} = 1$ to get γ_{ij}, construct L_{it} and **estimate**:

$$\ln w_{it} = \kappa_t + \ln \theta_{it} + (\rho - 1) \ln L_{it}.$$

Borjas (2003): θ_{it} specified as education group-specific **time trends**.

Allow for **model specification error**: instrument labor inputs using the stock of immigrants in the group (makes little difference in this case).

The race between technology and skills

The race between technology and skills

Common application in Labor and Macro: account for the determinants of the **increasing wage inequality**.

The race between technology and skills

Common application in Labor and Macro: account for the determinants of the **increasing wage inequality**.

The "canonical model" considers the following **production function**:

$$Y_t = \left(\alpha_{Lt} L^\rho + \alpha_{Ht} H^\rho\right)^{\frac{1}{\rho}} .$$

The race between technology and skills

Common application in Labor and Macro: account for the determinants of the **increasing wage inequality**.

The "canonical model" considers the following **production function**:

$$Y_t = \left(\alpha_{Lt} L^{\rho} + \alpha_{Ht} H^{\rho}\right)^{\frac{1}{\rho}}.$$

Tinbergen's race between technology and skills:

$$\ln \frac{w_H}{w_L} = \rho \ln \left(\frac{\alpha_{Ht}}{\alpha_{Lt}}\right) + (\rho - 1) \ln \frac{H_t}{L_t}.$$

The race between technology and skills

Common application in Labor and Macro: account for the determinants of the **increasing wage inequality**.

The "canonical model" considers the following **production function**:

$$Y_t = \left(\alpha_{Lt} L^{\rho} + \alpha_{Ht} H^{\rho}\right)^{\frac{1}{\rho}} .$$

Tinbergen's race between technology and skills:

$$\ln \frac{w_H}{w_L} = \rho \ln \left(\frac{\alpha_{Ht}}{\alpha_{Lt}}\right) + (\rho - 1) \ln \frac{H_t}{L_t}.$$

Katz and Murphy (1992) first estimate this by OLS with time trend.

The race between technology and skills

Common application in Labor and Macro: account for the determinants of the **increasing wage inequality**.

The "canonical model" considers the following **production function**:

$$Y_t = \left(\alpha_{Lt} L^{\rho} + \alpha_{Ht} H^{\rho}\right)^{\frac{1}{\rho}} .$$

Tinbergen's race between technology and skills:

$$\ln \frac{w_H}{w_L} = \rho \ln \left(\frac{\alpha_{Ht}}{\alpha_{Lt}}\right) + (\rho - 1) \ln \frac{H_t}{L_t}.$$

Katz and Murphy (1992) first estimate this by OLS with time trend.

Implicit assumption: time trend is SBTC, everything else measurement error/uncorrelated shocks (alternative: spatial approach).

Beyond Katz and Murphy

Beyond Katz and Murphy

This simple model **fits the data** well until mid-1990s.

Beyond Katz and Murphy

This simple model **fits the data** well until mid-1990s.

Out-of-sample predictions after which over-predicts the increase in college-high school wage gap.

Beyond Katz and Murphy

This simple model **fits the data** well until mid-1990s.

Out-of-sample predictions after which over-predicts the increase in college-high school wage gap.

Richer trends: decreasing SBTC in recent years $\Rightarrow$ counter to common perception.

Beyond Katz and Murphy

This simple model **fits the data** well until mid-1990s.

Out-of-sample predictions after which over-predicts the increase in college-high school wage gap.

Richer trends: decreasing SBTC in recent years $\Rightarrow$ counter to common perception.

Other papers: Card and Lemieux (2001) and Jeong, Kim, and Manovskii (2015).

Beyond Katz and Murphy

This simple model **fits the data** well until mid-1990s.

Out-of-sample predictions after which over-predicts the increase in college-high school wage gap.

Richer trends: decreasing SBTC in recent years $\Rightarrow$ counter to common perception.

Other papers: Card and Lemieux (2001) and Jeong, Kim, and Manovskii (2015).

Jeong, Kim, and Manovskii (2015) use a **similar estimation method** as in the application of this chapter (Albert, Glitz, and Llull, 2020).

Capital-labor substitution and biased TC

Capital-labor substitution and biased TC

Capital-labor elasticity of substitution is a very important parameter in economics.

Capital-labor substitution and biased TC

Capital-labor elasticity of substitution is a very important parameter in economics.

Many assume it to be one (**Cobb-Douglas**) $\Rightarrow$ Hicks-neutral TC.

Capital-labor substitution and biased TC

Capital-labor elasticity of substitution is a very important parameter in economics.

Many assume it to be one (**Cobb-Douglas**) $\Rightarrow$ Hicks-neutral TC.

Others test whether it is one $\Rightarrow$ Antràs (2004) show that assuming neutral TC biases the results towards Cobb-Douglas.

Capital-labor substitution and biased TC

Capital-labor elasticity of substitution is a very important parameter in economics.

Many assume it to be one (**Cobb-Douglas**) $\Rightarrow$ Hicks-neutral TC.

Others test whether it is one $\Rightarrow$ Antràs (2004) show that assuming neutral TC biases the results towards Cobb-Douglas.

Diamond, McFadden, and Rodríguez (1978) argue that this elasticity and biased TC are **not simultaneously identified**.

Capital-labor substitution and biased TC

Capital-labor elasticity of substitution is a very important parameter in economics.

Many assume it to be one (**Cobb-Douglas**) $\Rightarrow$ Hicks-neutral TC.

Others test whether it is one $\Rightarrow$ Antràs (2004) show that assuming neutral TC biases the results towards Cobb-Douglas.

Diamond, McFadden, and Rodríguez (1978) argue that this elasticity and biased TC are **not simultaneously identified**.

León-Ledesma, McAdam, and Willman (2010) use Monte-Carlo to asses **when they are well identified and robust**.

León-Ledesma, McAdam, and Willman (2010)

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt}K_t)^\rho + (1-\pi)(\Gamma_{Lt}L_t)^\rho\right)^{\frac{1}{\rho}} .$$

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt} K_t)^\rho + (1-\pi)(\Gamma_{Lt} L_t)^\rho \right)^{\frac{1}{\rho}} .$$

where:

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt}K_t)^\rho + (1-\pi)(\Gamma_{Lt}L_t)^\rho\right)^{\frac{1}{\rho}} .$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1-\rho)$.

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt} K_t)^\rho + (1-\pi)(\Gamma_{Lt} L_t)^\rho \right)^{\frac{1}{\rho}} .$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1-\rho)$.
- Γ_{Kt} and Γ_{Lt} denote efficiency (and, its evolution, **technical progress**).

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi (\Gamma_{Kt} K_t)^{\rho} + (1 - \pi)(\Gamma_{Lt} L_t)^{\rho} \right)^{\frac{1}{\rho}} .$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1 - \rho)$.
- Γ_{Kt} and Γ_{Lt} denote efficiency (and, its evolution, **technical progress**).
- ζ is an efficiency parameter, and $\pi \in [0, 1]$ is the capital intensity.

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt}K_t)^\rho + (1-\pi)(\Gamma_{Lt}L_t)^\rho \right)^{\frac{1}{\rho}} .$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1-\rho)$.
- Γ_{Kt} and Γ_{Lt} denote efficiency (and, its evolution, **technical progress**).
- ζ is an efficiency parameter, and $\pi \in [0,1]$ is the capital intensity.

Embeds the **Cobb-Douglas** case when $\rho = 0$, the **Lenotieff** case when $\rho \to \infty$, and the **linear** case when $\rho \to 1$.

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt}K_t)^{\rho} + (1-\pi)(\Gamma_{Lt}L_t)^{\rho}\right)^{\frac{1}{\rho}}.$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1-\rho)$.
- Γ_{Kt} and Γ_{Lt} denote efficiency (and, its evolution, **technical progress**).
- ζ is an efficiency parameter, and $\pi \in [0,1]$ is the capital intensity.

Embeds the **Cobb-Douglas** case when $\rho = 0$, the **Lenotieff** case when $\rho \to \infty$, and the **linear** case when $\rho \to 1$.

Usual assumptions: $\Gamma_{Kt} = e^{-\gamma_K t}$ and $\Gamma_{Lt} e^{-\gamma_L t}$

León-Ledesma, McAdam, and Willman (2010)

Consider the following **CES framework**:

$$Y_t = \zeta \left(\pi(\Gamma_{Kt} K_t)^\rho + (1-\pi)(\Gamma_{Lt} L_t)^\rho\right)^{\frac{1}{\rho}} .$$

where:

- **Elasticity of substitution** between capital and labor is $1/(1-\rho)$.
- Γ_{Kt} and Γ_{Lt} denote efficiency (and, its evolution, **technical progress**).
- ζ is an efficiency parameter, and $\pi \in [0, 1]$ is the capital intensity.

Embeds the **Cobb-Douglas** case when $\rho = 0$, the **Lenotieff** case when $\rho \to \infty$, and the **linear** case when $\rho \to 1$.

Usual assumptions: $\Gamma_{Kt} = e^{-\gamma_K t}$ and $\Gamma_{Lt} e^{-\gamma_L t}$ $\Rightarrow$ Hicks-neutral TC ($\gamma_K = \gamma_L$), Solow-neutral ($\gamma_L = 0$), Harrod-neutral ($\gamma_K = 0$), capital-augmenting ($\gamma_K > \gamma_L > 0$) or labor-augmenting ($\gamma_L > \gamma_K > 0$).

León-Ledesma, McAdam, and Willman (2010)

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow\uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself)

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself) $\Rightarrow$ one, two, or three equation estimation.

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself) $\Rightarrow$ one, two, or three equation estimation.

Results:

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself) $\Rightarrow$ one, two, or three equation estimation.

Results:

- Factor neutral TC when factor shares are relatively stable $\Rightarrow$ **bias estimates towards Cobb-Douglas** (Antràs, 2004).

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself) $\Rightarrow$ one, two, or three equation estimation.

Results:

- Factor neutral TC when factor shares are relatively stable $\Rightarrow$ **bias estimates towards Cobb-Douglas** (Antràs, 2004).
- Estimates from the **labor FOC** larger than those based on the **capital FOC** or **PF**.

León-Ledesma, McAdam, and Willman (2010)

Identification problem:

$$\text{Capital-labor ratio} = \frac{\pi}{1-\pi}\left(\frac{\Gamma_{Kt}K_t}{\Gamma_{Lt}L_t}\right)^{\rho}.$$

$\Rightarrow \uparrow$ capital-labor explained by i) a **capital-augmenting** TC if **substitutes**, or ii) a **labor-augmenting** TC if **complements**.

Estimation: three possible equations (two FOCs plus the PF itself) $\Rightarrow$ one, two, or three equation estimation.

Results:

- Factor neutral TC when factor shares are relatively stable $\Rightarrow$ **bias estimates towards Cobb-Douglas** (Antràs, 2004).
- Estimates from the **labor FOC** larger than those based on the **capital FOC** or **PF**.
- **Single equation** not good in identifying capital-labor elasticity in the presence of biased TC $\Rightarrow$ **system estimation** much better!

Latent factor models

Latent factor models

Krusell, Ohanian, Ríos-Rull, and Violante (2000) explore what is behind the SBTC: capital-skill complementarity + $\downarrow$ price equipment.

Latent factor models

Krusell, Ohanian, Ríos-Rull, and Violante (2000) explore what is behind the SBTC: capital-skill complementarity + $\downarrow$ price equipment.

Consider the following **production function**:

$$Y_t = \zeta_t K_{St}^{\alpha} \left[\theta L_t^{\rho} + (1-\theta) \left(\pi H_t^{\gamma} + (1-\pi) K_{Et}^{\gamma} \right)^{\frac{\rho}{\gamma}} \right]^{\frac{1-\alpha}{\rho}},$$

with $L_t \equiv \psi_{Lt} h_{Lt}$ and $H_t \equiv \psi_{Ht} h_{Ht}$, where h_{Lt} and h_{Ht} are total hours worked, and ψ_{Lt} and ψ_{Ht} are efficiency units.

Latent factor models

Krusell, Ohanian, Ríos-Rull, and Violante (2000) explore what is behind the SBTC: capital-skill complementarity + $\downarrow$ price equipment.

Consider the following **production function**:

$$Y_t = \zeta_t K_{St}^{\alpha} \left[\theta L_t^{\rho} + (1-\theta) \left(\pi H_t^{\gamma} + (1-\pi) K_{Et}^{\gamma} \right)^{\frac{\rho}{\gamma}} \right]^{\frac{1-\alpha}{\rho}},$$

with $L_t \equiv \psi_{Lt} h_{Lt}$ and $H_t \equiv \psi_{Ht} h_{Ht}$, where h_{Lt} and h_{Ht} are total hours worked, and ψ_{Lt} and ψ_{Ht} are efficiency units.

Given that they consider a closed economy:

$$Y_t = C_t + I_{St} + \frac{I_{Et}}{q_t},$$

Latent factor models

Krusell, Ohanian, Ríos-Rull, and Violante (2000) explore what is behind the SBTC: capital-skill complementarity + $\downarrow$ price equipment.

Consider the following **production function**:

$$Y_t = \zeta_t K_{St}^{\alpha} \left[\theta L_t^{\rho} + (1-\theta) \left(\pi H_t^{\gamma} + (1-\pi) K_{Et}^{\gamma} \right)^{\frac{\rho}{\gamma}} \right]^{\frac{1-\alpha}{\rho}},$$

with $L_t \equiv \psi_{Lt} h_{Lt}$ and $H_t \equiv \psi_{Ht} h_{Ht}$, where h_{Lt} and h_{Ht} are total hours worked, and ψ_{Lt} and ψ_{Ht} are efficiency units.

Given that they consider a closed economy:

$$Y_t = C_t + I_{St} + \frac{I_{Et}}{q_t},$$

The **skill premium** is given by:

$$\ln \frac{w_{Ht}}{w_{Lt}} \approx C + (\rho - 1) \ln \frac{H_t}{L_t} + (\rho - \gamma) \frac{1-\pi}{\pi} \left(\frac{K_{Et}}{H_t} \right)^{\gamma}.$$

Latent factor models

Krusell, Ohanian, Ríos-Rull, and Violante (2000) explore what is behind the SBTC: capital-skill complementarity + $\downarrow$ price equipment.

Consider the following **production function**:

$$Y_t = \zeta_t K_{St}^{\alpha} \left[\theta L_t^{\rho} + (1-\theta)\left(\pi H_t^{\gamma} + (1-\pi) K_{Et}^{\gamma}\right)^{\frac{\rho}{\gamma}}\right]^{\frac{1-\alpha}{\rho}},$$

with $L_t \equiv \psi_{Lt} h_{Lt}$ and $H_t \equiv \psi_{Ht} h_{Ht}$, where h_{Lt} and h_{Ht} are total hours worked, and ψ_{Lt} and ψ_{Ht} are efficiency units.

Given that they consider a closed economy:

$$Y_t = C_t + I_{St} + \frac{I_{Et}}{q_t},$$

The **skill premium** is given by:

$$\ln \frac{w_{Ht}}{w_{Lt}} \approx C + (\rho - 1) \ln \frac{H_t}{L_t} + (\rho - \gamma) \frac{1-\pi}{\pi} \left(\frac{K_{Et}}{H_t}\right)^{\gamma}.$$

Last term is SBTC if $\rho > \gamma$ (capital-skill complementarity).

Latent factor models

Latent factor models

Three-equation factor model.

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Equations:

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Equations:

- Labor share.

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Equations:

- Labor share.
- Relative high-low skill wage bills.

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Equations:

- Labor share.
- Relative high-low skill wage bills.
- Dynamic equipment-structures relative demand/no-arbitrage.

Latent factor models

Three-equation factor model.

Factors include q_t and $\ln \psi_{it} = \varphi_{0i} + \varphi_{i1} t + \epsilon_{it}$ for $i \in \{H, L\}$, with $(\epsilon_{Ht}, \epsilon_{Ht})'$ is i.i.d. normal.

Equations:

- Labor share.
- Relative high-low skill wage bills.
- Dynamic equipment-structures relative demand/no-arbitrage.

Estimation through **simulated pseudo-maximum likelihood**, taking into account the potential endogeneity of hours worked to technology and efficiency shocks.

Spatial approach

Spatial approach

Spatial variation $\Rightarrow$ **endogeneity concerns** (similar to those at the firm-level) are more apparent.

Spatial approach

Spatial variation ⇒ **endogeneity concerns** (similar to those at the firm-level) are more apparent.

Often ignore **capital** (unobservable at local level), estimating the following **CES production function**:

$$Y_{it} = \zeta_{it} \left(\sum_j \theta_{ijt} L_{ijt}^{\rho} \right)^{\frac{1}{\rho}},$$

where j typically denotes different skill groups or industries.

Spatial approach

Spatial variation ⇒ **endogeneity concerns** (similar to those at the firm-level) are more apparent.

Often ignore **capital** (unobservable at local level), estimating the following **CES production function**:

$$Y_{it} = \zeta_{it} \left(\sum_j \theta_{ijt} L_{ijt}^{\rho} \right)^{\frac{1}{\rho}},$$

where j typically denotes different skill groups or industries.

ζ_{it} **cancels out** (or captured by location-time fixed effects) in FOCs.

Spatial approach

Spatial variation $\Rightarrow$ **endogeneity concerns** (similar to those at the firm-level) are more apparent.

Often ignore **capital** (unobservable at local level), estimating the following **CES production function**:

$$Y_{it} = \zeta_{it} \left(\sum_j \theta_{ijt} L_{ijt}^{\rho} \right)^{\frac{1}{\rho}},$$

where j typically denotes different skill groups or industries.

ζ_{it} **cancels out** (or captured by location-time fixed effects) in FOCs.

θ_{ijt} are random $\Rightarrow$ **simultaneity bias** in relative demands.

Spatial approach

Spatial approach

Common solution: **Bartik instrument**.

Spatial approach

Common solution: **Bartik instrument**.

Consider the following **demands** (from the previous PF):

$$\ln w_{ijt} = (\rho - 1)\ln L_{ijt} + \delta_{it} + \ln \theta_{ijt},$$

Spatial approach

Common solution: **Bartik instrument**.

Consider the following **demands** (from the previous PF):

$$\ln w_{ijt} = (\rho - 1)\ln L_{ijt} + \delta_{it} + \ln \theta_{ijt},$$

The **Bartik instrument** for L_{ijt}, denoted by $\Delta\hat{L}_{ijt}$, is:

$$\Delta\hat{L}_{ijt} = \frac{L_{ij0}}{\sum_j L_{ij0}} \sum_{-i} \Delta L_{ijt},$$

where Δ indicates over-time differences, and $\sum_{-i}$ denotes sum across all local markets excluding the market i.

Spatial approach

Common solution: **Bartik instrument**.

Consider the following **demands** (from the previous PF):

$$\ln w_{ijt} = (\rho - 1) \ln L_{ijt} + \delta_{it} + \ln \theta_{ijt},$$

The **Bartik instrument** for L_{ijt}, denoted by $\Delta \hat{L}_{ijt}$, is:

$$\Delta \hat{L}_{ijt} = \frac{L_{ij0}}{\sum_j L_{ij0}} \sum_{-i} \Delta L_{ijt},$$

where Δ indicates over-time differences, and $\sum_{-i}$ denotes sum across all local markets excluding the market i.

Often used on the estimation in **first differences**.

Spatial approach

Common solution: **Bartik instrument**.

Consider the following **demands** (from the previous PF):

$$\ln w_{ijt} = (\rho - 1) \ln L_{ijt} + \delta_{it} + \ln \theta_{ijt},$$

The **Bartik instrument** for L_{ijt}, denoted by $\Delta \hat{L}_{ijt}$, is:

$$\Delta \hat{L}_{ijt} = \frac{L_{ij0}}{\sum_j L_{ij0}} \sum_{-i} \Delta L_{ijt},$$

where Δ indicates over-time differences, and $\sum_{-i}$ denotes sum across all local markets excluding the market i.

Often used on the estimation in **first differences**.

Widely used in many applications (not only production function/labor demand estimation), but also **often criticized**.

Application: Albert, Glitz, and Llull (2021)

Introduction

In the context of the **recent refugee crisis** $\Rightarrow$ renewed interest in understanding the process through which immigrants assimilate in the labor market.

The degree of immigrant labor market **assimilation** is typically measured in terms of **relative wages** compared to natives.

Years spent in the host country and relative wages positively correlated. Traditional discussion: disentangle **assimilation** from **composition effects**?

Unexplored mechanism: if immigrants and natives are imperfect substitutes $\Rightarrow$ relative wages also depend on **labor market equilibrium effects**.

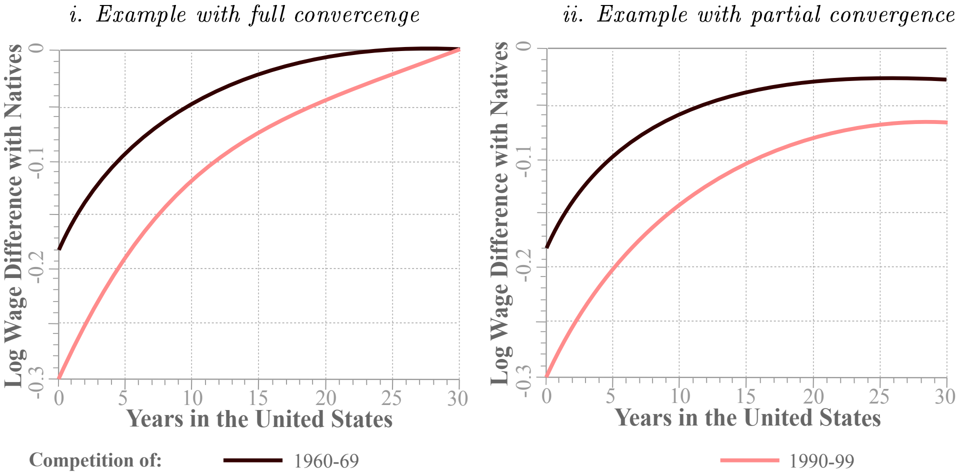
Main intuition

Natives and immigrants tend to work in **different occupations** $\Rightarrow$ **imperfect substitutes** in production.

Implication $\Rightarrow$ increasing **size of immigrant cohorts** change labor market competition for natives and for immigrants differently:

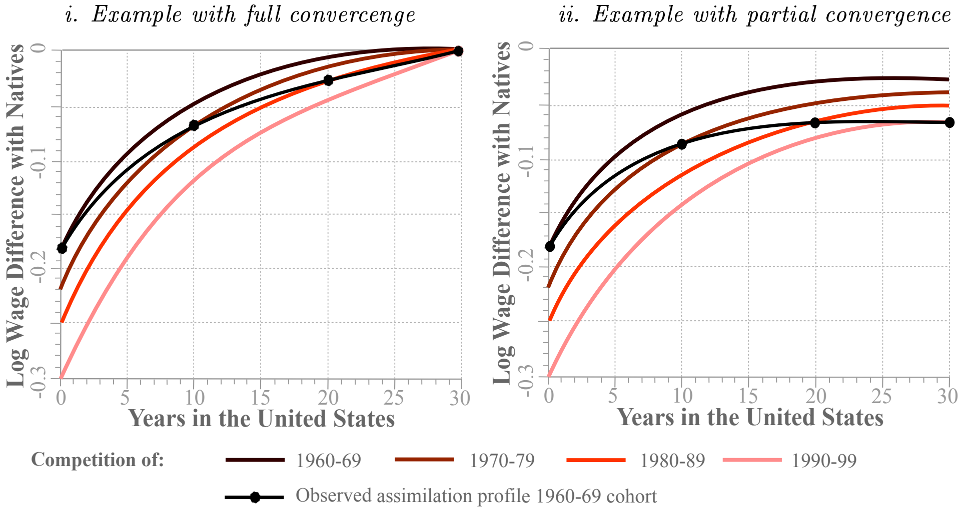
- Larger **wage gap** at arrival.
- Ambiguous effect on **speed of convergence**.

Figure III. – Dynamic Competition Effect: An Example



Note: The figure plots two hypothetical convergence paths for different levels of competition when the size of the immigrant inflows increase across arrival cohorts, and the implied assimilation curve we would observe in the data for a cohort that arrived in 1960s. The left figure shows an example with full wage convergence, and the right figure shows one without full convergence.

Figure III. – Dynamic Competition Effect: An Example



Note: The figure plots two hypothetical convergence paths for different levels of competition when the size of the immigrant inflows increase across arrival cohorts, and the implied assimilation curve we would observe in the data for a cohort that arrived in 1960s. The left figure shows an example with full wage convergence, and the right figure shows one without full convergence.

Our contribution

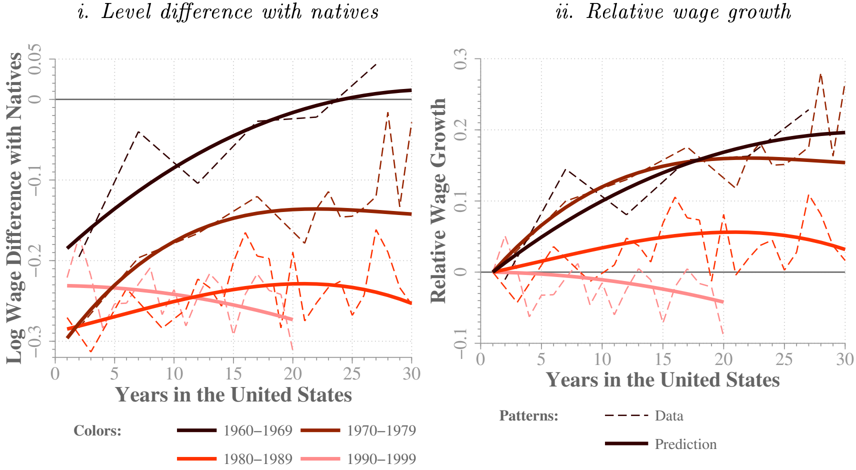
Study the interrelation between **immigrant wage assimilation** and the **wage impacts of immigration**.

Provide a **simple framework** that explicitly links them.

Structurally estimate the parameters of the model and use it to **decompose** the observed wage dynamics into:

- **Competition** effects (our new mechanism)
- **Composition** effects driven by:
  - **Education**
  - Country of **origin**
  - **Unobservable** skills

FIGURE I. – WAGE GAP BETWEEN NATIVES AND IMMIGRANTS AND YEARS IN THE U.S.



Note: Dashed lines represent the raw data, which is the result from year-by-year regressions of log wages on a third order polynomial in age and dummies for the number of years since migration. Solid lines represent fitted values of:

$$\ln w_i = \beta_{0c(i)} + \beta_{1t(i)} + \sum_{\ell=1}^{3} \beta_{2\ell t(i)} age_i^{\ell} + \sum_{\ell=1}^{3} \beta_{3\ell c(i)} ysm_i^{\ell} + \nu_i,$$

where $c(i)$ and $t(i)$ indicate immigration cohort and the census year for individual i, age_i indicates age, and ysm_i indicates years since migration.

Theoretical framework

Two types of **imperfectly substitutable skills**: "general" and "U.S.-specific".

Observationally equivalent natives and immigrants supply the same **general skills**.

Immigrants arrive with only a fraction of the **specific skills** of comparable natives (e.g. language skills). However, once in the United States, they start accumulating (assimilation).

Skills are **accumulated mechanically** (no investment decision).

Workers are **paid** their marginal product.

Production technology

Let G_t denote the aggregate supply of **general skill units** in year t, and let S_t denote the **supply of specific skills**.

Output, Y_t, is produced according to:

$$Y_t = A_t \left(G_t^{\frac{\sigma-1}{\sigma}} + S_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where:

- σ is **elasticity of substitution** between general and specific skills.
- A_t is **total factor productivity**.

Workers are paid their **marginal product**:

$$r_{Gt} = A_t \left(\frac{Y_t}{A_t G_t} \right)^{\frac{1}{\sigma}} \quad \text{and} \quad r_{St} = A_t \left(\frac{Y_t}{A_t S_t} \right)^{\frac{1}{\sigma}} \quad \Rightarrow \quad \frac{r_{St}}{r_{Gt}} = \left(\frac{G_t}{S_t} \right)^{\frac{1}{\sigma}}.$$

Skill supply and wages

All individuals in the economy supply one **general skill unit** and s **specific skill units** (shifted by the skill index $h_t(E,x)$ below):

$$s(n,y,o,c,E,x) \equiv \begin{cases} 1 & \text{if } n=1 \\ \theta_{1o} + \sum_{\ell=1}^{3} \theta_{2o\ell} y^{\ell} + \theta_{3e} + \sum_{\ell=1}^{3} \theta_{4e\ell} y^{\ell} & \\ + \sum_{\ell=1}^{3} \theta_{5\ell} (x-y)^{\ell} + \theta_{6c} + \sum_{\ell=1}^{3} \theta_{7c\ell} y^{\ell} & \text{if } n=0 \end{cases}$$

where:

- $n=1$ denotes **natives** and $n=0$ denotes **immigrants**,
- k denotes country of **origin**,
- j denotes **cohort** of entry,
- E denotes years of **education**, and e, education group,
- x denotes **potential experience** (age minus education),
- y denotes **years in the United States**.

Skills supply and wages

General and specific skills shifted by the following **skill index**:

$$h_t(E, x) \equiv \exp\left(\eta_{0et} + \eta_{1t}E + \sum_{i=1}^{3} \eta_{2it}x^i\right).$$

Therefore, **wages** are:

$$w_t(n, y, o, c, E, x) = \left[r_{Gt} + r_{St}s(n, y, o, c, E, x)\right] h_t(E, x).$$

Relative wages of immigrants compared to equivalent natives are:

$$\begin{aligned}\frac{w_t(0, y, o, c, E, x)}{w_t(1, \cdot, \cdot, \cdot, E, x)} &= \frac{r_{Gt} + r_{St}s(n, y, o, c, E, x)}{r_{Gt} + r_{St}} \\ &= \frac{1 + s(n, y, o, c, E, x)(G_t/S_t)^{\frac{1}{\sigma}}}{1 + (G_t/S_t)^{\frac{1}{\sigma}}}.\end{aligned}$$

Discussion

Model features:

- The **competition** effect discussed above if $\sigma < \infty$.
- **Imperfect substitutability** between natives and immigrants if $\sigma < \infty$.
- **Downgrading** of immigrants at entry (Dustmann et al., 2013) if $s < 1$ at entry.
- Embeds the **traditional** assimilation model when $\sigma = \infty$:

$$\ln w_t(n, y, o, c, E, x) = \ln A_t + \ln[1 + s(n, y, o, c, E, x)] + \ln h_t(E, x)$$
$$\approx \delta_t + \eta_{0et} + \eta_{1t}E + \sum_{\ell=1}^{3} \eta_{2\ell t}x^\ell + (1-n)\left[\begin{array}{c} \theta_{1o} + \sum_{\ell=1}^{3}\theta_{2o\ell}y^\ell + \theta_{3e} + \sum_{\ell=1}^{3}\theta_{4e\ell}y^\ell \\ + \sum_{\ell=1}^{3}\theta_{5\ell}(x-y)^\ell + \theta_{6c} + \sum_{\ell=1}^{3}\theta_{7c\ell}y^\ell \end{array}\right],$$

Identification and Estimation

1. From **native wages**, OLS estimate:

$$\ln w_i = \gamma_{j(i)t(i)} + \eta_{0e(i)t(i)} + \eta_{1t(i)} E_i + \sum_{\ell=1}^{3} \eta_{2\ell t(i)} x_i^{\ell} + \epsilon_i,$$

where $\gamma_{j(i)t(i)} = \ln\left(r_{Gj(i)t(i)} + r_{Sj(i)t(i)}\right)$ is a set of **state-year dummies**.

Identification and Estimation

1. From **native wages**, OLS estimate:

$$\ln w_i = \gamma_{j(i)t(i)} + \eta_{0e(i)t(i)} + \eta_{1t(i)} E_i + \sum_{\ell=1}^{3} \eta_{2\ell t(i)} x_i^\ell + \epsilon_i,$$

where $\gamma_{j(i)t(i)} = \ln\left(r_{Gj(i)t(i)} + r_{Sj(i)t(i)}\right)$ is a set of **state-year dummies**.

2. From **immigrant wages**, NLS estimate:

$$\ln w_i - \ln(\widehat{r_{Gj(i)t(i)} + r_{Sj(i)t(i)}}) - \widehat{h_{t(i)}(E_i, x_i)} = -\ln\left[1 + \left(\frac{G_{j(i)t(i)}(\hat{\boldsymbol{\eta}})}{S_{j(i)t(i)}(\boldsymbol{\theta}, \hat{\boldsymbol{\eta}})}\right)^{\frac{1}{\sigma}}\right]$$

$$+ \ln\left[1 + \left(\frac{G_{j(i)t(i)}(\hat{\boldsymbol{\eta}})}{S_{j(i)t(i)}(\boldsymbol{\theta}, \hat{\boldsymbol{\eta}})}\right)^{\frac{1}{\sigma}} \begin{pmatrix} \theta_{1o(i)} + \sum_{\ell=1}^{3} \theta_{2o(i)\ell} y^\ell + \theta_{3e(i)} + \sum_{\ell=1}^{3} \theta_{4e(i)\ell} y^\ell \\ + \sum_{\ell=1}^{3} \theta_{5\ell}(x-y)^\ell + \theta_{6c(i)} + \sum_{\ell=1}^{3} \theta_{7c(i)\ell} y^\ell \end{pmatrix}\right] + \epsilon_i$$

Data

The sample consists of **men aged 25-64** from the **Census 1970-2000** and **ACS 2009-2011** (downloaded from IPUMS).

We drop workers that are **unemployed**, **self-employed**, living in **group quarters**, enrolled **in school** or working for the **government**.

Immigrants are defined as **foreign-born without US parents**.

Hourly wages are computed by dividing the annual wage and salary income by annual hours worked, and deflated to 1999US$.

Estimation results

Returns to **education and potential experience** in line with the literature.

Heterogeneous assimilation patterns by origin, education, cohort.

The model **fits the data** well.

Same level of imperfect substitutability between natives and immigrants as in the literature (with very different production function!).

Table IV. – Elasticity of Substitution Parameter, σ

A. Estimated elasticity of substitution between general and specific skills

	Point estimate	Standard error	Confidence interval
Elasticity of substitution (σ)	0.021	(0.002)	[0.025,0.018]

B. Implied elasticity of substitution between natives and immigrants

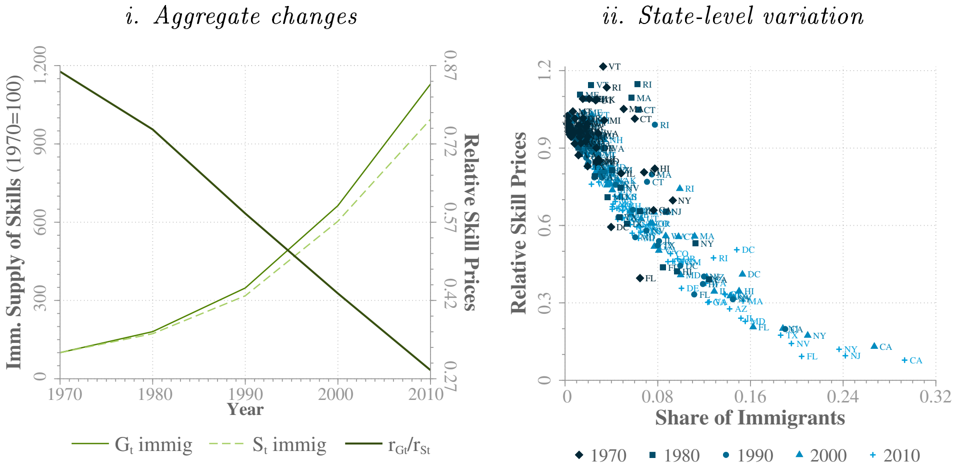
	Elasticity
Natives vs immigrants	29.3

C. Implied elasticity of substitution between immigrants and different groups

	Natives	Immigrants by years in the United States:		
Years in the United States:		30-39 years	20-29 years	10-19 years
0-9 years	15.6	39.2	63.9	137.6
10-19 years	27.9	81.4	176.9	
20-29 years	79.8	483.6		
30-39 years	5,894.1			

Note: Panel (A) presents estimates for the elasticity of substitution between general and specific skills parameter σ. Sample weights, rescaled by annual hours worked are used in estimation. The 95% confidence interval is based on the estimates for $1/\sigma$. Panel (B) provides the implied elasticity of substitution between natives and immigrants, based on (15). This elasticity of substitution is computed with mean values for the period 1990-2010 comparable to Ottaviano and Peri (2012), implying $\bar{s} = 0.821$ and $m = 0.105$. Panel (C) shows the implied elasticities of substitution between immigrants and different groups, based on (15) and (16), for s evaluated at 0.764, 0.817, 0.885, and 0.975 for the 0–9, 10–19, 20-29, and 30-39 years-in-the-U.S. groups respectively, and the values of m_1 are 0.046, 0.041, 0.022, and 0.008 respectively.

Figure VI. – Changes in Relative Supplies and Relative Skill Prices



Note: The figure shows the predicted supplies of aggregate skill units of each type by immigrants in each year (left plot, left axis), the relative skill prices implied by these aggregate supplies (left plot, right axis), and the predicted relative skill prices at the state-year level. Aggregate supplies are normalized to 100 in year 1970.

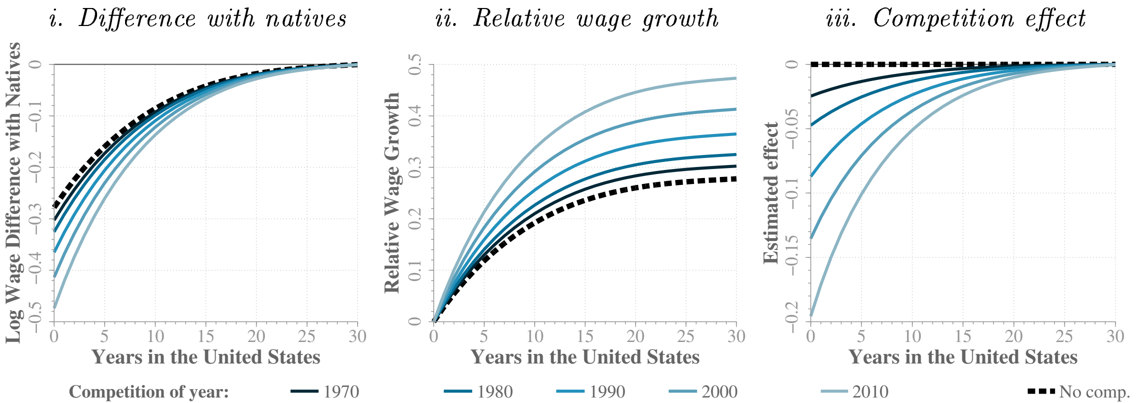
Competition and composition effects

Baseline individual: Mexican high school dropout who arrived in the U.S. in the 1970s cohort with 10 years of potential experience at arrival (except otherwise noted).

For this individual we compute:

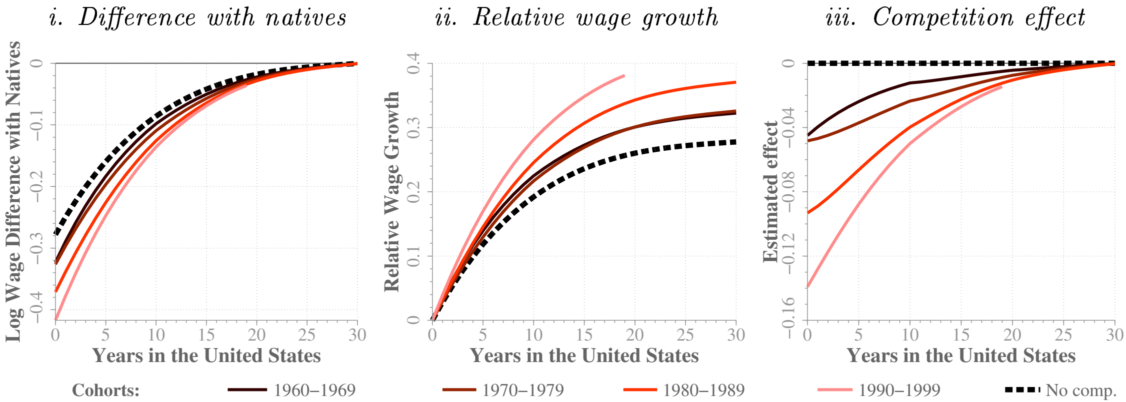
- **Competition** effect.
- **Composition** effects for education and origin.
- Changes in **unobservable skills** across cohorts.

Figure IV. – The Labor Market Competition Effect, One-time Increase



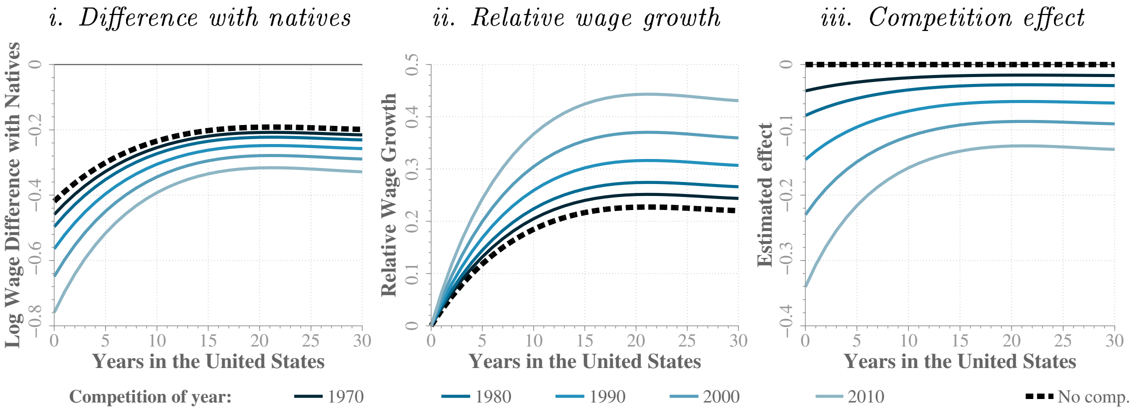
Note: The figure shows wage assimilation profiles of a baseline individual under different counterfactual scenarios. Our baseline individual is a Mexican high school dropout who arrived in the United States in (or with the skills of) the cohort of 1970s with 10 years of potential experience prior to arrival. The thick dashed line assumes relative skill prices are one. Solid lines are counterfactual scenarios in which the relative skill prices are maintained constant to the level of the indicated years based on the results in Figure VIi. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between the assimilation profiles in the counterfactual scenario and the no-competition benchmark.

Figure IV. – The Labor Market Competition Effect, Dynamic Effect



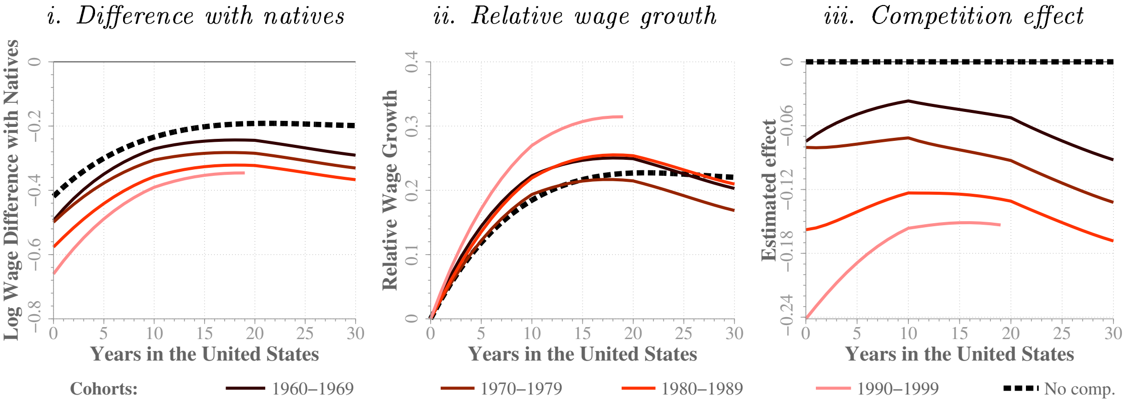
Note: The figure shows wage assimilation profiles of a baseline individual under different counterfactual scenarios. Our baseline individual is a Mexican high school dropout who arrived in the United States in (or with the skills of) the cohort of 1970s with 10 years of potential experience prior to arrival. The thick dashed line assumes relative skill prices are one. Solid lines show the predicted assimilation curves for the baseline individual (averaged over states) if he experienced the sequence of relative skill prices experienced by each of the indicated cohorts according to the results in Figure VIii. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between the assimilation profiles in the counterfactual scenario and the no-competition benchmark.

Figure C1. – The Labor Market Competition Effect (Alternative Immigrant)



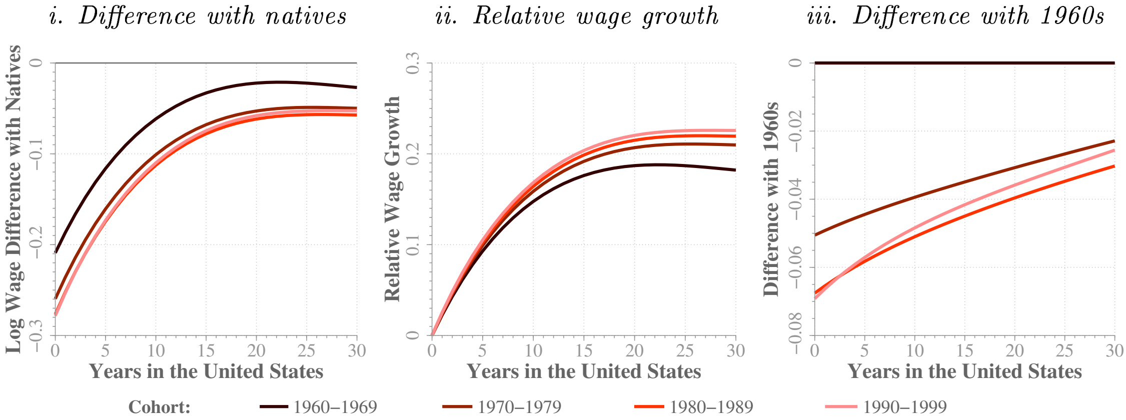
Note: The figure shows wage assimilation profiles of a baseline individual under different counterfactual scenarios. Our baseline individual is a Mexican high school graduate who arrived in the United States in (or with the skills of) the cohort of 1970s with 10 years of potential experience prior to arrival. The thick dashed line assumes relative skill prices are one. Solid lines are counterfactual scenarios in which the relative skill prices are maintained constant to the level of the indicated years based on the results in Figure VIi. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between the assimilation profiles in the counterfactual scenario and the no-competition benchmark.

Figure C1. – The Labor Market Competition Effect (Alternative Immigrant)



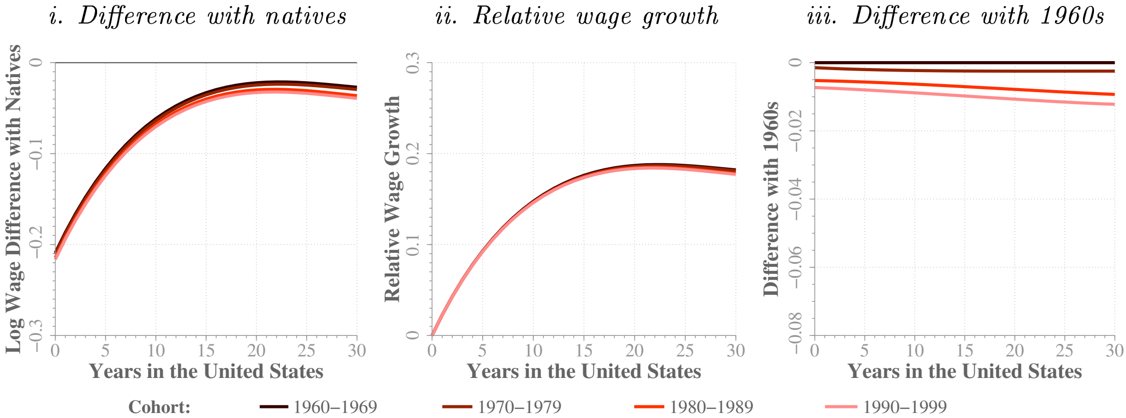
Note: The figure shows wage assimilation profiles of a baseline individual under different counterfactual scenarios. Our baseline individual is a Mexican high school graduate who arrived in the United States in (or with the skills of) the cohort of 1970s with 10 years of potential experience prior to arrival. The thick dashed line assumes relative skill prices are one. Solid lines show the predicted assimilation curves for the baseline individual (averaged over states) if he experienced the sequence of relative skill prices experienced by each of the indicated cohorts according to the results in Figure VIii. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between the assimilation profiles in the counterfactual scenario and the no-competition benchmark.

FIGURE VIII. – COMPOSITION EFFECTS, ORIGIN



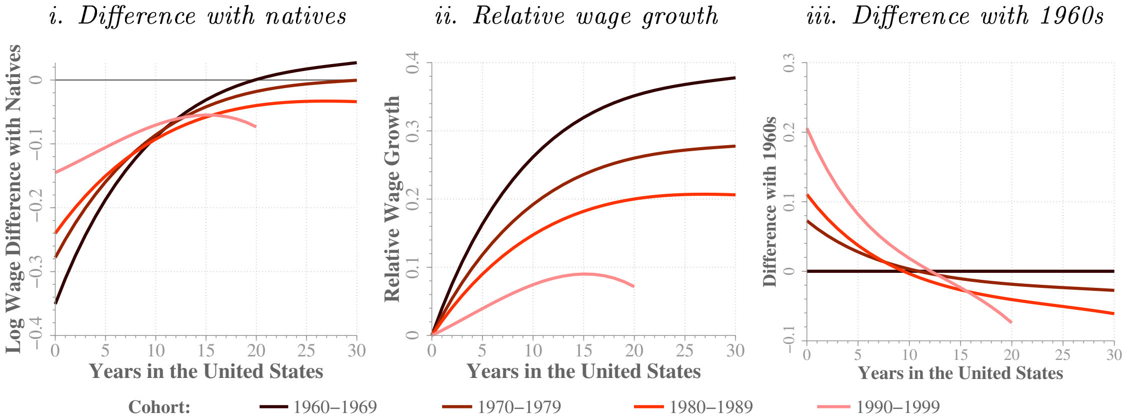
Note: The figure shows wage assimilation profiles for a counterfactual scenario in which we assume no competition effects (relative skill prices equal one), and we keep the distribution of immigrants across education groups constant to 1960s for each region of origin, and adjust the proportion of immigrants from each region of origin as we observe them changing in the data for the different cohorts. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between each cohort and the cohort of 1960s.

FIGURE VIII. – COMPOSITION EFFECTS, EDUCATION



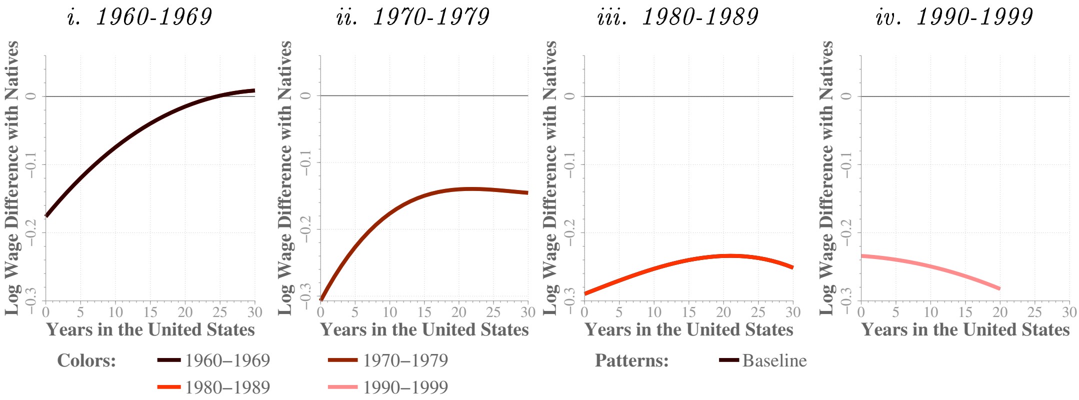
Note: The figure shows wage assimilation profiles for a counterfactual scenario in which we assume no competition effects (relative skill prices equal one), and we keep the distribution of regions of origin within each education group constant to 1960s, but adjust the distribution of immigrants in each education group across cohorts as we observe them changing in the data. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between each cohort and the cohort of 1960s.

Figure IX. – Changes in Unobservable Skills across Cohorts



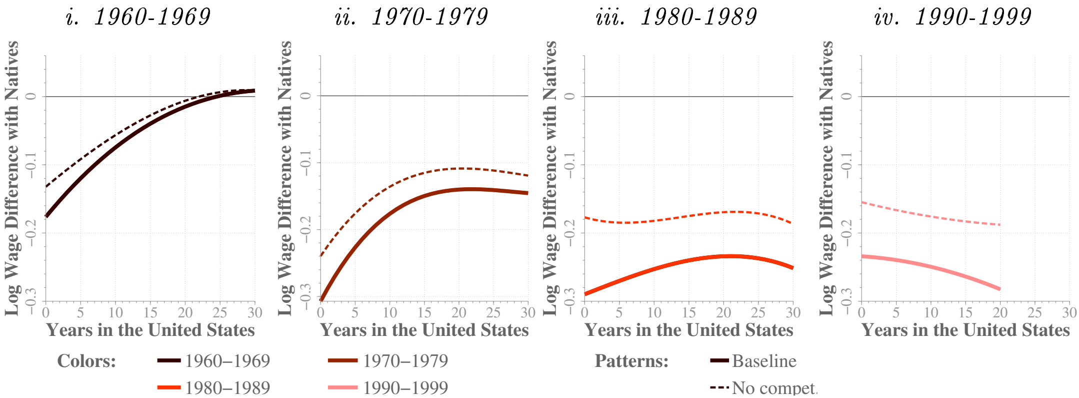
Note: The figure shows wage assimilation profiles for a counterfactual scenario in which we assume no competition effects (relative skill prices equal one), and we keep the distribution of regions of origin within each education group constant to 1960s, but adjust the distribution of immigrants in each education group across cohorts as we observe them changing in the data. Plots (i) and (ii) show the wage gap relative to natives and the relative wage growth as in Figure I. Plot (iii) shows the difference between each cohort and the cohort of 1960s.

FIGURE X. – COMPETITION EFFECTS, COMPOSITION EFFECTS, AND OBSERVED ASSIMILATION



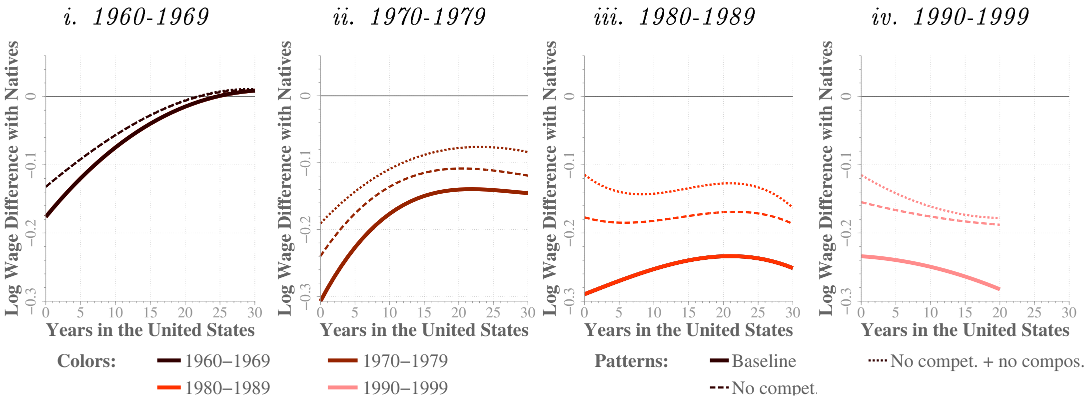
Note: The figure shows baseline and counterfactual predictions of the wage gap between natives and immigrants for different cohorts as they spend time in the United States. The baseline lines (solid) correspond to the model predictions in Figure V. The counterfactuals represent assimilation profiles in the absence of competition effects (dashed), in the absence of competition and composition effects (dotted), and in the absence of competition effects with education of immigrants evolving as that of natives (long-dashed). Each plot represents one cohort. The assimilation profiles are regression lines analogous to those presented in Figures I, fitted on simulated data under the baseline and the different counterfactual scenarios. Both counterfactuals set relative skill prices to one. The no-composition effects and native education counterfactuals adjust regression sample weights for immigrants to keep the composition in terms of education and region of origin as in the cohort of 1960 (no composition) or to adjust education in the same way that native education adjusts (native education).

Figure X. – Competition Effects, Composition Effects, and Observed Assimilation



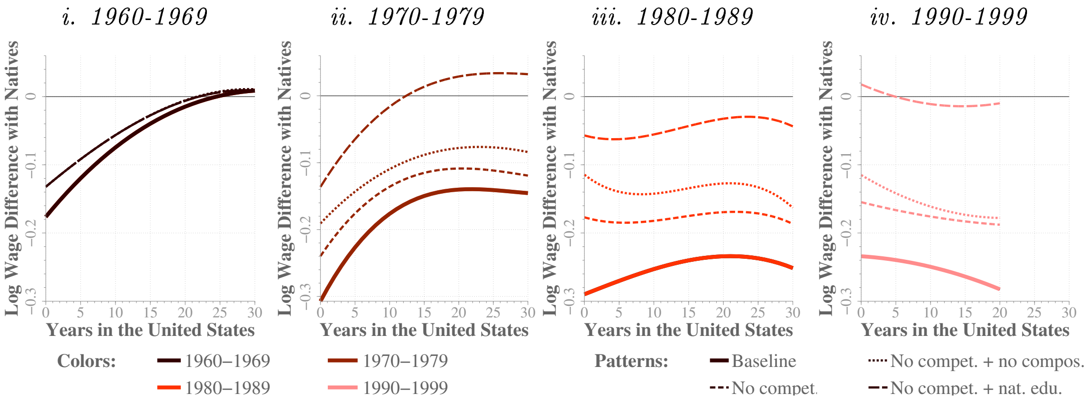
Note: The figure shows baseline and counterfactual predictions of the wage gap between natives and immigrants for different cohorts as they spend time in the United States. The baseline lines (solid) correspond to the model predictions in Figure V. The counterfactuals represent assimilation profiles in the absence of competition effects (dashed), in the absence of competition and composition effects (dotted), and in the absence of competition effects with education of immigrants evolving as that of natives (long-dashed). Each plot represents one cohort. The assimilation profiles are regression lines analogous to those presented in Figures I, fitted on simulated data under the baseline and the different counterfactual scenarios. Both counterfactuals set relative skill prices to one. The no-composition effects and native education counterfactuals adjust regression sample weights for immigrants to keep the composition in terms of education and region of origin as in the cohort of 1960 (no composition) or to adjust education in the same way that native education adjusts (native education).

Figure X. – Competition Effects, Composition Effects, and Observed Assimilation



Note: The figure shows baseline and counterfactual predictions of the wage gap between natives and immigrants for different cohorts as they spend time in the United States. The baseline lines (solid) correspond to the model predictions in Figure V. The counterfactuals represent assimilation profiles in the absence of competition effects (dashed), in the absence of competition and composition effects (dotted), and in the absence of competition effects with education of immigrants evolving as that of natives (long-dashed). Each plot represents one cohort. The assimilation profiles are regression lines analogous to those presented in Figures I, fitted on simulated data under the baseline and the different counterfactual scenarios. Both counterfactuals set relative skill prices to one. The no-composition effects and native education counterfactuals adjust regression sample weights for immigrants to keep the composition in terms of education and region of origin as in the cohort of 1960 (no composition) or to adjust education in the same way that native education adjusts (native education).

FIGURE X. – COMPETITION EFFECTS, COMPOSITION EFFECTS, AND OBSERVED ASSIMILATION



Note: The figure shows baseline and counterfactual predictions of the wage gap between natives and immigrants for different cohorts as they spend time in the United States. The baseline lines (solid) correspond to the model predictions in Figure V. The counterfactuals represent assimilation profiles in the absence of competition effects (dashed), in the absence of competition and composition effects (dotted), and in the absence of competition effects with education of immigrants evolving as that of natives (long-dashed). Each plot represents one cohort. The assimilation profiles are regression lines analogous to those presented in Figures I, fitted on simulated data under the baseline and the different counterfactual scenarios. Both counterfactuals set relative skill prices to one. The no-composition effects and native education counterfactuals adjust regression sample weights for immigrants to keep the composition in terms of education and region of origin as in the cohort of 1960 (no composition) or to adjust education in the same way that native education adjusts (native education).

Robustness checks

Results are **robust** to:

- **Network effects**: allowing stock or share of immigrants from the same country of origin to affect s.
- **Relative demand shifts**: changing the production function to

$$Y_t = A_t \left(G_t^{\frac{\sigma-1}{\sigma}} + \delta_t S_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

  for different specifications of δ_t (log-linear and log-quadratic and time dummies).
- **Different definitions of labor markets**: state-education and census division.

Table V. – Selected Parameter Estimates from Robustness Checks

A. Effects of network size (immigrants from the same country) on assimilation

	Intercept	Interaction with years since migration: Linear	Quadratic ($\times 10^2$)	Cubic ($\times 10^3$)
Share of state's population (%)	-0.211	-0.015	0.024	-0.010
	(0.148)	(0.036)	(0.241)	(0.045)
Stock in the state ($\times 10^6$)	-0.032	-0.009	0.050	-0.009
	(0.021)	(0.005)	(0.032)	(0.006)

B. Demand shifter for relative skill prices

	Intercept/ 1970 dummy	Trend/ 1980 dummy	Quadratic ($\times 10^2$)/1990 dummy	2000 dummy	2010 dummy
Linear specification	-0.958	0.026			
	(0.068)	(0.002)			
Quadratic specification	-1.059	0.035	-0.024		
	(0.090)	(0.005)	(0.012)		
Time dummies	-0.949	-0.954	-0.306	-0.206	-0.023
	(0.095)	(0.078)	(0.099)	(0.103)	(0.123)

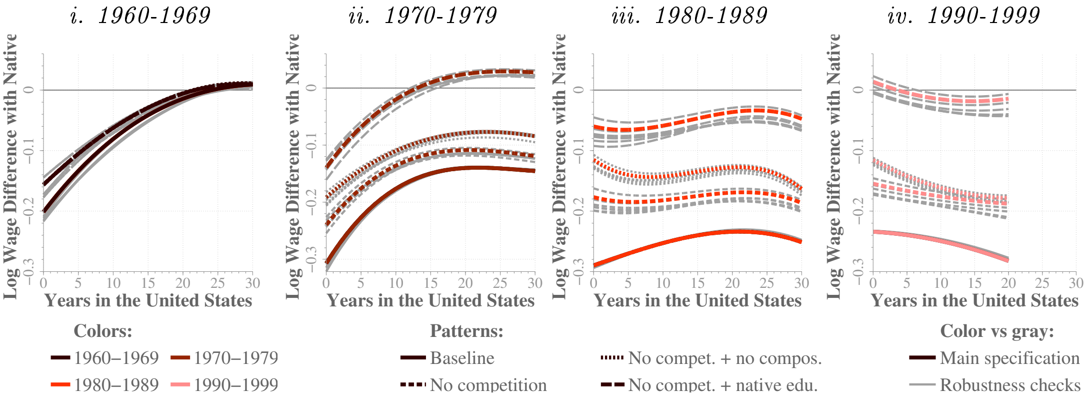
C. Elasticity of substitution between general and specific skills

	Networks:		Demand factors:		
	Share	Stock	Linear	Quadratic	Dummies
Elasticity of substitution (σ)	0.022	0.015	0.046	0.050	0.049
	(0.002)	(0.001)	(0.006)	(0.008)	(0.007)

	Market definitions:	
	State-education	Census division
Elasticity of substitution (σ)	0.039	0.014
	(0.002)	(0.001)

Note: Panel (A) of this table presents estimates for the parameters associated to the two specifications of the networks robustness check. These two specifications respectively account for the share and stock of immigrants from the same origin country living in the state of the reference person, which enters the specific skills functions both additively and interacted with a third order polynomial of years since migration. Panel (B) shows the demand shifter parameters for the relative demand shifters' counterfactual estimated in the second robustness check. Three specifications are presented, in which relative demand of specific skills is controlled for with a log-linear trend, log-quadratic trend, and time dummies. Panel (C) shows the estimated elasticities of substitution between general and specific skills (σ) for the different robustness checks. Standard errors in parentheses.

Figure XI. – Robustness: Wage Decomposition under Alternative Specifications



Note: The figure reproduces the counterfactual assimilation profiles described in Figure X for the different robustness checks described in the text: controlling for networks in the assimilation profiles (shares and stocks), controlling for relative demand shifters (linear, quadratic, and time dummies), and re-defining labor markets (state-education and census division).

Conclusions

We explore the role of **labor market competition** in explaining the observed wage assimilation patterns for different cohorts of immigrants in the United States.

Provide an analytic **framework** to analyze it $\Rightarrow$ counterfactual **simulations** with the structurally estimated model.

Simulations show:

- The **competition effect** explains about **one third** of the wage gap with natives (diverging **education** explains roughly the other two thirds).
- Large contribution on **widening** the initial wage gap, **positive effect** on speed of convergence.
- Recent immigrants arrived with **higher** amounts of specific skills (e.g. English) and hence have a **smaller gap** at arrival but converge at a **slower** rate.