Chapter 2: Applications of discrete and dynamic choice models

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Structural Econometrics for Labor Economics
(and Industrial Organization)

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Introduction

Introduction

This chapter:

- **Spatial equilibrium**: Diamond (2016)
- Married woman's labor force **participation**.
- **Human capital accumulation**: Heckman, Lochner, and Taber (1998).

Spatial equilibrium: Diamond (2016)

Spatial Equilibrium Models

Date back from **Rosen (1979)** and **Roback (1982)**.

The Rosen-Roback model is **static** (similar to the random utility models described in Chapter I, one time migration decisions).

These models typically **feature** (some of) the following:

- Perfectly **mobile labor market** (with moving costs?).
- Fixed land and endogenous **housing** markets.
- Local **amenities**.
- **Productivity differentials** across cities.
- Local **price** differences.

They are at the intersection of **urban economics** and **labor economics** (cities vs individuals as the subject of interest).

Diamond (2016)

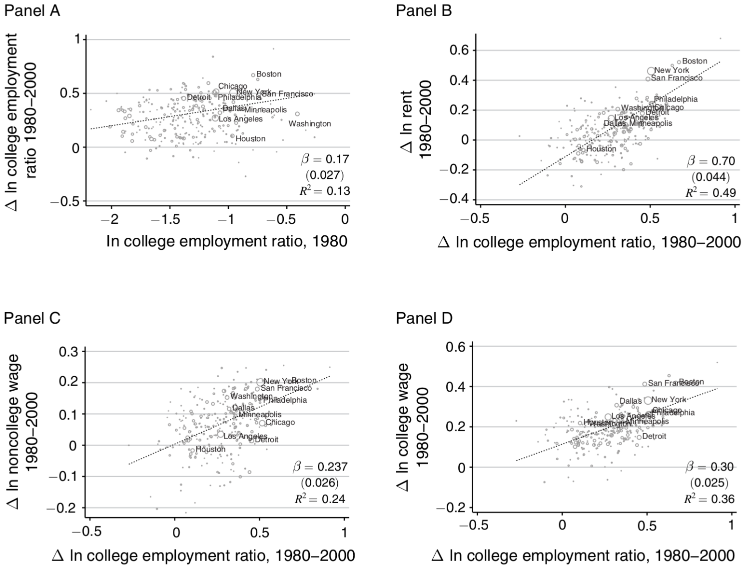
I am going to present a stylized version of **Diamond (2016)** as a canonical example.

Spatial equilibrium model to determine causes and welfare consequences of **increased skill sorting**.

The model **features**:

- Perfectly **mobile labor market** (one time settlement).
- Fixed land and endogenous **housing** markets.
- **Endogenous amenities** (main novelty).
- **Productivity differentials** across cities.

Motivation


Panel A
Δ ln college employment ratio 1980–2000
ln college employment ratio, 1980
Boston
Chicago
New York
San Francisco
Detroit
Philadelphia
Dallas
Minneapolis
Los Angeles
Washington
Houston
β = 0.17
(0.027)
R² = 0.13
Panel B
Δ ln rent 1980–2000
Δ ln college employment ratio, 1980–2000
β = 0.70
(0.044)
R² = 0.49
Panel C
Δ ln noncollege wage 1980–2000
Δ ln college employment ratio, 1980–2000
β = 0.237
(0.026)
R² = 0.24
Panel D
Δ ln college wage 1980–2000
Δ ln college employment ratio, 1980–2000
β = 0.30
(0.025)
R² = 0.36

Model

Aggregate firm in each city:

$$Y_j = K_j^{1-\alpha}(\theta_{Uj} U_j^\rho + \theta_{Sj} S_j^\rho)^{\frac{\alpha}{\rho}}.$$

Wage rates: workers' **marginal product** W_{kj} for $k \in \{S, U\}$.

Workers of skill k choose **consumption** of housing services and goods, as well as **location** to maximize utility:

$$\max_{\{j,c,h\}} \zeta \ln c + (1-\zeta) \ln h + G(A_j(U_j, S_j), \boldsymbol{X}_j, \boldsymbol{z}) + \varepsilon_j$$

$$\text{s.t.} \qquad Pc + R_j h \leq W_{kj}.$$

Housing supply:

$$R_j = F(C_j, L_j).$$

Estimation

Except for amenity parameters associated to $\boldsymbol{z}$, all relevant variation in the model is at the **city-education group level**.

Two-stage estimation:

1. **MLE/Logit** estimation of parameters for $\boldsymbol{z}$, collapsing city-education group-specific parts in **city-education dummies**.

2. **Joint estimation** using two-step GMM (in diffs) of all other parameters, using city level variation.

Instruments: exogenous variables plus Bartik shocks and their interactions with variables that affect the housing supply elasticity.

GMM equations

Amenities: construct an amenity index A_j as $\Delta A_j = \gamma \Delta \ln \frac{S_j}{U_j} + \epsilon_j$.

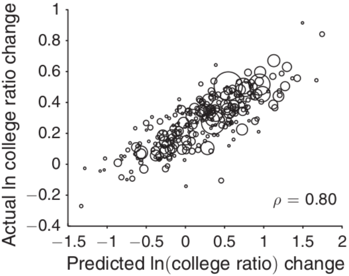
Production function: standard expressions from FOCs.

Housing: equilibrium expression linking changes in rents with regulation and land availability indexes, interest rates, and endogenous housing demand variables.

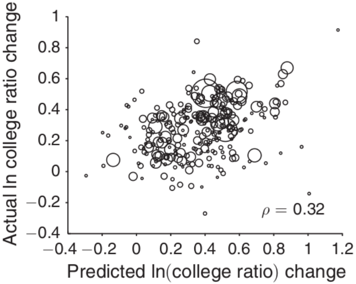
Location choice: once the combination of all city-education group-specific elements of the utility are estimated as δ_{jk}, a linear expression relates them to primitives.

Results

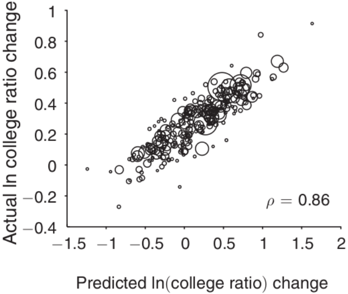
Panel A. Predicted change in ln college ratio due only to productivity changes



Panel B. Predicted change in ln college ratio due to observed wage and rent changes



Panel C. Predicted change in ln college ratio due to observed changes in wage, rent, and endogenous amenities



Married woman's labor force participation

Utility

Unitary household model to describe female labor force participation decisions.

Single decision unit that takes into account the **utilities of the two** members of the couple in the decision process.

The **couple's utility** is: $U(c, d, n, \boldsymbol{x}, \varepsilon(1-d))$, with:

- $\partial U / \partial c > 0$.
- $\partial^2 U / \partial c^2 < 0$.
- $U(c, 1, n, \boldsymbol{x}, \varepsilon) > U(c, 0, n, \boldsymbol{x}, 0)$ for some values of ε.
- Typically, $U(c, 1, n, \boldsymbol{x}, \varepsilon) > U(c, 1, n', \boldsymbol{x}, \varepsilon)$ for $n > n'$ as well.

Budget constrain and distr. of unobservables

The **husband** is assumed to work, generating income y.

The **wife** receives a wage offer $\omega(\boldsymbol{x}, \upsilon)$ and decides whether to work or not accordingly.

If the wife works, the household incurs in **child care cost** of π per child.

The **budget constraint** is:

$$c = y + [\omega(\boldsymbol{x}, \upsilon) - \pi n]d.$$

Unobservables ε and υ are **serially uncorrelated** and are **jointly distributed** as $F(\varepsilon, \upsilon | y, \boldsymbol{x}, n)$. The probability that the wife participates is:

$$\begin{aligned}
&\Pr(d = 1 | \boldsymbol{x}, n, y) \\
&= \int \mathbb{1}\{U(y + \omega(\boldsymbol{x}, \upsilon) - \pi n, 1, n, \boldsymbol{x}, \varepsilon) - U(y, 1, n, \boldsymbol{x}, 0) > 0\} dF(\varepsilon, \upsilon | y, \boldsymbol{x}, n) \\
&\equiv G(y, \boldsymbol{x}, n).
\end{aligned}$$

Estimation approaches

Primitives to recover: $U(\cdot)$, $\omega(\cdot)$, and $F(\cdot)$.

Four **estimation methods**: structural vs non-structural + parametric vs non-parametric.

We evaluate the convenience of these approaches considering **three goals**, to what extent the following elements affect participation:

1. wages.
2. husband's earnings.
3. childcare costs.

Non-structural approaches

In a **non-structural non-parametric** approach, we do not need to make further assumptions: estimate $G(\cdot)$ non-parametrically.

First goal requires further assumptions: **exclusion restriction**.

Let $\boldsymbol{x}_1$ denote the **partition** of the vector $\boldsymbol{x}$ that affects wages but does not enter the utility function directly.

The **effect of wage** changes on participation can be inferred from $\partial G/\partial \boldsymbol{z}'$.

The **second goal** is clearly feasible (within sample) without further assumptions, since $\partial G/\partial y$ is identified.

The **third goal**, on the contrary, is unfeasible without further assumptions, because G and π cannot be separately identified.

Parametric specification of $G(\cdot)$ (e.g. probit or logit): similar results but $\partial G/\partial y$ is also identified out of sample.

Non-parametric structural

The **non-parametric structural** approach requires identifying $U(\cdot)$, $\omega(\cdot)$, and $F(\cdot)$ separately without imposing additional assumptions about functional forms.

This is **infeasible** provided wages are only observed for the individuals who work.

With **further non-parametric assumptions** and data on wages for the women who work, one could go a bit further.

For example, if $\omega(\cdot)$ is assumed to be **additively separable** $\Rightarrow$ deterministic part of the wage function is identified (y, n, and potentially some elements in $\boldsymbol{x}$ not included in $\boldsymbol{x}_1$, denoted by $\boldsymbol{x}_2$ are **exclusion restrictions**).

Further assumptions on $F(\cdot)$ could also lead to partial identification of $U(\cdot)$.

Parametric structural

Consider the following very standard **parametric assumptions**:

$$U(c, d, n, \boldsymbol{x}, n, \varepsilon(1-d)) \equiv c + (1-d)[\boldsymbol{x}_2'\boldsymbol{\beta} + \gamma n + \varepsilon],$$

$$\omega(\boldsymbol{x}, \upsilon) = \boldsymbol{x}'\boldsymbol{\delta} + \upsilon,$$

and:

$$(\varepsilon, \upsilon)' | y, \boldsymbol{x}, n \sim \mathcal{N}(\mathbf{0}, \Sigma).$$

Given this parameterization, the **difference in utilities** is:

$$\begin{aligned} U(y + \omega(\boldsymbol{x}, \upsilon) - \pi n, 1, n, \boldsymbol{x}, \varepsilon) - U(y, 1, n, \boldsymbol{x}, 0) > 0\} dF(\varepsilon, \upsilon | y, \boldsymbol{x}, n) \\ = \boldsymbol{x}'\boldsymbol{\delta} - [\pi + \gamma] n - \boldsymbol{x}_2'\boldsymbol{\beta} + \upsilon - \varepsilon. \end{aligned}$$

Parametric structural

Data on **choices** $\Rightarrow \pi+\gamma$, $\boldsymbol{\delta}_1$, and $\boldsymbol{\delta}_2-\boldsymbol{\beta}$, where $\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$ are the partitions of δ associated, respectively, to $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$.

Further data on **wages for women who work** $\Rightarrow \boldsymbol{\beta}$ and $\boldsymbol{\delta}$ are separately identified (Heckman selection approach).

Wage data and exclusion restrictions $\Rightarrow \Sigma$ could also be identified ($\sigma_{\nu\varepsilon}$ from Heckman, σ_ν^2 from variance in wages, and σ_ε^2 from coefficient of $\boldsymbol{x}_1\boldsymbol{\delta}_1$ the probit).

Goals:

1. Elasticity of labor supply with respect to **wages** is only identified if there are exclusion restrictions.
2. Effect of **husband's income** is assumed to be zero in this case; other utility functions would lead to different effects.
3. Effect of changing the **cost of child care** can also be identified, even though only $\gamma+\pi$ is identified.

Human capital accumulation: Heckman, Lochner, and Taber (1998)

Equilibrium model for wage inequality

Literature on inequality: partial equilibrium.

Heckman, Lochner, and Taber (1998):

- General equilibrium.
- Overlapping generations.
- Human capital accumulation (education and on the job).
- New methods for estimating these models.

Using their estimated model, evaluate the mechanisms behind **increasing wage inequality**.

Model

Consider the following **life-cycle maximization** problem:

$$V(h_a, b_a, e, i_t, r_{et}) \equiv \max_{c,g} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta V(h_{a+1}, b_{a+1}, e, i_{t+1}, r_{et+1}) \right\},$$

$$\text{s.t. } b_{a+1} \leq b_a[1 + (1-\tau)i_t] + (1-\tau)r_{et}h_a(1-g) - c.$$

On-the-job human capital accumulates as:

$$h_{a+t}(\omega, e) = \omega g^{\eta_e} h_a(\omega, e)^{\psi_e} + (1-\delta)h_a(\omega, e),$$

with $0 < \eta_e < 1$ and $0 < \psi_e < 1$ for $e \in \{S, U\}$.

Discrete distribution for ω, with eight points of support (four observable types, denoted by k, and based on quartiles of AFQT test, two education groups).

Individuals are assumed to have **perfect foresight** of future prices and interest rates in equilibrium (no aggregate shocks in the economy).

Individuals **work** until age a_R, when are forced to retire, and afterwards **live** until age $\bar{a}$ without perceiving labor income.

Model

Education decision is taken at the front end:

$$\max_e \left[V^E(\omega, e, t) - \pi_e + \varepsilon_e\right].$$

Output Y_t is determined by the following nested CES technology:

$$Y_t = \left\{\alpha K_t^\phi + (1-\alpha)[\theta_t L_{St}^\rho + (1-\theta_t) L_{Ut}^\rho]^{\frac{\phi}{\rho}}\right\}^{\frac{1}{\phi}}.$$

Skill-biased technical change, determined by the evolution of θ_t is given by:

$$\ln\left(\frac{\theta_t}{1-\theta_t}\right) = \ln\left(\frac{\theta_0}{1-\theta_0}\right) + \varphi t.$$

Equilibrium is given by the sequence of interest rates $\{i_t\}_{t=0}^\infty$ and skill prices $\{r_{Ut}, r_{St}\}_{t=0}^\infty$ that **clear the market** subject to aggregate firm profit maximization, and workers' lifetime utility maximization.

Estimation

They **assume values** for β, γ, $\delta(=0)$, and τ.

The **tuition costs** π_e are estimated from the data.

The estimation of the remaining parameters is carried with a **step-wise procedure**.

First step: production function.

At **old ages**, say $a > a^*$ for some a^*, individuals no longer invest in human capital (that is, $g \approx 0$). Therefore:

$$w(a^* + 1, t + 1, h_{a^*+1}) \equiv r_{et+1} h_{a^*+1} = r_{et+1} h_{a^*}(1 - \delta),$$

which implies:

$$\frac{w(a^* + \ell, t + \ell, h_{a^*+\ell})}{w(a^*, t, h_{a^*})} = \frac{r_{et+\ell}(1 - \delta)^{\ell}}{r_{et}}.$$

Normalizing $r_{e0} = 1$, **skill prices** are identified up to a scale $(1 - \delta)^t$.

Estimation

Given these skill prices, the **aggregate stocks** of skill units can be recovered from the skill prices:

$$\frac{wage\ bill_{et}}{r_{et}(1-\delta)^t} = \frac{L_{et}}{(1-\delta)^t}.$$

Relative demands of the two labor inputs give:

$$\ln\frac{r_{St}}{r_{Ut}} = \frac{\theta_t}{1-\theta_t} + (\phi-1)\ln\frac{L_{St}}{L_{Ut}} = \ln\left(\frac{\theta_0}{1-\theta_0}\right) + \varphi t + (\phi-1)\ln\frac{L_{St}}{L_{Ut}}.$$

$\Rightarrow$ θ_0, φ, and ϕ can be recovered by **OLS**.

The **remaining aggregate PF parameters** estimated analogously.

Second step: lifetime maximization problem.

NLS for wages (g is unobserved, replaced by the solution of the dynamic problem). This solution is computed by **backwards induction**.

To estimate $h_0(k,e)$, parameterize $h_{a_R}(k,e)$ and recover backwards.

Estimation

Third step: education decision.

First estimate an **auxiliary probit model** to recover a non-parametric estimate of $\{(1-\tau)[V^E(\omega,S,t)-V^E(\omega,U,t)]+\mu_k\}/\sigma$ and a coefficient associated to estimated tuition costs π_e.

Then, they **recover the structural parameters** from these estimates:

- σ is recovered as the coefficient associated to π_e.
- μ_k is recovered comparing the non-parametric estimates to the values predicted by the model (given the parameters estimated in steps one and two).