Chapter 6. Auctions

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Motivation

Auctions are often used as **mechanisms for allocation** of resources to bidders in an as efficient as possible way.

Many interesting real-world **applications** $\Rightarrow$ Important and growing literature in empirical micro.

Aim: estimate the parameters of **bidders' valuations**.

Simulation of efficiency results under different types of auction mechanism $\Rightarrow$ optimal design.

Types of auctions

To illustrate structural estimation of auctions, we focus on two **types of auctions**: first and second price sealed bid.

We assume **one good**, N **players**, and **independent** valuations v_i for $i = 1, ..., N$.

Paradigm: each player simultaneously submits a bid, and the object is assigned to the highest bidder, who pays the price she bid (first price) or the bid of the second highest bidder (second price).

Other types of auctions that have been studied (but we don't not review here):

- English auctions.
- Dutch auctions.
- Japanese auctions.
- ...

Setup

First and **second price sealed bid** auctions.

N **risk-neutral** bidders (indexed by $i = 1, ..., N$).

Valuations v_i, independently drawn from a common distribution $F(\cdot)$.

The **data** consists of the outcomes observed across **independent auctions** $k = 1, ..., K$ that follow the same paradigm.

Individual observes her **own valuation** v_i, but not other players' valuations (literature also focus on signal $x_i \neq v_i$ and common valuations).

Players submit a **single bid** $b_i \in \mathbb{R}^+$ and do not observe other players'.

Econometrician only observes bids, either all of them or only the winning bid/price.

Equilibrium bids from **Perfect Bayesian Equilibria** in weakly undominated pure strategies.

Equilibrium responses

In **second price sealed bid** auctions, it is a weakly dominant strategy for every individual to **bid her expected valuation**, i.e. $b_i = v_i$.

In a **first price sealed bid** auction, best responses solve:

$$b_i = \arg\max_b (v_i - b) p_i(b),$$

where $p(b)$ denote the probability of winning the auction with bid b.

The **resulting bid** solves:

$$(v_i - b_i) p'(b_i) - p(b_i) = 0.$$

Totally differentiating this expression with respect to b and v, we obtain:

$$\frac{db_i}{dv_i} = \frac{-p'(b_i)}{(v_i - b_i) p''(b_i) - 2p'(b_i)} > 0.$$

Therefore, if players are in pure strategy equilibrium with an interior solution, then b_i is **increasing** in v_i. This ensures invertibility, and, hence, identification.

Identification, second price sealed bid

In a second price sealed bid auction, the distribution of valuations is trivially identified **if all bids are observed** because, as noted before, all players bid their valuation.

If only the **winning price is observed**, the distribution of the second highest valuation, denoted by $F_{N-1,N}(v)$ is identified.

Let $f_{N-1,N}(v)$ denote the corresponding density. Given **symmetry and independence**, this density equals:

$$f_{N-1,N}(v) = N(N-1)F(v)^{N-2}(1-F(v))f(v).$$

Given a **boundary condition** $F_{N-1,N}(\underline{v}) = F(\underline{v}) = 0$, and noting that $f(v) > 0$ above the boundary condition, the identification of $F(v)$ comes from solving the differential equation above.

Identification, first price sealed bid

In first price sealed bid auctions, **if all bids are observed**, identification comes from the first order condition above:

$$v_i = b_i + \frac{p(b_i)}{p'(b_i)} = b_i + \frac{G(b_i)}{(N-1)g(b_i)},$$

where $G(\cdot)$ and $g(\cdot)$ are respectively the cdf and pdf of observed bids.

If **only the winning bid** is recorded, the distribution of winning bids, $H(b)$ is identified from the outcomes observed in the different auctions.

$H(b)$: probability that all the bids in all the auctions are less than or equal to b, such that:

$$H(b) = \Pr(b_i^k \leq b \forall i = 1, ..., N) = G(b)^N.$$

Therefore:

$$G(b) = H(b)^{\frac{1}{N}}, \Rightarrow g(b) = \frac{1}{N} H(b)^{\frac{1}{N}-1} h(b),$$

where $h(b)$ is the density of winning bids. Replacing these two in the FOC, this shows that the bidding distribution is identified.

Estimation

Estimation strategies range from **minimum distance** to **maximum likelihood**.

In minimum distance estimation, once the distribution of bids is derived, up to parameter values, these parameters are estimated comparing **sample moments** of the observed bids against **theoretical moments** of the $G(\cdot)$ distribution for each parameter value.

Some times, this moments are trivial functions of the parameters, and **standard minimum distance** methods are easy to implement.

Other times, it is too costly to derive the theoretical moments from the distribution, and we proceed with **simulated method of moments**, using Monte Carlo approaches.

Maximum likelihood approaches: upper bound of the support often depends on parameter values $\Rightarrow$ consistent but not asymptotically normal estimates.