Chapter 5. Dynamic discrete games with incomplete information

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Introduction

Introduction

Among **the most popular topics** in empirical IO.

Estimation of dynamic **oligopolistic games**.

Motivational example: **entry/stay/exit** in oligopolistic markets.

Complication: each firm **best-reacts** to other firms' behavior.

Data: M markets, N firms per market, T observed periods (infinite horizon).

Motivating example: market entry and exit

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The profit function is given by:

$$U_{it}(1, \boldsymbol{x}_{it}, \boldsymbol{\omega}_i, \boldsymbol{\varepsilon}_{it}) = \theta_{RS} \ln S_{m(i)t} - \theta_{RN} \ln\left(1 + \sum_{\{j: j \in m(i), j \neq i\}} d_{jt}\right)$$
$$-\omega_{Fi} - (1 - d_{it-1})\omega_{Ei} + \varepsilon_{1it},$$

$$U_{it}(0, \boldsymbol{x}_{it}, \boldsymbol{\omega}_i, \boldsymbol{\varepsilon}_{it}) = \omega_{Ni} + \varepsilon_{0it}.$$

ω_{Ni} is **normalized to zero** for identification.

$\boldsymbol{\varepsilon}_{it}$ is **private information** of firm i.

This embeds Rust when $\theta_{RN} = 0$. If not, $d^*_{it}(\boldsymbol{x}_{it}, \boldsymbol{\omega}_i, \boldsymbol{\varepsilon}_{it})$ are **best response functions**, and d^*_{it} is given by the Nash equilibrium.

NFXP is too costly here. **Aguirregabiria and Mira** (2007) propose a CCP estimator.

Potential complication: **multiple equilibria**.

General structure

General structure

Let d_{it} denote the action of individual i, and $\boldsymbol{d}_{-it}$ denote the actions of all other $I-1$ players. The **flow utility** of individual i choosing j is:

$$u_{ij}(\boldsymbol{x}_t, \boldsymbol{d}_{-it}) + \varepsilon_{ijt},$$

where $\boldsymbol{\varepsilon}_{it} \equiv (\varepsilon_{i1t}, ..., \varepsilon_{iJt})'$ is an i.i.d. random variable **privately observed** by individual i but not by the others.

The vector $\boldsymbol{x}_t$, on the contrary, is **observed by all individuals**, and, therefore, includes the state variables of all individuals.

The dependence of $u_{ij}(\cdot)$ on i is a reflection of the possibility of different state variables affecting payoffs differently (e.g. own state variables vs other individuals'); the absence of t in it reflects that we are in a stationary environment.

Markov Perfect Equilibria

Choices are taken **simultaneously** in each period $\Rightarrow$ we concentrate on **rational stationary Markov perfect equilibria**.

Given that $\boldsymbol{\varepsilon}_{it}$ is i.i.d. across individuals, individual i **expects** other agents to make choices $\boldsymbol{d}_{-it}$ with probabilities:

$$\Pr(\boldsymbol{d}_{-it}|\boldsymbol{x}_t) = \prod_{j \neq i} \Pr(d_{jt}|\boldsymbol{x}_t).$$

These CCPs represent the **best-response** probability functions.

Uniqueness is rather unlikely to hold. However, these CCPs uniquely identify the **beliefs** of agents.

Re-writing the problem

Taking expectations of $u_{ij}(\boldsymbol{x}_t, \boldsymbol{d}_{-it})$ over $\boldsymbol{d}_{-it}$, we obtain:

$$\tilde{u}_{ij}(\boldsymbol{x}_t) = \sum_{\boldsymbol{d}_{-it} \in \mathcal{D}^{I-1}} \Pr(\boldsymbol{d}_{-it} | \boldsymbol{x}_t) u_{ij}(\boldsymbol{x}_t, \boldsymbol{d}_{-it}).$$

Likewise:

$$\tilde{F}_i(\boldsymbol{x}_{t+1} | \boldsymbol{x}_t, d_{it}) = \sum_{\boldsymbol{d}_{-it} \in \mathcal{D}^{I-1}} \Pr(\boldsymbol{d}_{-it} | \boldsymbol{x}_t) F_x(\boldsymbol{x}_{t+1} | \boldsymbol{x}_t, d_{it}, \boldsymbol{d}_{-it}).$$

Given all this, the conditional value functions can be expressed as:

$$v_{ij}(\boldsymbol{x}_t) = \tilde{u}_{ij}(\boldsymbol{x}_t) + \beta \int V(\boldsymbol{x}_{t+1}) d\tilde{F}_i(\boldsymbol{x}_{t+1} | \boldsymbol{x}_t, d_{it}),$$

and then $V(\boldsymbol{x}_{t+1})$ can be replaced the standard CCP representation, i.e.:

$$v_{ij}(\boldsymbol{x}_t) = \tilde{u}_{ij}(\boldsymbol{x}_t) + \beta \int \left[v_{ik}(\boldsymbol{x}_t) + \psi_k(\boldsymbol{p}_i(\boldsymbol{x}_t)) \right] d\tilde{F}_i(\boldsymbol{x}_{t+1} | \boldsymbol{x}_t, d_{it}).$$

Identification and Estimation

Identification

The use of **CCP estimation** methods made the estimation of these models **feasible**.

Full solution maximum likelihood approaches would **not be tractable**, because it would require solving for the equilibrium of the game (on top of solving for the dynamic problem).

Identifying assumptions:

1. Every observation in the sample comes from the same MPE.
2. There are no unobserved common-knowledge variables.

Given the first assumption, the multiplicity of equilibria in the model does not play any role in the identification of structural parameters.

For non-parametric identification $\Rightarrow$ **exclusion restriction**:
$\boldsymbol{x}_t \equiv (s_{1t}, ..., s_{Nt}, \boldsymbol{w}_t)$ is such that $u_{ij}(\boldsymbol{x}_t, d_{-it}) = u_{ij}(s_{it}, \boldsymbol{w}_t, d_{-it})$.

Estimation

Estimation follows any of Chapter 4's techniques.

Aguirregabiria and Mira (2007) use Aguirregabiria and Mira (2002).

Bajari, Benkard, and Levin (2007) use the Hotz, Miller, Sanders, and Smith (1994) forward simulation technique.

Estimation based on conditions that are satisfied by **every Markov perfect equilibrium**, which skips the complication generated by the existence of **multiple equilibria**.