

CHAPTER 7. DIFFERENCE IN DIFFERENCES

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Difference in differences setup

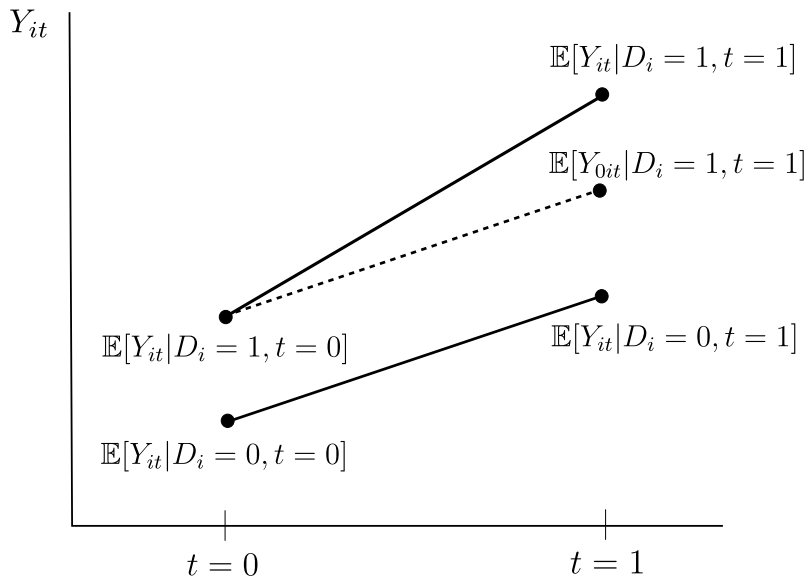
Randomized experiment \Rightarrow simple comparison of the mean outcome in treatment and control groups (“difference” estimator), unbiased and consistent estimate of the ATE (Chapter 2).

The approach in this chapter: like in matching or sharp RD, adjusts somehow to **compensate confounders**.

Linking Chapter 6 to treatment effects approaches, we propose an alternative method to eliminate confounders that are **fixed over time** (like a fixed effect), using repeated observations.

Key assumption: common trend.

Difference in differences



Formal discussion

Figure suggests to use **trend observed for untreated** to predict the **counterfactual trend** for treated individuals in the absence of treatment:

$$\begin{aligned}\mathbb{E}[Y_{0it}|D_i = 1, t = 1] &= \underbrace{\mathbb{E}[Y_{it}|D_i = 0, t = 1]}_{\text{level for controls at } t=1} \\ &+ \underbrace{\{\mathbb{E}[Y_{it}|D_i = 1, t = 0] - \mathbb{E}[Y_{it}|D_i = 0, t = 0]\}}_{\text{difference in levels at } t=0 \text{ difference}}.\end{aligned}$$

Fundamental DD assumption: common trend:

$$\mathbb{E}[Y_{0i1} - Y_{0i0}|D_i = 1] = \mathbb{E}[Y_{0i1} - Y_{0i0}|D_i = 0].$$

Hence, the difference in differences coefficient (which is an average treatment effect on the treated) is:

$$\begin{aligned}\beta &= \{\mathbb{E}[Y_{it}|D_i = 1, t = 1] - \mathbb{E}[Y_{it}|D_i = 1, t = 0]\} \\ &\quad - \{\mathbb{E}[Y_{it}|D_i = 0, t = 1] - \mathbb{E}[Y_{it}|D_i = 0, t = 0]\}.\end{aligned}$$

Diff-in-diff and regression

The difference in differences coefficient can be obtained as the β coefficient in the following **regression**:

$$Y_{it} = \beta_0 + \beta_D D_i + \beta_T T_{it} + \beta D_i T_{it} + U_{it},$$

where $T_{it} = 1$ if individual i is treatment period $t = 1$, and $T_{it} = 0$ otherwise.

With similar arguments as in previous chapters:

- $\beta_0 = \mathbb{E}[Y_{it} | D_i = 0, t = 0]$,
- $\beta_0 + \beta_D = \mathbb{E}[Y_{it} | D_i = 1, t = 0]$,
- $\beta_0 + \beta_T = \mathbb{E}[Y_{it} | D_i = 0, t = 1]$,
- and β is the difference in differences coefficient.

Diff-in-diff and regression

This regression model can be expanded in several ways:

- **Further periods:** In such case, T_{it} is not a time dummy but, instead, a dummy that equals one in the post-treatment period. One could additionally include time effects, but the interaction term should be with the “post” dummy only.
- **Controls:** the regression allows for controls, X_{it} (works as in regression vs matching).
- **Panel data:** there is actually no need for panel data. However, in the repeated cross-section context, the researcher needs to sustain the assumption that the sample composition does not vary over time, which is satisfied by construction with panel data (also individual fixed effects)
- **Placebo analysis:** a regression that simulates the difference in differences analysis but for a point in time or group of individuals that resemble the treatment period or group but that was actually not treated.

Triple differences

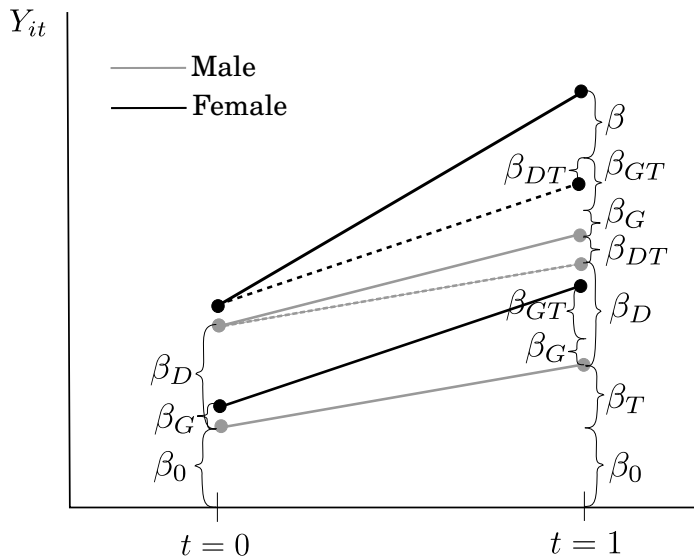
Triple difference: the difference in differences assumption does not hold, but the change in trends is assumed to be the same across sub-groups, some of which should be more affected than others.

Let G_i denote the (say sociodemographic) group to which individual i belongs. Then, the **triple-differences** model is:

$$Y_{it} = \beta_0 + \beta_D D_{it} + \beta_T T_{it} + \beta_G G_i + \beta_{GD} G_i D_{it} \\ + \beta_{GT} G_i T_{it} + \beta_{DT} D_{it} T_{it} + \beta_{GDT} G_i D_{it} T_{it} + U_{it}.$$

Example: Maternity leave policies combined with a tax reform that affects young and old differently.

Difference in differences



Synthetic Control Methods

Synthetic control methods: use longitudinal data to build the weighted average of non-treated units that best reproduces the characteristics of the treated unit over time prior to the treatment.

Thus, we build an **artificial control** that has the best possible pre-trend possible, and then we compute the difference in differences estimate using such synthetic control group.