

# Disaster Risk through Investors' Eyes: a Yield Curve Analysis\*

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## Abstract

This paper presents a model for extracting disaster risk from yield curve data. I employ an asset pricing model with disasters, which influence bond prices through changes in consumption growth, inflation, and sovereign default risk. I decompose bond prices into a non-disaster theoretical price and a disaster wedge. Using high-frequency yield curve data, I estimate disaster probabilities of around 50 countries over two decades. The model can identify key stylized facts such as the non-anticipation of the Russian-Ukrainian war, the impact of Mario Draghi's "whatever it takes" speech, and the effect of the COVID-19 pandemic.

**Keywords:** Disaster Risk, Yield Curve, Asset Pricing, Macroeconomic Shocks

**JEL Classification:** E20, G01, G12, G15, G17

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# 1 Introduction

Asset prices are inherently forward-looking. If investors expect high inflation, prices of instruments with nominal payments decrease to compensate for the loss in purchasing power. Beyond regular economic fluctuations, asset prices are also affected by the risk of economic disasters, such as wars, depressions, and defaults, capable of altering economic paths. Ross (2015) refers to disaster risk as *dark matter*: “It is unseen and not directly observable but exerts a force that can change over time and profoundly influence markets.”

This study provides a novel approach to extract investors’ disaster risk from yield curve data. I use a classic asset pricing model, based on Rietz (1988) and Barro (2006), that incorporates time-varying disaster probabilities. Disasters can affect consumption growth, inflation, and sovereign default risk. Their probability of occurring influences bond prices by altering investors’ expectations on these variables. I define three types of disasters—consumption shocks, interstate wars, and sovereign defaults. The model decomposes bond prices into a theoretical non-disaster price and a disaster wedge. I calibrate the model to a panel of 50 countries which allows to compute theoretical prices over the past two decades. I then apply a multiple fixed-effect regression to high-frequency yield curve data from Datastream to estimate the disaster wedge for each country over time. Finally, by specifying the disaster type based on each country’s context, I infer the probability of a disaster.

The model can identify several key stylized facts. First, it reveals that financial markets largely failed to anticipate the Russian-Ukrainian war. The estimated disaster probability remained low in the months before the conflict, then jumped abruptly after the invasion began. Second, the model captures the significant impact of Mario Draghi’s “whatever it takes” speech in 2012. For Italy and Spain, which were at the peak of default risk the day before the speech, the disaster risk declined on the day of the speech, reversing the upward trend. Lastly, the model can also be used to identify the “type” of a disaster once it occurs, as with the COVID-19 pandemic. The pandemic led to a general increase in perceived default risk across multiple countries.

This paper mainly relates to the macroeconomic literature on “(rare) disasters” or “tail events”. The early disaster literature was theoretical, addressing asset pricing puzzles—such as the risk-free rate premium—by introducing the concept of a low-probability of a “consumption” disaster (Rietz 1988; Barro 2006; Gabaix 2008; Backus, Chernov, and Martin 2011; Gourio 2012; Gabaix 2012; Wachter 2013; Farhi and Gabaix 2016).<sup>1</sup> More recently, advancements in econometric techniques and the availability of richer datasets have fueled a new wave of research focused on empirically identifying disaster risk from the data (Berkman, Jacobsen, and Lee 2011; Ross 2015; Schreindorfer 2020; Barro and Liao 2021). The most similar approach to this paper, which combines a theoretical model with fixed effect estimation, is found in Barro and Liao (2021) that employs option prices. This paper offers a theoretical-based struc-

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1. Julliard and Ghosh (2012) argue that rare events alone cannot adequately explain asset pricing puzzles like the equity premium.

tural model that not only explains the corresponding yield curve puzzles but also attempts to estimate disaster risk directly from the panel data on the yield curve. Unlike earlier models that primarily focus on “consumption disasters” (Rietz 1988; Barro 2006; Barro and Ursua 2008), my model is calibrated to specific disaster types tailored to the country’s context. Furthermore, using yield curve data is particularly insightful because it consolidates investors’ expectations over different time horizons, providing a comprehensive measure of both short and long-term risks. Substantial empirical evidence suggests that the yield curve is one of the most informative indicators, particularly for forecasting economic downturns (Estrella and Hardouvelis 1991; Estrella and Mishkin 1998; Ang, Piazzesi, and Wei 2006).<sup>2</sup> Empirical studies show that the yield curve responds to economic policy uncertainty (Leippold and Matthys 2022), political uncertainty (Pástor and Veronesi 2013; Smales 2016) and international political risk (Huang et al. 2015).<sup>3</sup>

This research also aligns with a broader literature that delves into the economic effects of disruptive events like disasters (Barro and Ursúa 2008), and economic and political crises (Cerra and Saxena 2008; Mueller 2012). This paper contributes by providing estimates of how a set of events affects consumption growth, inflation, and default risk.

The financial literature has explored the predictive power of asset prices in forecasting economic outcomes. Beyond the yield curve, the spread between corporate bonds and treasury notes has also been explored to predict economic activity (Gilchrist and Zakrajšek 2012; Gilchrist et al. 2016). This paper enhances the existing literature by offering insights into leveraging the yield curve for analyzing disaster events through an economically founded structural model. Unlike reduced-form approaches, a model provides a structured framework to incorporate the underlying mechanisms through which fundamentals influence prices. Additionally, there is related literature in corporate finance that tries to disentangle the valuation of political risks using sovereign yield spreads (Clark 1997; Bekaert et al. 2014, 2016).

This paper is structured as follows. The next section outlines the asset pricing model. The calibration of the model and some preliminary results are described in Section 3. In Section 4, I present the methodology for estimating the disaster risk and the probability of a disaster. In Section 5, I discuss the results, followed by the conclusions in the final section.

## 2 Model Setup

The model follows Rietz (1988) and Barro (2006), which I extend by including time-varying probabilities of disasters and inflation.

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2. Even the Federal Reserve Bank of New York has a webpage dedicated to the yield curve and its predictive power for recessions. See [https://www.newyorkfed.org/research/capital\\_markets/yieldcurveFAQ.html](https://www.newyorkfed.org/research/capital_markets/yieldcurveFAQ.html) and [https://www.newyorkfed.org/medialibrary/media/research/capital\\_markets/Prob\\_Rec.pdf](https://www.newyorkfed.org/medialibrary/media/research/capital_markets/Prob_Rec.pdf)

3. International political risk refers as the risk of a political conflict like military escalation or a war.

The representative consumer maximizes a time-additive utility function:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_{t+i}), \quad (1)$$

where

$$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta} \quad (2)$$

$\beta$  is the time discount factor and  $\theta$  is the coefficient of relative risk aversion. At each period, agents can invest in government nominal zero-coupon bonds, each of which will pay out one unit of currency when reaching its maturity.  $Q_{Nt}$  is the price of a bond that matures in  $N$  periods, and  $X_{Nt}$  is the amount bought. The government can partially default on its obligations, where  $F_{Nt}$  represents the fraction of the bond that is repaid ( $F_{Nt} = 1$  when no default occurs). The budget constraint of the agents is given by

$$P_t C_t = W_t - \sum_{N=1}^H Q_{Nt} X_{Nt} \quad \forall t \quad (3)$$

where  $P_t$  is the price of consumption and  $W_t$  corresponds to the wealth if no bond is bought.<sup>4</sup>  $H$  represents the maximum maturity. Using the usual first-order conditions, I derive the fundamental asset pricing equation:

$$Q_{Nt} = \beta^N \mathbb{E}_t \left[ \frac{U'(C_{t+N}) P_t}{U'(C_t) P_{t+N}} \right] \quad (4)$$

Substituting in the functional form of the marginal utilities of consumption, the previous equation can be rewritten as

$$Q_{Nt} = \beta^N \mathbb{E}_t \left[ \frac{F_{Nt}}{\prod_{j=1}^N G_{t+j}^{\theta} \Pi_{t+j}} \right] \quad (5)$$

with  $G_{t+1} = C_{t+1}/C_t$  being consumption growth and  $\Pi_{t+1} = P_{t+1}/P_t$  being inflation. Note that the Stochastic Discount Factor (SDF), and therefore the bond price, decreases in expected consumption growth and inflation. Since the bond is a mechanism to transfer consumption to the future, there are fewer incentives to buy the bond if consumption is expected to be high. Higher expected inflation diminishes the real expected value of the bond.

Following the conventional approach in asset pricing research, I will examine this equilibrium price equation using an exogenous process for consumption.<sup>5</sup>

In every period, a disaster ( $D$ ) can happen or not ( $ND$ ). For simplicity, each disaster is considered independent from the others.  $\delta_{\tau,t}$  denotes the probability at  $t$  of a disaster happening in  $\tau$  periods s.t.

$$\delta_{\tau+1,t} = \phi_{\delta} \delta_{\tau,t} \quad (6)$$

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4. The model shows a closed economy, where all that is produced is consumed. The BIS report Fang, Hardy, and Lewis (2022) shows that the majority of government bonds are held by domestic investors, especially during crises.

5. See Cochrane (2009).

with  $\phi_\delta \in [0, 1]$  being the persistence parameter of the disaster probability. This allows expressing all disaster probabilities in terms of  $\delta_{1,t}$  since  $\delta_{\tau,t} = \phi_\delta^{\tau-1} \delta_{1,t}$ .

The law of motion of consumption growth is

$$G_{t+1} = \alpha_G G_t^{\phi_G} \varepsilon_{t+1} V_{t+1} \quad (7)$$

where  $\alpha_G$  is a constant term,  $\phi_G$  represents a persistence parameter,  $\varepsilon_{t+1} \stackrel{\text{iid}}{\sim} \log N(0, \sigma_\varepsilon^2)$  is white noise, and

$$V_{t+\tau} = \begin{cases} 1 & \text{with prob } 1 - \delta_{\tau,t} (ND) \\ J_G & \text{with prob } \delta_{\tau,t} (D) \end{cases} \quad (8)$$

Therefore, the disaster affects consumption growth through  $V_{t+1}$ . When the disaster does not occur, the process defaults to the log of consumption growth following an AR(1) process.  $J_G > 0$  represent the “jump” in consumption growth induced by the disaster. A  $J_G = 0.98$  implies that disaster lowers consumption growth by 2%.

Analogously, the process of inflation is

$$\Pi_{t+1} = \alpha_\Pi \Pi_t^{\phi_\Pi} \eta_{t+1} W_{t+1} \quad (9)$$

where  $\alpha_\Pi$  is a constant term,  $\phi_\Pi$  is the persistence parameter,  $\eta_{t+1} \stackrel{\text{iid}}{\sim} \log N(0, \sigma_\eta^2)$  is white noise, and

$$W_{t+\tau} = \begin{cases} 1 & \text{with prob } 1 - \delta_{\tau,t} (D) \\ J_\Pi & \text{with prob } \delta_{\tau,t} (ND) \end{cases} \quad (10)$$

When a disaster occurs, there is also a probability  $\gamma$  of partial default, resulting in an equal haircut across all bonds. This implies that

$$F_{Nt} = 1 \cdot \prod_{\tau=1}^N Z_{t+\tau} \quad (11)$$

with

$$Z_{t+\tau} = \begin{cases} 1 & \text{with prob } 1 - \delta_{\tau,t} (ND) \\ 1 & \text{with prob } (1 - \gamma) \delta_{\tau,t} (D \text{ but no partial default}) \\ 1 - J_F & \text{with prob } \gamma \delta_{\tau,t} (D \text{ and partial default}) \end{cases} \quad (12)$$

$J_F \in [0, 1]$  denotes the size of the haircut, being 1 full repudiation.

Note that independence between disasters implies that  $V_t \perp V_{t'}, W_{t'}, Z_{t'}$  for  $t' \neq t$ . However,  $V_t, W_t$ , and  $Z_t$  are perfectly correlated through the disaster event. Considering this, equation (2) can be rewritten

as

$$Q_{Nt} = Q_{Nt}^{ND} DW_{Nt} = Q_{Nt}^{ND} \prod_{\tau=1}^N \underbrace{(1 + \delta_{\tau,t} (J_{\tau,N} - 1))}_{DW_{\tau,Nt}} \quad (13)$$

where

$$Q_{Nt}^{ND} = \beta^N \frac{e^{\frac{1}{2}(\sum_{i=1}^N (\sum_{j=0}^{i-1} \phi_G^j)^2 \theta^2 \sigma_\varepsilon^2 + \sum_{i=1}^N (\sum_{j=0}^{i-1} \phi_\Pi^j)^2 \sigma_\eta^2)}}{\left( \alpha_G^{\sum_{i=1}^N i \phi_G^{N-i}} G_t^{\sum_{i=1}^N \phi_G^i} \right)^\theta \alpha_\Pi^{\sum_{i=1}^N i \phi_\Pi^{N-i}} \Pi_t^{\sum_{i=1}^N \phi_\Pi^i}} \quad (14)$$

and

$$J_{\tau,N} = \frac{1 - \gamma J_F}{J_G^{\sum_{j=1}^{N+1-\tau} \theta \phi_G^{j-1}} J_\Pi^{\sum_{j=1}^{N+1-\tau} \phi_\Pi^{j-1}}} \quad (15)$$

Therefore, the price of a bond is composed of its price when there are no disasters,  $Q_{Nt}^{ND}$ , times the product of the disaster wedge,  $DW_{Nt}$ .  $DW_{\tau,Nt}$  represents the specific disaster wedge from the disaster in  $\tau$  periods.  $J_{\tau,N}$  represents the overall effect of a disaster happening in  $\tau$  periods to a bond that matures in  $N$  periods.  $Q_{Nt}^{ND}$  accounts for expectations driven by the current business cycle, since it internalizes the effect of current consumption growth and inflation on the bond price.

This model provides tractable solutions that allow us to analyze how beliefs regarding disasters influence bond prices, which can be later translated to the yield curve. Ignore for a moment that the disaster probabilities are connected through Equation 6. An increase in  $\delta_{\tau,t}$  increases prices if and only if  $J_{\tau,N} < 1$ , which implies

$$\log(1 - \gamma J_F) - \log(J_\Pi) \sum_{\tau=1}^{N+1-\tau} \phi_\Pi^{\tau-1} - \theta \log(J_G) \sum_{\tau=1}^{N+1-\tau} \phi_G^{\tau-1} < 0 \quad (16)$$

This means that bond prices increase with the likelihood of disaster when the overall effect of the disaster has a nature that increases the SDF. The overall effect depends on the magnitude of the jumps, but also on the persistencies of the fundamentals. So short-term and long-term bonds may move in opposite directions. Note also that the summation in the exponent of the  $J_G$  and  $J_\Pi$  includes more components as the difference between the period when the disaster occurs ( $\tau$ ) and when it matures ( $N$ ) increases. This implies that long-term bonds accumulated more effects due to the persistence of the fundamentals' process.

Accounting for the persistence of the disaster probabilities,

**Proposition 1**  $Q_{Nt}$  is increasing in  $\delta_{1,t}$  if and only if

$$\sum_{\tau=1}^N \frac{\phi_\delta^{\tau-1} (J_{\tau,N} - 1)}{1 + \phi_\delta^{\tau-1} (J_{\tau,N} - 1)} > 0 \quad (17)$$

The proof is in the Appendix A. This implies that the effect also hinges on  $\phi_\delta$ .

Finally,

$$Y_{Nt} = \frac{1}{Q_{Nt}^{1/N}} - 1 \quad (18)$$

represents the bond's yield. This equation allows us to compute the yield curve and subsequently derive observed prices from the yield curve data.

### 3 Calibration and preliminary results

Before estimating the disaster probabilities, it is valuable to explore the baseline calibration of the model, which allows for deriving comparative statics exercises and theoretical insights related to the disaster literature. The empirical analysis is conducted using a panel of 50 countries for the period 2000-2023.

#### 3.1 Utility Function Parameters

I derive the utility function parameters from established literature. Following the methodology posited by Barro (2006), I set the discount factor,  $\beta$ , to 0.97 per year, and the coefficient of relative risk aversion,  $\theta$ , to 4, which are common to all the countries.

#### 3.2 Law of Motion of Fundamentals

The laws of motion of consumption growth and inflation are country-specific. To estimate them, I use annual data from the World Bank's World Development Indicators (WB/WDI) from 1989 to 2021. By taking logs on Equation 7 and 9, the law motions are transformed into a linear equation. In the absence of disaster shocks, these equations represent an AR(1) process. Using OLS on countries' time series, I estimate the constant parameters ( $\alpha_G$  and  $\alpha_{\Pi}$ ), the persistence parameters ( $\phi_G$  and  $\phi_{\Pi}$ ), and the standard deviations ( $\sigma_{\varepsilon}$  and  $\sigma_{\eta}$ ), for each country.

#### 3.3 Disaster Parameters

The disaster parameters to be calibrated are  $J_G$ ,  $J_{\Pi}$ ,  $\gamma$ , and  $J_F$ . These parameters are calibrated specifically for each type of disaster. I define three distinct types of disasters: consumption disaster, inter-state war, and sovereign default.

The consumption disaster is widely used in macroeconomic literature to explain asset pricing puzzles (Rietz 1988; Barro 2006; Gabaix 2012; Barro and Liao 2021). It represents a severe drop in consumption growth, which also carries default risk. Following Barro (2006), I set  $J_G$  to 0.71,  $\gamma$  to 0.4, and  $J_F$  to 0.29.

For inter-state wars, I calibrate  $J_F$  to 0.56, based on the haircut analysis from Luckner et al. (2023), which uses historical data on sovereign defaults triggered by geopolitical disasters. Given that they recorded 45 defaults resulting from 95 inter-state wars, I set  $\gamma$  to 0.5. For the jumps in consumption

growth and inflation, I conducted a two-way fixed-effects analysis using WDI/WB data.<sup>6</sup> The results are  $J_G = 0.98$  and  $J_{\Pi} = 1.02$ .

For sovereign defaults, the objective is to capture the probability of default through  $\delta_{1,t}$ . Therefore, I set  $\gamma$  to 1, so it does not impact the model.  $J_F$  is set to 0.44, as the average haircut for sovereign defaults is 56% (Meyer, Reinhart, and Trebesch 2022).

Using the calibrated values for the disaster type along with the country's law of motion parameters as described in the previous section, I can construct  $\hat{J}_{\tau,cN}$ . Table 1 provides a summary of the disaster definitions. Finally, I set  $\hat{\phi}_{\delta} = 0.5$  for all disaster types.

Table 1: Summary of Disaster Definitions and Calibrations

| Disaster Type        | $\hat{J}_G$ | $\hat{J}_{\Pi}$ | $\hat{\gamma}$ | $\hat{J}_F$ | Source  |
|----------------------|-------------|-----------------|----------------|-------------|---|
| Consumption Disaster | 0.71        | -               | 0.4            | 0.29        | Barro (2006)  |
| Inter-State War      | 0.98        | 1.02            | 0.5            | 0.56        | Von Laer & Bartels (2023) and author's calculations |
| Sovereign Default    | -           | -               | -              | 0.44        | Meyer et al. (2022)                                 |

*Notes:* The jumps in consumption growth and inflation are denoted by  $J_G$  and  $J_{\Pi}$ , respectively. The haircut for partial default is  $J_F$ . The default probability is  $\gamma$ .

*Source:* Barro (2006), Luckner et al. (2023), and Meyer, Reinhart, and Trebesch (2022) and author's calculations.

### 3.4 Comparative Statics on the US

Figure 1 shows the simulated impact of each shock on the log price curve for each disaster type, using a US calibration with  $\delta_{1,t} = 0.25$ . The figure shows that the consumption disaster has the largest impact and is the only one that raises prices. The sharp drop in consumption makes the recessionary effect outweigh default concerns, prompting individuals to invest to smooth consumption. In contrast, for inter-state war, the inflationary/default risk outweighs the recessionary effect, leading to a price drop across all maturities. Lastly, default has the strongest negative impact.

6. To match the conflict size with Luckner et al. (2023), I define war as having more than 1,000 deaths per year using UCDP data.

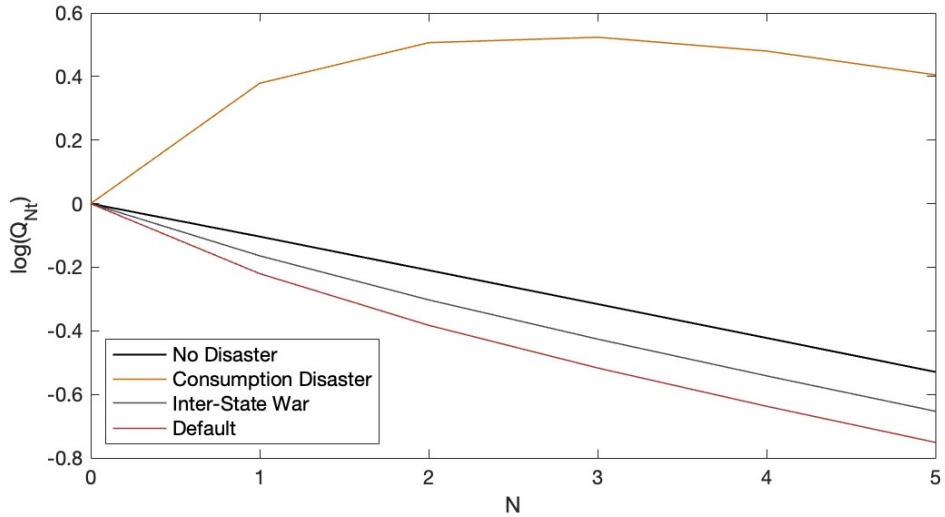


Figure 1: Comparative Statics of Disaster Shocks

Notes: The figure presents simulated price curves for each disaster type, using a calibration for the US with  $\delta_{1,t} = 0.25$ ,  $\phi_\delta = 0.5$ , and the remaining disaster parameters taken from Table 1. I set  $G_t = \Pi_t = 1.02$ . Source: Author's calculations.

### 3.5 Preliminary results

Using the calibration for the US economy,<sup>7</sup> the model captures two key phenomena: the upward-sloping yield curve puzzle and the inversion of the yield curve before recessions.

The puzzle involving the yield curve, described by Gabaix (2012), is that the nominal yield curve slopes upwards on average, with long-term yields' premium being higher than what traditional RBC models can explain. This mirrors the bond version of the equity premium puzzle noted by Campbell (2003). Following the theoretical literature on macroeconomic disasters (Barro 2006; Gabaix 2012), I set the probability of a consumption disaster to 0.02, which is constant over time. The model successfully addresses the puzzle, even without disasters. Following Gabaix (2012)'s methodology, I calculate the ratio between 5-year and 1-year bonds ( $Y_{5t}/Y_{1t}$ ). The resulting ratio is close to the 0.57% obtained in Gabaix (2012), for both non-disaster (0.4%) and disaster scenarios (0.33%).

The other phenomenon is the inversion of the yield curve. The yield curve inverts when short-term interest rates are higher than long-term rates, which is often interpreted as a signal of an upcoming recession. The slope of the yield is measured by the spread between the 3-month and 10-year bonds. Approximating the 3-month yield using an interpolation between the 0-maturity bond and the 1-year bond, the model can replicate this behavior when a sufficiently high disaster probability and a recessionary jump are introduced. With parameters set at  $J_G = 0.875$  and  $\delta_{1,t} = 1$ , the resulting spread is -1.4%.

7. For this part of the analysis, I set  $G_t = \Pi_t = 1.02$ .

## 4 Estimating Investors' Disaster Risk

The model, summarized by Equation 13, shows that government bond prices can be decomposed by a non-disaster theoretical price and a disaster wedge. To estimate disaster probabilities, I first compute the theoretical price for each country and period. Then, I use a fixed effect regression to attribute part of the difference between the observed prices and the theoretical ones to the disaster wedge. Finally, specifying the disaster type, I determine the probability of the disaster.

### 4.1 Computing Theoretical Prices

Let  $Q_{Nct}^{ND}$  represent the theoretical price of a bond that matures in  $N$  periods for country  $c$  at time  $t$ . According to Equation 14, this price is determined by the utility parameters, the parameters governing the law of motion for inflation and consumption growth, and the current levels of consumption growth and inflation. The utility parameters and the law of motion for these fundamentals are calibrated as detailed in the previous section. For the period-specific consumption growth and inflation,  $G_t$  and  $\Pi_t$ , I use data from the IMF's International Financial Statistics (IMF/IFS), which provides quarterly year-over-year rates. This allows for calculating annual maturity bond prices with quarterly updates.

Importantly, theoretical prices do not incorporate disaster risk; they reflect prices based purely on expectations driven by the current values of the economic fundamentals and their projected evolution according to the country's laws of motion.

### 4.2 Disaster Wedge Estimation

To extract the disaster wedge, I exploit the difference between the computed theoretical prices and the observed prices.

Observed prices are sourced from Datastream, which provides daily data for a wide range of countries, though there is considerable variation in both the period and maturity coverage. This results in an unbalanced panel dataset covering 50 countries from 2000 to 2023. I restrict the data to bonds with annual maturities ranging from 1 to 10 years to align with the theoretical model, as inflation and consumption growth are calculated on a year-over-year basis. Additionally, as theoretical prices are updated quarterly, I average the daily observed prices to match the quarterly frequency. For some applications, I interpolate economic data to perform the analysis at a daily frequency.

To account for the panel data structure, I introduce the country index  $c$  into Equation 13, which, given the calibration of disaster parameters, can be rewritten as:

$$q_{Nct} = q_{Nct}^{ND} + \sum_{\tau=1}^N \log \left( 1 + \hat{\phi}_\delta^{\tau-1} \delta_{1,ct} \left( \hat{J}_{\tau,cN} - 1 \right) \right) \quad (19)$$

with  $q_{Nct} = \log(Q_{Nct})$  and  $q_{Nct}^{ND} = \log(Q_{Nct}^{ND})$ .

I bring this equation to the data by estimating the following regression:

$$q_{Nct} = \beta q_{Nct}^{ND} + \chi_N + \chi_c + \chi_t + \kappa_{Nc} + \kappa_{Nt} + \kappa_{ct} + u_{Nct} \quad (20)$$

where  $\chi_N, \chi_c, \chi_t$  are fixed effects for maturity, country, and time, respectively.  $\kappa_{Nc}, \kappa_{Nt}$  and  $\kappa_{ct}$  are interaction terms, and  $u_{Nct}$  is the error term. The interpretation of this equation is that observed prices can be explained by the theoretical price, which accounts for expectations driven by the current business cycle, plus a set of unobserved factors that vary at different levels. Table 2 presents the regression results for the full sample and when applied exclusively to the USA time series.

Table 2: Fixed Effect Regression

|                                       | Observed Price ( $q_{Nct}$ ) | USA Observed Price ( $q_{NUSA}$ ) |
|---------------------------------------|------------------------------|-----------------------------------|
| Non-Disaster Price ( $q_{Nct}^{ND}$ ) | 0.099***<br>(0.010)          | 1.388***<br>(0.524)               |
| Country FE                            | Y                            | -                                 |
| Maturity FE                           | Y                            | Y                                 |
| Time FE                               | Y                            | Y                                 |
| Country-Time FE                       | Y                            | -                                 |
| Country-Maturity FE                   | Y                            | -                                 |
| Maturity-Time FE                      | Y                            | -                                 |
| Observations                          | 26,811                       | 528                               |
| Adjusted R <sup>2</sup>               | 0.972                        | 0.885                             |

Note: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Source: Author's calculations using IMF and Datastream data.

The first testable hypothesis is on the model's ability to explain observed bond prices. If the model perfectly captures the data-generating process of bond prices, the estimated  $\beta$  should be close to 1. In both specifications, the estimated  $\beta$  is positive and significant, indicating that the theoretical price moves in the expected direction. For the full sample with all fixed effects,  $\hat{\beta} = 0.1$ , which is notably far from 1, justifying the inclusion of additional fixed effects to capture unobserved factors not accounted for by the theoretical model. However, given that bond prices are typically close to 1, a  $\hat{\beta}$  of 0.1 does not imply major deviations. In the USA-only specification,  $\hat{\beta} = 1.388$ , which is closer to 1, but the adjusted R<sup>2</sup> is substantially lower than in the full sample. Another observation is that the coefficient of the fixed effect for maturity decreases over the horizon, indicating that the theoretical model tends to overestimate prices more for long-term bonds.

Given  $\delta_{1,ct}$  varies at the country-time level, it is captured in  $\kappa_{ct}$  as the common unobserved factor at  $ct$  level. Its interpretation is as follows: if  $\hat{\kappa}_{ct}$  is significantly positive, it indicates that there is an unobserved factor at the country-time level causing bond prices to be higher than what the current business cycle alone would suggest. Conversely, a significantly negative  $\hat{\kappa}_{ct}$  implies that this unobserved factor is reducing bond prices. I assume this unobserved factor is disaster risk. Specifically, it is the average

effect of disaster risk across all maturities, i.e.,

$$\hat{\kappa}_{ct} = \frac{\sum_{N \in \mathcal{N}(c,t)} q_{Nct} - \hat{\beta} \hat{q}_{Nct}^{ND} - \hat{\chi}_N - \hat{\chi}_c - \hat{\chi}_t - \hat{\kappa}_{Nc} - \hat{\kappa}_{Nt}}{|\mathcal{N}(c,t)|} \quad (21)$$

where  $\mathcal{N}(c,t)$  is the set of maturities available for country  $c$  at time  $t$ , and  $|\mathcal{N}(c,t)|$  is the number of them. Figure 2 plots the evolution of  $\hat{\kappa}_{ct}$  for Switzerland, Spain, Greece, Ireland, Italy, Portugal, Ukraine, and the USA with a 95% confidence interval. The red line at -0.1 represents a reference threshold. While the US and Switzerland managed to keep the disaster risk above the threshold, the other European countries failed during the European debt crisis, and Ukraine when the war started. To see the evolution of  $\hat{\kappa}_{ct}$  overtime for the rest of the countries, see Figure A1, A2, A3, and A4 in the Appendix B.

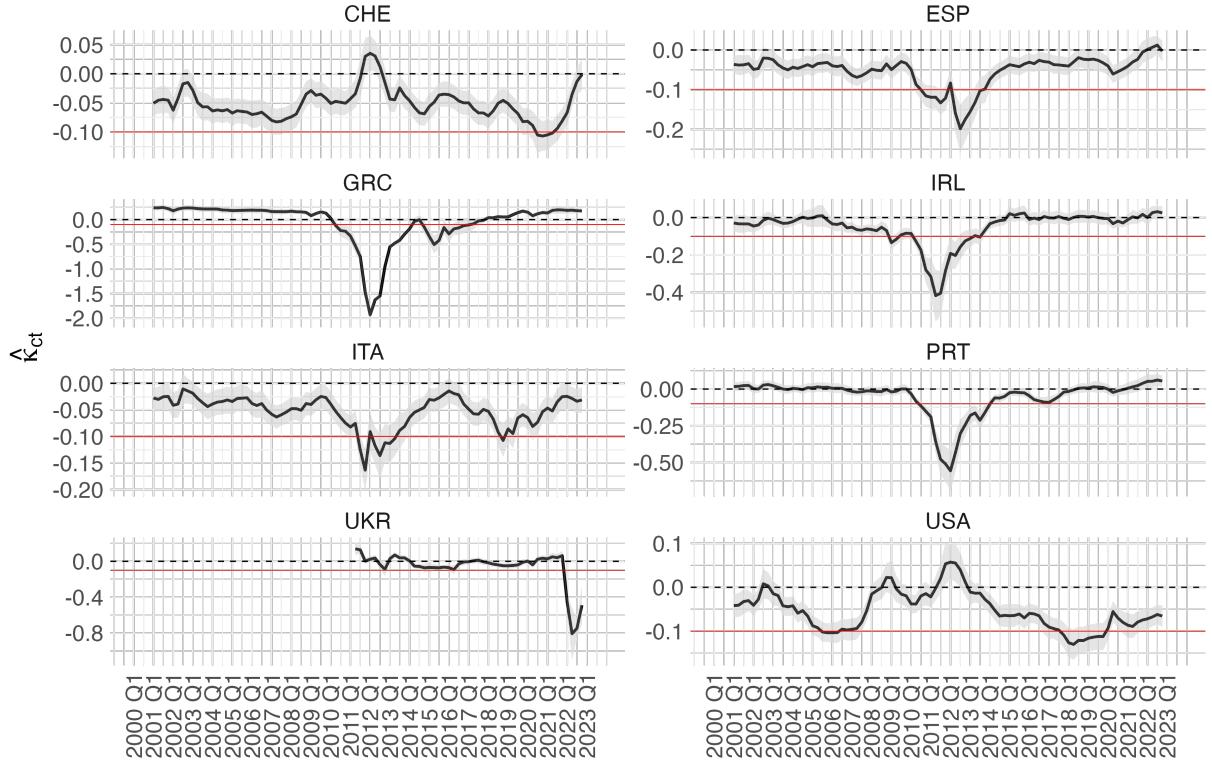


Figure 2:  $\hat{\kappa}_{ct}$  for Selected Countries

*Notes:* The figure shows the disaster wedge with 95% confidence intervals for the selected countries (Spain, Greece, Italy, Portugal, Ukraine, and the USA). The red line at -0.1 represents a reference threshold. The values are estimated based on quarterly data.

*Source:* Author's calculations using IMF and Datastream data.

Given that the elements in the summation are the specific impact on each maturity, the effect of disaster risk on each maturity,  $\hat{\kappa}_{ct(N)}$ , is given by

$$\hat{\kappa}_{ct(N)} = q_{Nct} - \hat{\beta} \hat{q}_{Nct}^{ND} - \hat{\chi}_N - \hat{\chi}_c - \hat{\chi}_t - \hat{\kappa}_{Nc} - \hat{\kappa}_{Nt} \quad \forall N \in \mathcal{N}(c,t) \quad (22)$$

According to the assumption that  $\hat{\kappa}_{ct}$  is the disaster wedge, the theoretical form of the disaster wedge is linked to the estimation through

$$\hat{\kappa}_{ct(N)} = \sum_{\tau=1}^N \log \left( 1 + \hat{\phi}_\delta^{\tau-1} \delta_{1,ct} \left( \hat{J}_{\tau,cN} - 1 \right) \right) \quad \forall N \in \mathcal{N}(c, t) \quad (23)$$

### 4.3 Likelihood of disaster

Equation 23 represents a system of  $|\mathcal{N}(c, t)|$  with  $\delta_{1,ct}$  being the unknown. I estimate it by minimizing the square distance between  $\hat{\kappa}_{ct(N)}$  and the disaster wedge, i.e.,

$$\hat{\delta}_{1,ct} = \underset{\delta_{1,ct}}{\operatorname{argmin}} \sum_{N=1}^{\mathcal{N}(c,t)} \left( \hat{\kappa}_{ct(N)} - \sum_{\tau=1}^N \log \left( 1 + \hat{\phi}_\delta^{\tau-1} \delta_{1,ct} \left( \hat{J}_{\tau,cN} - 1 \right) \right) \right)^2 \quad (24)$$

Note that the disaster probability is estimated based on a specific disaster effect calibration. If two different disasters have a similar effect on bond prices, the model cannot differentiate between them. Therefore, identifying the type of disaster ultimately relies on the country's specific context. To enhance this identification process, an NLP (Natural Language Processing) model could be used in parallel to analyze news articles, reports, and other text data, providing insights into the specific events of concern for each country and moment in time.

## 5 Results

Based on the yield curve model estimation process detailed in the previous section, I can now estimate the disaster probabilities for each country and period for a given disaster type. This allows us to draw several key insights.

### 5.1 Non-anticipation of the Ukrainian-Russian War

Financial markets are well-suited to reveal true perceptions of war risk, as they aggregate the opinions of participants who have a financial incentive to estimate risks accurately. However, while existing literature shows that market fluctuations can serve as early warning signals (Schneider and Troeger 2006), the accuracy of investors' war predictions has only recently gained attention.

I estimated the probability of an inter-state war in Ukraine. Figure 3 illustrates the evolution of the estimated probability from December 2021 to March 2022, with key events marked by dashed vertical lines. The figure reveals that, despite years of geopolitical tensions, investors largely did not anticipate the likelihood of an inter-state war two months before the invasion, and only showed some concern in the last month and a half. Even after the Belarus military drills on February 10, and Russia's recognition of the independence of Donetsk and Luhansk on February 21, investors' perceptions of an imminent

conflict remained relatively low. It was only after the invasion began on February 24, that the estimated probability of an inter-state war rose sharply, indicating a rapid adjustment in investor expectations in response to the unfolding events.

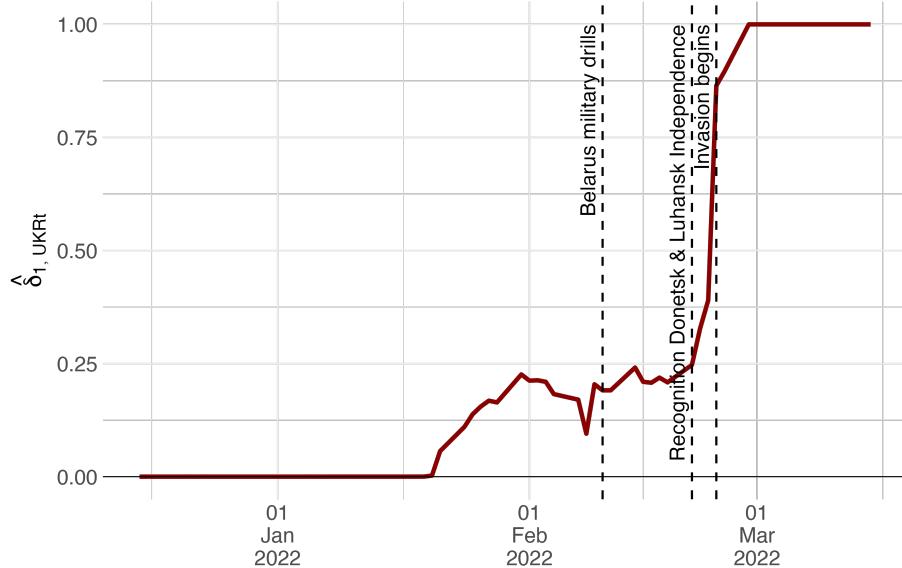


Figure 3: Evolution of Probability of Inter-State War

*Notes:* The figure shows the evolution of the probability of an inter-state war for Ukraine. The values are estimated by interpolating economic quarterly data.

*Source:* Author's calculations using IMF and Datastream data.

This finding indicates that the market largely failed to anticipate the outbreak of the Ukrainian-Russian war, as demonstrated by the model's estimation of a low probability before the conflict and the strong reaction that followed its onset. If financial markets accurately anticipated the onset of war, there would be no strong reaction once it starts because the risk would have already been priced in. The results are consistent with Chadeaux (2017), who conducted a reduced-form analysis using government bond data and found that financial markets tend to underestimate the risk of war based on a strong yield reaction. This model extends the analysis by not just examining yield reactions, but by estimating the underlying investors' probability of war, accounting for the business cycle through country-specific laws of motion and other factors using fixed effects.

## 5.2 The Impact of Mario Draghi's "Whatever it Takes" Speech

After the 2008 recession, the Eurozone faced a severe sovereign debt crisis that threatened the stability of the currency union, particularly in Spain, Italy, Portugal, Greece, and Ireland (also known as the PIIGS). These countries underwent multiple rounds of bailouts and austerity measures imposed by the IMF, the ECB, and the European Commission. I particularly analyze the case of Spain and Italy, which were in a different context than Portugal, Greece, and Ireland since they did not have an IMF bailout.

Figure 4 shows the evolution of the estimated probability of default for Spain and Italy from Greece's debt restructuring in March 2012 to January 2013. Several policy actions were taken to reduce this risk. On June 9, 2012, Spain requested a bank bailout to recapitalize its banking sector, aiming to contain the crisis and prevent further contagion. Later, on June 29, the Eurozone leaders agreed to establish the European Stability Mechanism (ESM) to provide financial assistance and stabilize the banking sector across the region. While these actions provided short-term relief, they did not reverse the overall upward trend in perceived default risk, which continued to rise. The risk reached its peak just before Draghi's speech on July 26, 2012. After his commitment to do "whatever it takes" to preserve the euro, the perceived risk dropped sharply, reversing the previously upward trend into a downward one. Finally, the ECB's announcement of the Outright Monetary Transactions (OMT) program on September 6, 2012, which enabled the ECB to purchase unlimited short-term government bonds, further reduced default probabilities and led to a sustained decline in risk perceptions for both Spain and Italy.

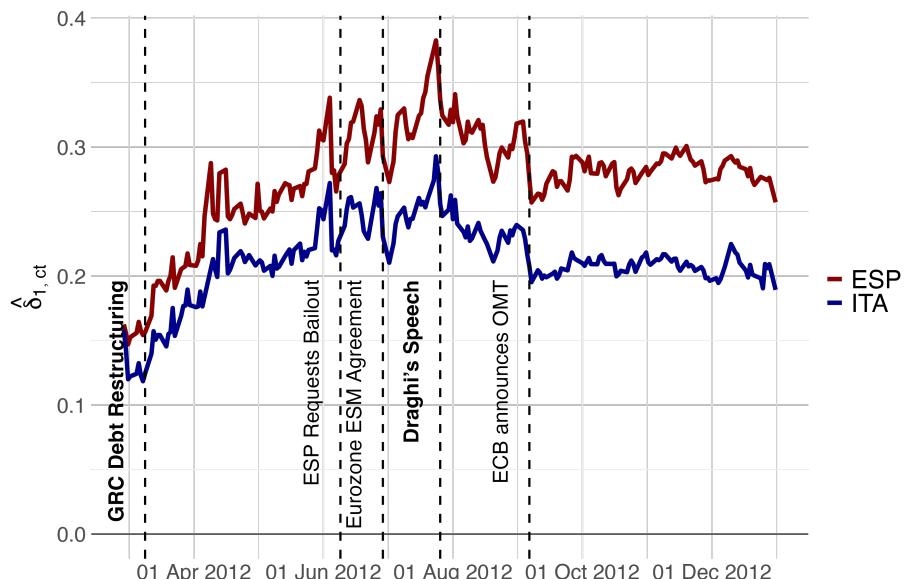


Figure 4: Evolution of Probability of Default for Spain and Italy

Notes: The figure shows the evolution of the probability of default for Spain and Italy. The values are estimated by interpolating economic quarterly data. Vertical lines represent relevant events.

Source: Author's calculations using IMF and Datastream data.

This analysis is consistent with the findings of Leombroni et al. (2021), which show that central bank communication can significantly reduce yields and risk premia, especially in times of crisis. While their study demonstrates this effect through shifts in risk premia, my analysis captures it by estimating the perceived probability of default, offering a more direct measure of investor sentiment and expectations.

### 5.3 COVID-19, a disaster of default risk

The COVID-19 pandemic significantly impacted global financial markets, creating uncertainty in bond prices that could either decrease or increase. One mechanism through which the crisis could reduce bond prices involves heightened government default risks, as fiscal pressures intensify with increased spending to manage the pandemic and reduced tax revenues. Additionally, inflationary concerns might arise if government stimulus leads to an oversupply of money, prompting investors to demand higher yields to offset expected losses in purchasing power, further depressing bond prices. On the other hand, the pandemic might trigger a recessionary shock that increases asset prices. Investors, anticipating low economic growth and limited consumption opportunities, may flock to the safety of government bonds. This surge in demand, coupled with increased savings channeled into securities, could drive up their prices.

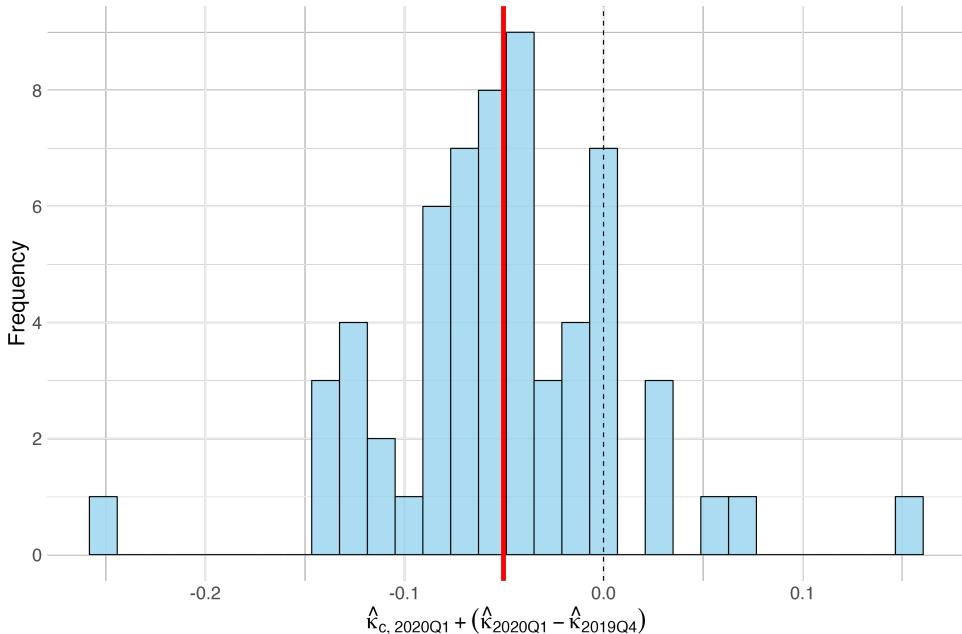


Figure 5: Distribution of Disaster Risk due to Covid-19

Notes: The figure shows the distribution of disaster risk due to COVID-19 for all countries in the sample. The values are estimated based on quarterly data. The red line at -0.05 represents the mean value.

Source: Author's calculations using IMF and Datastream data.

Figure 5 displays the distribution of estimated disaster risk across all countries during the first quarter of the COVID-19 pandemic (2020Q1). Given that the pandemic was a global event, its impact is likely reflected in the estimated time-fixed effect ( $\hat{\kappa}_t$ ) instead of in the country-time fixed effect ( $\hat{\kappa}_{ct}$ ). Consequently, I have adjusted the model to include the difference in time fixed effects between the pre-pandemic period and the onset of the pandemic, i.e.,  $\hat{\kappa}_{2020Q1} - \hat{\kappa}_{2019Q4}$ , with a minor positive correction factor of 0.0016. The mean disaster risk level, marked by a red line at -0.05, indicates that the COVID-19 pandemic was a shock that reduced bond prices, reflecting heightened default risk and inflationary con-

cerns. However, the sample shows significant variation, with 10 countries experiencing a disaster risk effect below -0.1, while a small subset showed an increase in bond prices.

My findings align with Arellano, Bai, and Mihalache (2024), which emphasizes the significant increase in default risk during pandemic crises. However, my results suggest that the COVID-19 pandemic was perceived as a less severe default shock compared to the more pronounced impacts identified in their analysis.

## 6 Conclusions

This paper presents a novel method to extract disaster risk from yield curve data, providing a structured approach to analyze investor expectations regarding macroeconomic shocks, such as wars, financial crises, and pandemics. By integrating a classic asset pricing model with time-varying disaster probabilities and using high-frequency yield curve data, the model captures how changes in investor expectations about consumption growth, inflation, and sovereign default risk influence bond prices.

The application of the model to a diverse panel of 50 countries over two decades reveals several key insights into investor behavior. Firstly, the model demonstrates that financial markets largely failed to anticipate the Russian-Ukrainian war, with disaster probabilities only spiking after the invasion began. This highlights the inherent challenges of predicting geopolitical risks in financial markets, despite years of rising tensions. Secondly, the model captures the significant impact of Mario Draghi's "whatever it takes" speech in 2012 on reducing perceived default risk in Southern European countries. The rapid decline in estimated disaster probabilities following this speech underscores the profound effect that central bank communication and credible policy commitments can have on investor sentiment, especially during periods of financial instability. Finally, the analysis shows that the COVID-19 pandemic led to a marked increase in perceived default risk across many countries, as reflected by a general decline in bond prices. While the pandemic's impact was less severe than that of previous crises, such as the European debt crisis, it nonetheless highlighted the vulnerability of sovereign debt markets to sudden and unexpected shocks.

Overall, this study contributes to the literature on disaster risk and financial markets by offering a theoretically grounded and empirically robust method for estimating investor expectations of disaster probabilities. By incorporating both macroeconomic fundamentals and disaster probabilities into a unified framework, the model provides a valuable tool for understanding how investors respond to extreme events. It also offers policymakers insights into how their actions, such as communication strategies and fiscal measures, can shape market perceptions during crises.

Further research could expand the model by incorporating additional asset classes, such as equities and corporate bonds, and developing a general equilibrium framework to more accurately capture the data-generating process, thereby improving the theoretical price estimations and enhancing identifica-

tion. Moreover, it would be beneficial to refine the model by addressing potential measurement errors in the estimation process and accounting for heterogeneity in disaster impacts across different countries and events. For instance, the severity of a conflict could be influenced by its outcome, which might be better captured through a contest function that reflects the varying impacts on economies depending on the results of the war. Additionally, integrating an NLP model to analyze news reports and text data could help identify not only the type of disaster but also its severity, allowing for a more precise estimation of disaster risk probabilities. This multi-faceted approach would improve the model's ability to differentiate between various disaster scenarios and their potential effects on investor expectations.

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## Appendices

### A Derivations and Proofs

#### A.1 Derivation of the Price Equation

Take 1-year bond, then

$$\begin{aligned}
Q_t^{(1)} &= \beta \mathbb{E}_t \left[ \frac{1}{G_{t+1}^\theta \Pi_{t+1}} \right] = \beta \mathbb{E}_t \left[ \frac{1}{(C_G G_t^{\phi_G} \varepsilon_{t+1} V_{t+1})^\theta C_\Pi \Pi_t^{\phi_\Pi} \eta_{t+1} W_{t+1}} \right] \\
&= \beta \frac{1}{(C_G G_t^{\phi_G})^\theta C_\Pi \Pi_t^{\phi_\Pi}} \mathbb{E}_t \left[ \frac{1}{\varepsilon_{t+1}^\theta} \right] \mathbb{E}_t \left[ \frac{1}{\eta_{t+1}} \right] \mathbb{E}_t \left[ \frac{1}{V_{t+1}^\theta W_{t+1}} \right] \\
&= \beta \frac{1}{(C_G G_t^{\phi_G})^\theta C_\Pi \Pi_t^{\phi_\Pi}} \mathbb{E}_t \left[ e^{-\log(\theta \varepsilon_{t+1})} \right] \mathbb{E}_t \left[ e^{-\log(\eta_{t+1})} \right] \left( 1 - \delta_{t+1} + \delta_{t+1} \frac{1}{J_G^\theta J_\Pi} \right) \\
&= \beta \frac{1}{(C_G G_t^{\phi_G})^\theta C_\Pi \Pi_t^{\phi_\Pi}} e^{\frac{1}{2}((\theta \sigma_\varepsilon)^2 + \sigma_\eta^2)} \left( 1 + \delta_{t+1} \left( \frac{1}{J_G^\theta J_\Pi} - 1 \right) \right)
\end{aligned} \tag{25}$$

Take 2-year bond, then

$$\begin{aligned}
Q_t^{(2)} &= \beta^2 \mathbb{E}_t \left[ \frac{1}{G_{t+1}^\theta G_{t+2}^\theta \Pi_{t+1} \Pi_{t+2}} \right] \\
&= \beta^2 \mathbb{E}_t \left[ \frac{1}{G_{t+1}^\theta (C_G G_{t+1}^{\phi_G} \varepsilon_{t+2} V_{t+2})^\theta \Pi_{t+1} (C_\Pi \Pi_{t+1}^{\phi_\Pi} \eta_{t+2} W_{t+2})} \right] \\
&= \beta^2 \frac{1}{(C_G^{2+\phi_G} G_t^{\phi_G+\phi_G^2})^\theta C_\Pi^{2+\phi_\Pi} \Pi_t^{\phi_\Pi+\phi_\Pi^2}} \mathbb{E}_t \left[ \frac{1}{\varepsilon_{t+1}^{(1+\phi_G)\theta}} \right] \mathbb{E}_t \left[ \frac{1}{\varepsilon_{t+2}^\theta} \right] \mathbb{E}_t \left[ \frac{1}{\eta_{t+1}^{1+\phi_\Pi}} \right] \mathbb{E}_t \left[ \frac{1}{\eta_{t+2}} \right] \mathbb{E}_t \left[ \frac{1}{V_{t+2}^\theta W_{t+2}} \right] \\
&= \beta^2 \frac{1}{(C_G^{2+\phi_G} G_t^{\phi_G+\phi_G^2})^\theta C_\Pi^{2+\phi_\Pi} \Pi_t^{\phi_\Pi+\phi_\Pi^2}} e^{\frac{1}{2}((1+(1+\phi_G)^2)\theta^2 \sigma_\varepsilon^2 + (1+(1+\phi_\Pi)^2)\sigma_\eta^2)} \\
&\quad \left( 1 - \delta_{t+1} + \delta_{t+1} \frac{1}{J_G^{(1+\phi_G)\theta} J_\Pi^{1+\phi_\Pi}} \right) \left( 1 - \delta_{t+2} + \delta_{t+2} \frac{1}{J_G^\theta J_\Pi} \right)
\end{aligned} \tag{26}$$

Take 3-year bond, then

$$\begin{aligned}
Q_t^{(3)} &= \beta^3 \frac{1}{(C_G^{3+2\phi_G+\phi_G^2} G_t^{\phi_G+\phi_G^2+\phi_G^3})^\theta C_\Pi^{3+2\phi_\Pi+\phi_\Pi^2} \Pi_t^{\phi_\Pi+\phi_\Pi^2+\phi_\Pi^3}} \\
&\quad e^{\frac{1}{2}((1+(1+\phi_G)^2+(1+\phi_G+\phi_G^2)^2)\theta^2 \sigma_\varepsilon^2 + (1+(1+\phi_\Pi)^2+(1+\phi_\Pi+\phi_\Pi^2)^2)\sigma_\eta^2)} \\
&\quad \left( 1 - \delta_{t+1} + \delta_{t+1} \frac{1}{J_G^{(1+\phi_G+\phi_G^2)\theta} J_\Pi^{1+\phi_\Pi+\phi_\Pi^2}} \right) \left( 1 - \delta_{t+2} + \delta_{t+2} \frac{1}{J_G^{(1+\phi_G)\theta} J_\Pi^{1+\phi_\Pi}} \right) \left( 1 - \delta_{t+3} + \delta_{t+3} \frac{1}{J_G^\theta J_\Pi} \right)
\end{aligned} \tag{27}$$

Then, for the N-period bond

$$\begin{aligned}
Q_t^{(N)} = & \beta^N \frac{1}{\left( C_G^{\sum_{i=1}^N i \phi_G^{N-i}} G_t^{\sum_{i=1}^N \phi_G^i} \right)^\theta C_\Pi^{\sum_{i=1}^N i \phi_\Pi^{N-i}} \Pi_t^{\sum_{i=1}^N \phi_\Pi^i} } e^{\frac{1}{2} \left( \sum_{i=1}^N \left( \sum_{j=0}^{i-1} \phi_G^j \right)^2 \theta^2 \sigma_\varepsilon^2 + \sum_{i=1}^N \left( \sum_{j=0}^{i-1} \phi_\Pi^j \right)^2 \sigma_\eta^2 \right)} \\
& \prod_{i=1}^N \left( 1 - \delta_{t+i} + \delta_{t+i} \frac{1}{J_G^{\sum_{j=1}^{N+1-i} \theta \phi_G^{j-1}} J_\Pi^{\sum_{j=1}^{N+1-i} \phi_\Pi^{j-1}}} \right)
\end{aligned} \tag{28}$$

## A.2 Proof of Proposition 1

A

II

## B Appendix Figures

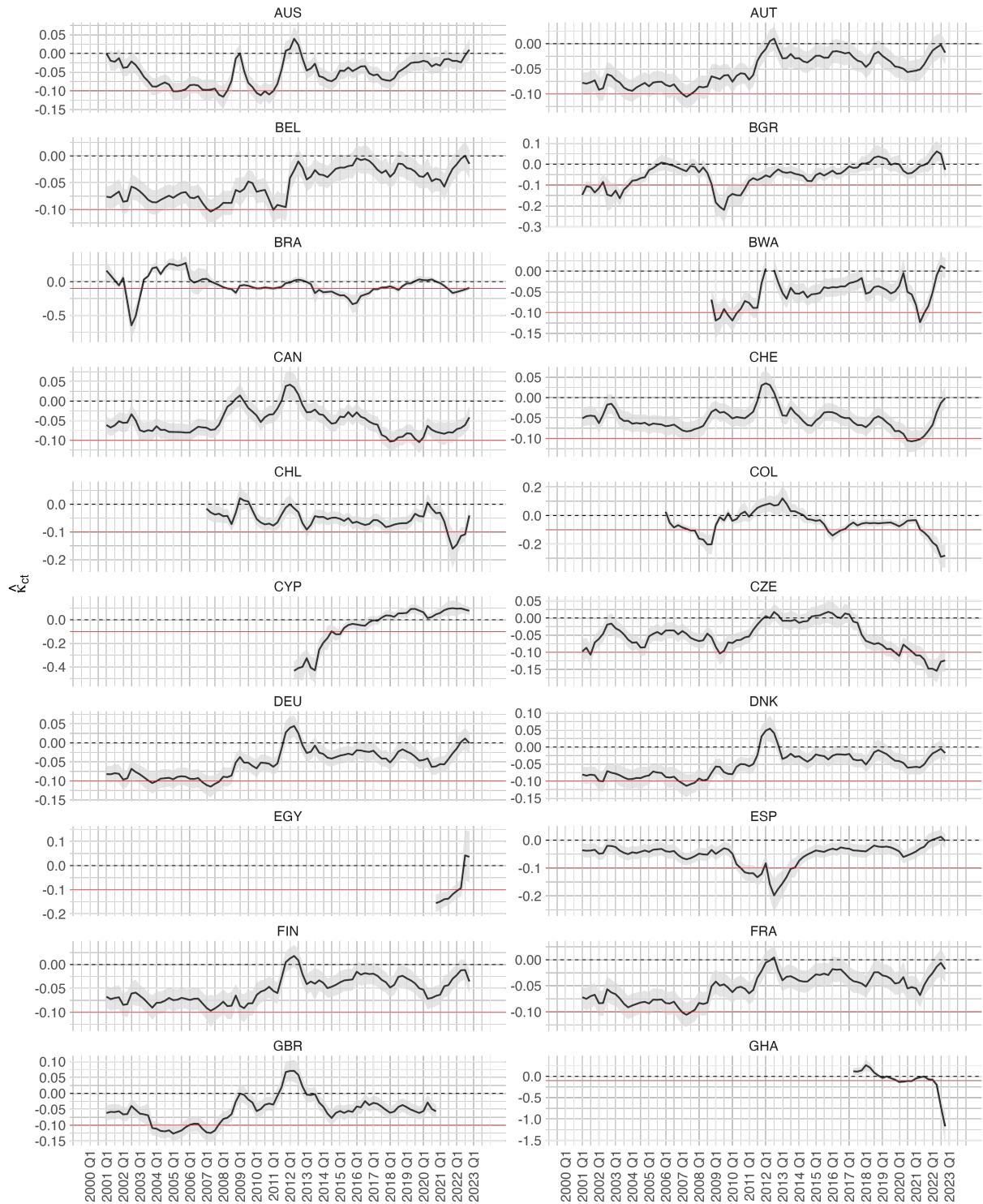


Figure A1:  $\hat{\kappa}_{ct}$

Notes: The figure shows the disaster wedge with 95% confidence intervals. The red line at -0.1 represents a reference threshold. The values are estimated based on quarterly data.

Source: Author's calculations using IMF and Datastream data.

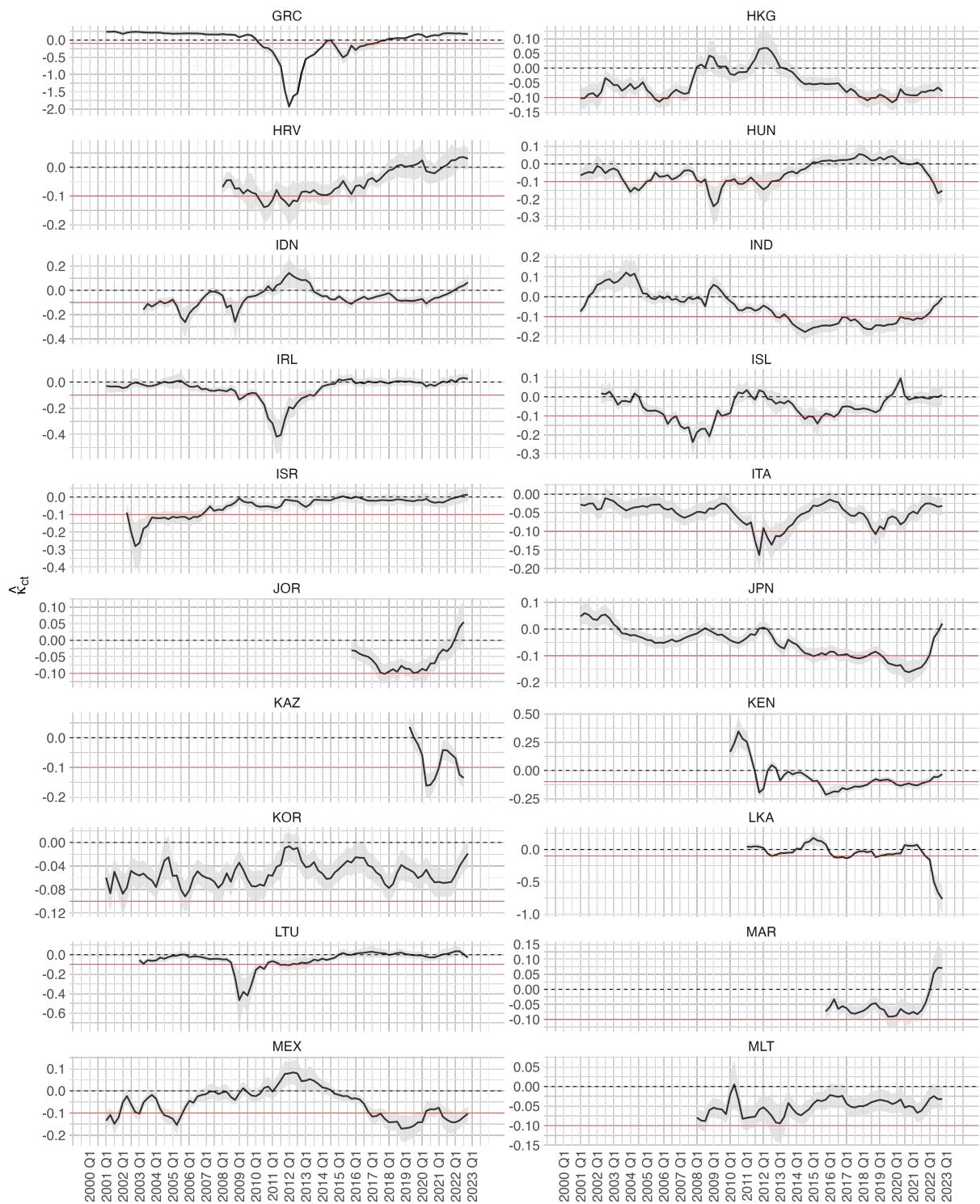


Figure A2:  $\hat{k}_{ct}$

Notes: The figure shows the disaster wedge with 95% confidence intervals. The red line at -0.1 represents a reference threshold. The values are estimated based on quarterly data.

Source: Author's calculations using IMF and Datastream data.

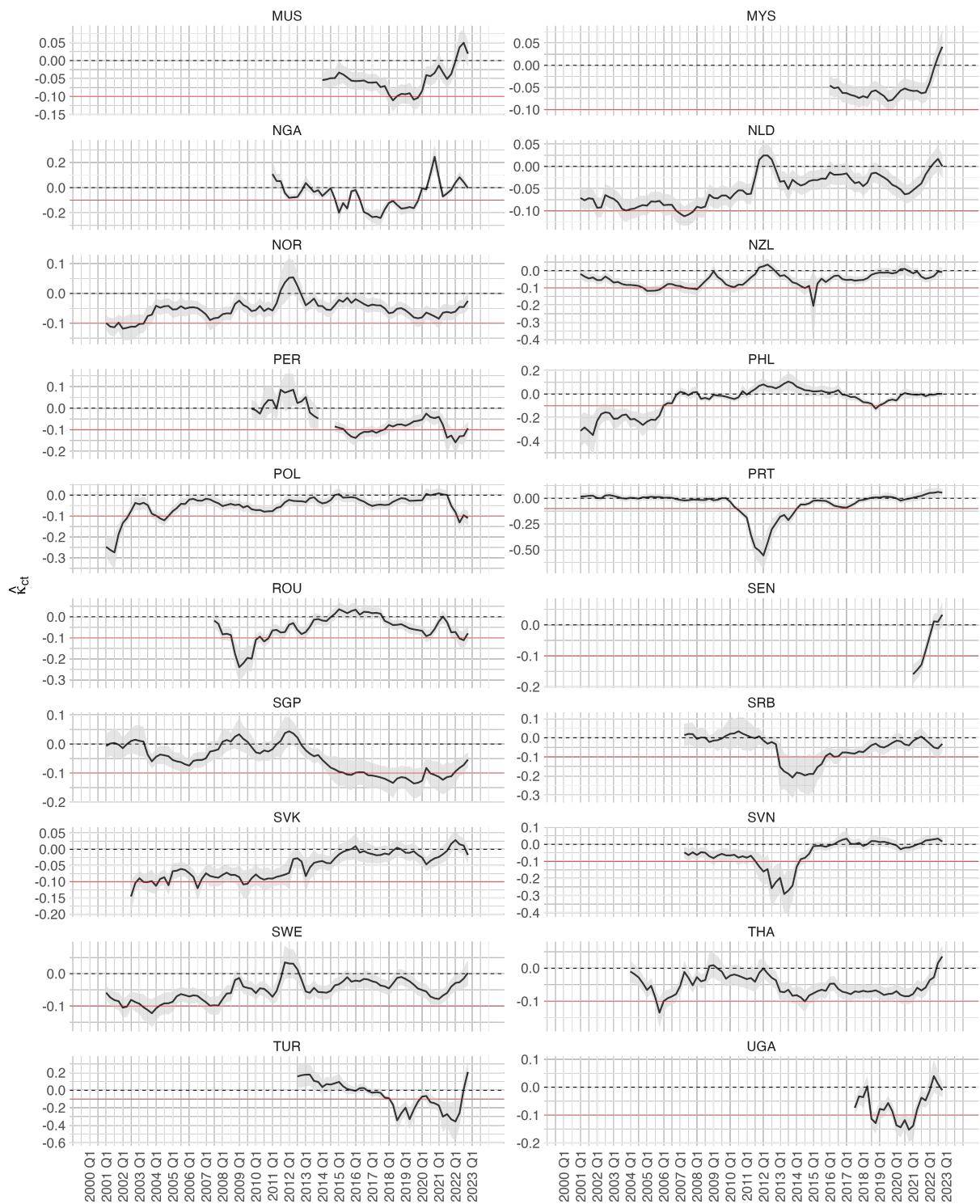


Figure A3:  $\hat{\kappa}_{ct}$

Notes: The figure shows the disaster wedge with 95% confidence intervals. The red line at -0.1 represents a reference threshold. The values are estimated based on quarterly data.

Source: Author's calculations using IMF and Datastream data.

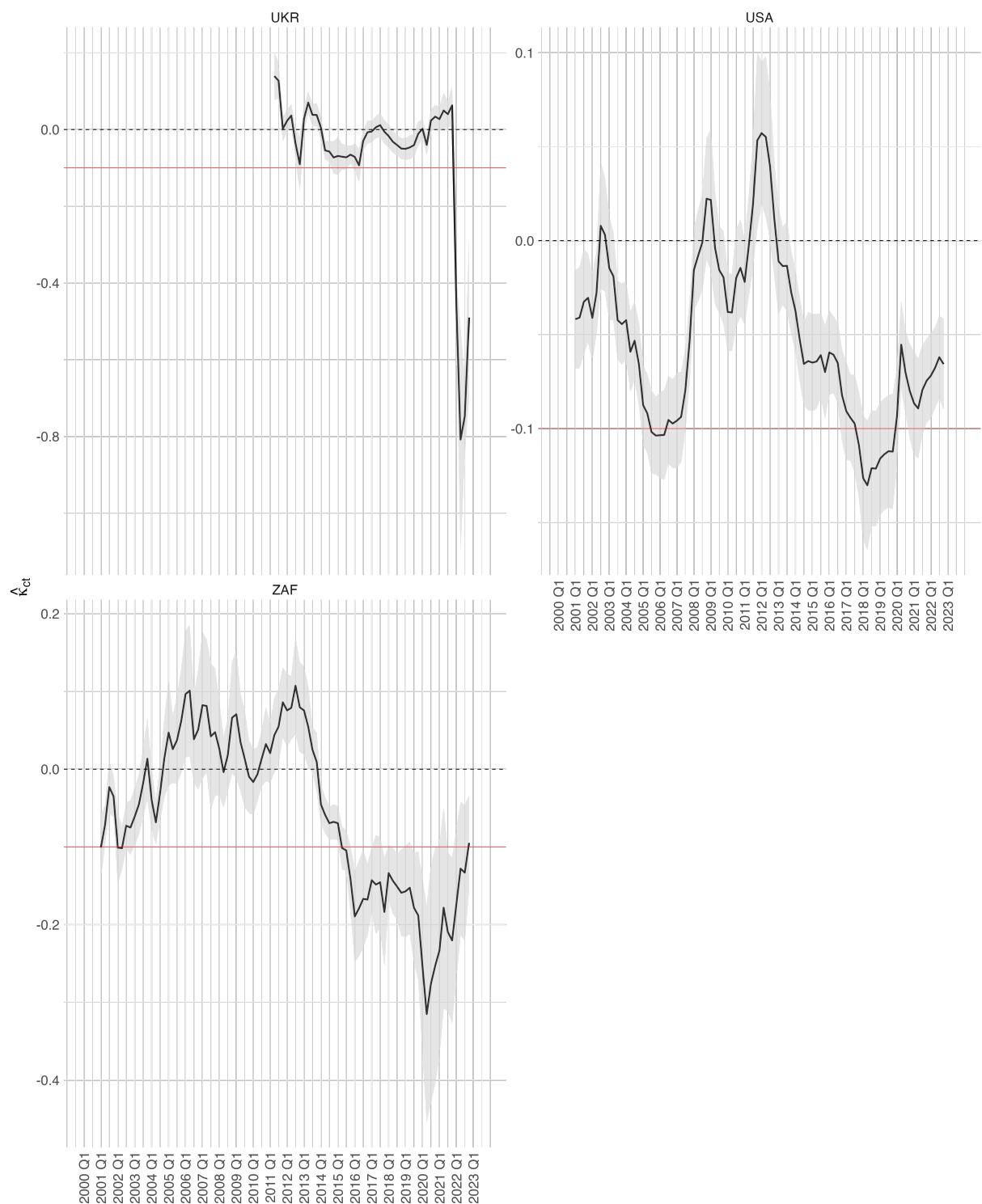


Figure A4:  $\hat{\kappa}_{ct}$

Notes: The figure shows the disaster wedge with 95% confidence intervals. The red line at -0.1 represents a reference threshold. The values are estimated based on quarterly data.

Source: Author's calculations using IMF and Datastream data.

## **C Appendix Tables**