(1) (1 Mark) Which of the following can't be a probability?

A. $-0.1$ B. 0 C. 0.001 D. $\frac{\sqrt{5}}{3}$ E. 0.999999

(2) (1 Mark) If two events (both with probability greater than 0) are mutually exclusive, then

A. They must be independent.

B. They could be independent.

C. They cannot be independent.

D. They cannot be dependent.

E. Cannot be determined from the information given.

mutually exclusive
$P(A \cap B) = 0$

independent
$P(A \cap B) = P(A) \cdot P(B)$

If A and B are independent events:

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

If A and B are mutually exclusive:

$$\boxed{P(A \cap B) = 0}$$

$\therefore P(A \cap B) = 0$ and $P(A) \cdot P(B)$
at the same time

(3) (1 Mark) Suppose that the probability of event A is 0.2 and the probability of event B is 0.4. Also, suppose that the two events are independent. Then $P(A|B)$ is:

A. 0.2 B. 0.4 C. 0.5 D. 0.08 E. Cannot be determined from the information given.

$P(A) = 0.2$
$P(B) = 0.4$

independent $\longrightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

$P(A|B) = \frac{P(A \cap B)}{}$ Conditional

$\left[ \overline{P(B)} \right]$ probability

$$= \frac{(0.2)\cdot\cancel{(0.4)}}{\cancel{(0.4)}} = 0.2$$

(4) (1 Mark) If $P(A) = 0.30$, $P(B) = 0.70$ and $P(A \cup B) = 0.60$, then $P(A \cap B)$ is:

A. 0.10  B. 0.12  C. 0.30  D. 0.40  E. 0.42

?

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.3 + 0.7 - 0.6$$

$$= 0.4$$

(5) (1 Mark) A batch of 300 containers of frozen orange juice includes 5 broken containers. Two containers are selected at random, **WITHOUT REPLACEMENT** from the batch. What is the probability that the second one selected is broken given that the first one was broken?

A. $\frac{4}{300}$  B. $\frac{5}{300}$  C. $\frac{4}{299}$  D. $\frac{5}{299}$  E. $\frac{4}{5}$

(6) (1 Mark) A printed circuit board has eight different locations in which a component can be placed. If five **IDENTICAL** components are to be placed on the board, how many different designs are possible?

A. $P_5^8$  B. $C_5^8$  C. $P_5^8$  D. $C_5^8$  E. Cannot be determined from the information given.

If they distinct, use permutations

(7) (1 Mark) A pair of fair dice is tossed. The probability of getting a total of 8 is

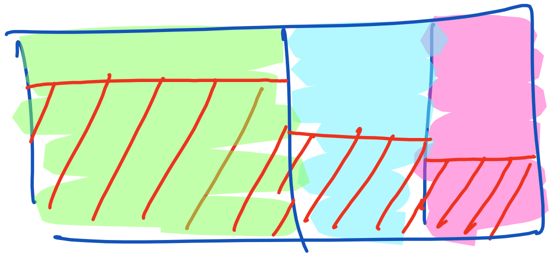
A. $\frac{3}{36}$  B. $\frac{5}{36}$  C. $\frac{6}{36}$  D. $\frac{8}{36}$  E. $\frac{12}{36}$

$S\{(2,6),(6,2),(3,5),(5,3),(4,4)\}$

(8) (1 Mark) Suppose that, in a particular city, airport A handles 50% of all airline traffic, and airport B and C handle 30% and 20% respectively. The detection rates for weapons at the three airports are 0.9, 0.5 and 0.4 respectively. If a passenger at one of the airports is found to be carrying a weapon through the boarding gate, what is the probability that the passenger is using airport A?

A. 0.12 B. 0.22 C. 0.30 D. 0.66 E. 0.68

$P(A) = 0.5$
$P(B) = 0.3$
$P(C) = 0.2$



$$\left[ P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D|A) \cdot P(A) + P(D|\bar{A}) \cdot P(\bar{A})} \right]$$ Bayes Theorem

$$= \frac{(0.9)(0.5)}{(0.9)(0.5) + [(0.5)(0.3) + (0.4)(0.2)]}$$

$= 0.6618$

Since they are not independent [they are mutually exclusive]

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{(P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A}))} = P(B)$$

(9) (1 Mark) If we choose 5 cards **without replacement** from a standard deck of 52 cards, then what is the probability that 4 of them come from the same suite and the 5th one comes from a different suite.

A. $\frac{13}{52}$ B. $(\frac{13}{52})(\frac{1}{48})$ C. $\frac{4(39)C_4^{13}}{C_5^{52}}$ D. $\frac{C_4^{13}C_1^{12}}{C_5^{52}}$ E. $\frac{13^4\ 12}{52^4}$

For one suit $\frac{13C4}{52C5}$ (probability 4 of same suit)

multiply by four, doesn't matter which suit

$52 - 13 = 39$ cards not of same suit

$$4 \cdot \frac{13C4}{52C5} \cdot 39C1$$

(10) (1 Mark) In a city, 3% of the residents believe that the property tax should be increased to build more schools in the city. A random sample of size 60 **with replacement** was chosen from the city's residents. Let $X$ be the number of residents in the sample who believe that property tax should be increased. The **approximate** distribution of $X$ and the **approximate** value of $P(X = 6)$, respectively, are

A. Binomial, 0.009  B. Normal, 0.0134  C. Normal, 0  D. Poisson, 0.0078  E. Poisson, 0.9974

$p = 0.03$   $n = 60$, with replacement

[since $n > 50$, check for binomial to poisson approximation]

$np = 60(0.03) < 5$

$\therefore$ valid approximation

$\mu = np = 1.8$   $\sigma = \sqrt{(1.8)(0.97)} = 1.321$

$$P(6) = \frac{e^{-1.8} \cdot (1.8)^6}{6!}$$

$= 0.0078$

(11) (1 Mark) A professor specified 10 questions for her class and announced that she will randomly choose 5 of them for the upcoming test. If Dave (a student in the class) already knows the answer to 8 of the questions and has no intention to study for the test, then what is the probability that he will answer 4 out of the 5 questions on the test correctly?

A. 0.92  B. 0.67  C. 0.86  D. 0.81  E. 0.56



No replacement, Hypergeometric

As opposed to replacement ↳ Binomial

$$P(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{}}$$

$m = 8 \quad N = 10$
$x = 4 \quad n = 5$

$$P(4) = \frac{\binom{8}{4}\binom{2}{1}}{\binom{10}{5}}$$

$= 0.56$

(12) (1 Mark) In the Winter in Ottawa, big potholes appear on the streets according to a poisson distribution with rate of 0.2 big potholes per 10 kilometers. Bank Street is 70 kilometers long, what is the probability that in a Winter there will be at least (inclusive) 2 big potholes on Bank Street.

A. 0.2466 B. 0.5918 C. 0.4082 D. 0.6767 E. 0.3233

$\mu = 0.2/10\,km \cdot 70\,km$
$= 1.4$

$p(x \geq 2) = 1 - p(x<2)$

$p(x \geq 2) = 1 - p(0) - p(1)$

$= 1 - 0.5918$

$= 0.4082$

$\mu$

| $x$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|---|---|---|---|---|
| 0 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 |
| $x \leq$ 1 | 0.6990 | 0.6626 | 0.6268 | $\boxed{0.5918}$ | 0.5578 |
| 2 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 |
| 3 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 |

**For QUESTIONS 13 AND 14, USE THE FOLLOWING NARRATIVE:**

A machine produces 100 yard fabric rolls. Experience has shown that this machine produces flaws in the pattern of the fabric according to a Poisson distribution with mean of 1 flaw per 50 yards.

$n = 100$ yards $\quad p = 1/50$ yards

(13) (1 Mark) What is the probability that a fabric roll produced by this machine will have at most

(inclusive) one flaw.

A. 0.4060 B. 0.1353 C. 0.2707 D. 0.3329 E. 0.6990

$\mu = 100 \cdot 1/50 = 2$

$p(x \leq 1) = p(0) + p(1)$

$= 0.4060$

(14) (1 Mark) If we choose 10 rolls produced by this machine independently, then what is the probability that 4 of them will have at most (inclusive) one flaw.

A. 0.633 B. 0.382 C. 0.9473 D. 0.251 E. 0.0902

for each roll, $p(x \leq 1) = 0.4060 = p$

binomial $n = 10$

$P(4) = \binom{10}{4} p^4 q^6 \quad (q = 1-p)$

$= 0.2506$

(15) (1 Mark) The lifetime (in years) of a particular brand of tire has the pdf $f(x) = 1/3e^{-x/3}$, when $x \geq 0$, and 0 otherwise. 80% of the tires of this brand last at most $D$ years. What is the value of $D$?

A. 0.33 years B. 0.67 years C. 3 years D. 3.8 years E. 4.83 years

$0.8 = \int_0^D \frac{1}{3} e^{-x/3} dx$

$0.8 = -e^{-x/3} \Big|_0^D$ // Don't forget negative signs

$0.8 = -e^{-D/3} - (-1)$

$0.2 = e^{-D/3}$

$-3 \ln 0.2 = D = 4.828$

**For QUESTIONS 16 AND 17, USE THE FOLLOWING NARRATIVE:**

The daily sales (in tons) of a company is a random variable whose **cdf** is

$$F(x) = \begin{cases} 0, & x < 0; \\ 6(\frac{x^2}{2} - \frac{x^3}{3}), & 0 \le x < 1; \\ 1, & x \ge 1. \end{cases}$$

(16) (1 Mark) What is the average daily sales of this company.

A. 0.6 tons per day B. 0.12 tons per day C. 0.18 tons per day D. 0.5 tons per day E. 1 ton per day

$$\mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

insignificant

$$\mu = \cancel{\int_0^1 x \cdot \left[6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\right] dx} + \int_1^{\infty} x\,dx$$

$$\simeq \int_1^{\infty} x\,dx = 1$$

(17) (1 Mark) What is the probability that their sales in a day, say, tomorrow, will be more than twice their average daily sales?

A. 0 B. 0.05 C. 1/3 D. 1/6 E. 1/12

The cdf does not allow for more than 1.

(18) (1 Mark) A machine in a production line fails to operate on 15% of the shifts. On average how many shifts (including those on which the machine fails and those on which it does not) it takes the machine to fail for the 5th time.

A. 21.22 shifts B. 22.33 shifts C. 40 shifts D. 30.22 shifts E. 33.33 shifts

How many trials (x) until r successes (failures of machine)

Negative Binomial

$r = 5 \quad p = 0.15$

$$E(x) = \frac{r(1-p)}{p} \text{ (number of failures)}$$

equation on formula sheet does not count

$$= \frac{5(0.85)}{0.15} + 5$$

0.15

$= 28.33 + 5 = 33.33$

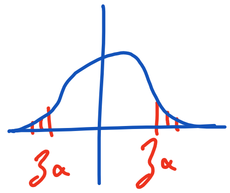
successes

(19) (1 Mark) The value of $z_{0.025}$

A. $-1.96$ B. 1.645 C. $-1.645$ D. 1.96 E. 2.56

$z_\alpha$ is critical area

$z_\alpha = 1 - \Phi(\alpha)$




$z_{0.025} = +1.96$

- find z value for 0.025
- flip sign

(20) (1 Mark) An oil company plans to drill one hole at each of 10 different locations. The probability of finding oil at each location is 0.08. Assuming that the outcomes at the locations are independent from each other, what is the probability the company will find oil in at least 2 locations?

A. 0.8121 B. 0.1879 C. 0.9599 D. 0.0401 E. 0.0782

$p = 0.08$ not on table

number of trials (independent) for x successes ↳ Binomial

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - (P(x \leq 1))$ could approximate from table ($p = 0.10$)

$= 1 - p(1) - p(0)$

$= 1 - \binom{10}{1} p^1 q^9 - \binom{10}{0} p^0 q^{10}$

$= 1 - 0.3778 - 0.4344$

$= 0.1879$

(21) (1 Mark) A quality control engineer is in charge of checking the quality of a shipment of 20 parts. To do this, he randomly selects a sample of 4 items and checks if they function properly. If all 4 selected items pass the test, he accepts the shipment, otherwise, he returns it to the supplier. If 5 of the 20 items in the shipment are actually defective, what is the probability that he accepts the shipment?

A. 0.31 B. 0.38 C. 0.28 D. 0.72 E. 0.69

$N = 20$
$n = 4$
$M = 5$
$x = 0$

No replacement
↓
Hypergeometric

$$P(0) = \frac{\binom{5}{0}\binom{15}{4}}{\binom{20}{4}}$$

$$= 0.2817$$

(22) (1 Mark) A normal random variable $X$ has $\sigma = 2$. If $P(X > 7.5) = 0.8023$, then what is the value of $\mu$ ?

A. 10.9 B. 5.8 C. 9.2 D. 8.6 E. 4.1

$$\left[P(X > 7.5) = 1 - \Phi\left(\frac{7.5-\mu}{2}\right)\right] \quad z = \frac{x-\mu}{\sigma}$$

$$1 - 0.8023 = \Phi\left(\frac{7.5-\mu}{2}\right)$$

$$0.1977 = \Phi\left(\frac{7.5-\mu}{2}\right) = \Phi(-0.86)$$

$$\rightarrow \frac{7.5-\mu}{2} = -0.86$$

$$7.5 + 1.71 = \mu$$

$$\mu = 9.21$$

(23) (1 Mark) The heights in a certain population are normally distributed with mean 160cm and variance 25cm. 95% of the population are of at most what height?

A. 155.8cm B. 165.0cm C. 168.2cm D. 169.8cm E. 209.0cm

$\mu = 160cm$ $\sigma^2 = 25cm^2$

$\rightarrow \sigma = 5cm$

$0.95 = \Phi(1.645)$

$\left[ z = \frac{x-\mu}{\sigma} \right]$

$1.645(5) + 160 = X_{max, 95\%}$

$= 168.225$

(24) (1 Mark) Suppose three companies, Company A, Company B, and company C, produce a certain type of microchips. Company A produces 50% of this microchip, Company B produces 35%, and Company C produces 15%. Suppose that the percentage of the defective microchips of this kind produced by Company A, Company B, and Company C are 20%, 15%, and 10%, respectively. If a microchip was chosen at random, what is the probability it is defective?

A. 0.1675 B. 0.45 C. 0.05 D. 0.10 E. 0.83

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

Bayes Theorem

not needed

$P(D|A) = 0.2$
$P(D|B) = 0.15$
$P(D|C) = 0.1$

$P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)$

$= 0.1675$

(25) (1 Mark) 5% of the computers produced by a manufacturer have a hardware issue. We purchased 200 computers from this manufacturer. Assume that the computers develop hardware issues independently from each other. Let $X$ be the number of computers with hardware issues amongst the 200 we purchased. Then, the **approximate** distribution of $X$ and the **approximate** probability

200 we purchased. Then, the **approximate** distribution of $X$ and the **approximate** probability that 13 of the purchased computers will have a hardware issue, respectively, are

A. Poisson, 0.01033 B. Normal, 0.01033 C. Poisson, 0.0819 D. Normal, 0.0819
E. Poisson, 5/200

$p = 0.05$
$n = 200$

$np \geq 10$ z-value ∴ Normal approximation
$nq \geq 10$

$P(X=13) = P(X \leq 13.5) - P(X \leq 12.5)$

$P(X \leq 13) = \Phi\left(\frac{13 - np}{\sqrt{npq}}\right)$

$= \Phi\left(\frac{13 - np}{\sqrt{npq}}\right) - \Phi\left(\frac{12 - np}{\sqrt{npq}}\right)$

$= \Phi\left(\frac{3}{3.08}\right)$

$= \Phi(3.5/3.08) - \Phi(2.5/3.08)$

$= \Phi(0.973)$

$= \Phi(1.14) - \Phi(0.81)$

$= 0.8729 - 0.7910$

$= 0.0819$