Problem Solving

CORE-UA 109.01, Joanna Klukowska

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your paycheck after graduation

NACE (National Association of Colleges and Employers) keeps track of average starting salaries of college graduates (BA/BS degrees) in US.

The overall **starting salary** for a Class of 2016 bachelor's degree graduate stands at **$50,359.**

Work with a partner (or two)

- Let's assume that these graduates will receive (on average) a 2% pay increase at the end of their first year and at the end of their second year.
- Calculate their average salaries after each of those years. Assume that the starting salary is $50,000 (it is easier to work with than $50,359).

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your paycheck after graduation

- Year one:
  - Calculating 2% of $50,000:

$$0.02 \times 50,000 = \frac{2}{100} \times 50,000 = \frac{2}{1} \times 500 = 1,000$$

  - salary increase: $1,000
  - new salary: $51,000

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  - new salary: $51,000

- **Is the salary increase at the end of the second year also $1,000?**

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- Year one:
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$$0.02 \times 50,000 = \frac{2}{100} \times 50,000 = \frac{2}{1} \times 500 = 1,000$$

  - salary increase: $1,000
  - new salary: $51,000

- **Is the salary increase at the end of the second year also $1,000?**

- No, because we are no longer computing 2% of $50,000, but rather 2% of $51,000!

- Year two:
  - Calculating 2% of $51,000:

$$0.02 \times 51,000 = \frac{2}{100} \times 51,000 = \frac{2}{1} \times 510 = 1,020$$

  - salary increase: $1,020
  - new salary: $52,020

your paycheck after graduation

Can we make this calculation more general?

To simplify our notation, let's establish some conventions:

- p is the percentage increase applied once a year

- S_0 is the initial salary

- S_1 is the salary at the end of year one, S_2 is the salary at the end of year two, etc.

- I_1 is the increase at the end of year one, I_2 is the increase at the end of year to, etc.

your paycheck after graduation

- p is the percentage increase applied once a year
- S_0 is the initial salary
- $S_1, S_2, \dots$ salaries at the end of year one, two, …
- $I_1, I_2, \dots$ increases at the end of year one, two, …

So far we know that the increase at the end of year 1 is

$$I_1 = S_0 \times p$$

and that the salary at the end of year 1 is

$$S_1 = S_0 + I_1$$

Can we rewrite the expression for S_1?

$$S_1 = S_0 + I_1 = S_0 + S_0 \times p = S_0 \times (1 + p)$$

your paycheck after graduation

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- S_0 is the initial salary
- $S_1, S_2, \dots$ salaries at the end of year one, two, …
- $I_1, I_2, \dots$ increases at the end of year one, two, …

How about the salary at the end of year two?

$$
\begin{aligned}
S_2 &= S_1 + I_2 \\
&= S_1 + S_1 \times p \\
&= S_1 \times (1 + p) \\
&= S_0 \times (1 + p) \times (1 + p) \\
&= S_0 \times (1 + p)^2
\end{aligned}
$$

And using our numbers we have:

$$S_2 = 50,000 \times 1.02 \times 1.02 = 52,020$$

your paycheck after graduation

$$S_1 = S_0 \times (1 + p)$$

$$S_2 = S_0 \times (1 + p) \times (1 + p)$$

What do you think the formula for S_3 might be?

How about S_{10}?

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your paycheck after graduation

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$$S_3 = S_0 \times (1 + p) \times (1 + p) \times (1 + p)$$

How about S_{10}?

$$S_{10} = S_0 \times (1 + p)^{10}$$

Now, how do we write a program that performs this computation?

Programming challenge

Write a program that prompts the user for

- their expected salary after graduation

- their expected yearly percentage increase

The program should produce a table showing the expected salary at the end of years 1, 2, 3, 4, 5, 10, 15, 20 (assuming that the percentage increase remains the same and that the user stays at the same job).

discounts and sales

- You have a preferred customer discount coupon from your favorite clothing store that gives you a 50% discount off of any item in the store.

- The store happens to also have an end of season sale and all items are being sold at a 50% discount.

- You decided to get a new shirt that was originally priced at $60.00.

- Do you get it for free after combining your discount coupon with the store sale?

Why not?

discounts and sales

- What is the discount due to the store's end of season sale?
  50% of 60.00 = 0.5 × 60.00 = 30.00

- What is the price of the shirt after the store's end of season sale discount is applied?
  60.00 - discount = 30.00

- What is the discount due to your coupon?
  50% of the price after the previous discount = 0.5 × 30.00 = 15.00

- What is the price you pay?
  60.00 − 30.00 − 15.00 = 15.00

discounts and sales

- Can we calculate the final price without computing the two discounts separately?

- Sure: we can use calculation similar to the one from pay raise calculations
  - just remember that for discounts we are going to be removing something
  - instead multiply by (1+p) for each year, we will multiply by (1-d) for each discount wher d is the discount percentage

- Since we have two discounts, we need to apply them both:

original_price × (1 - d_1)×(1 - d_2) = final_discounted_price

where d_1 and d_2 are the two discounts

Programming challenge

Write a program that prompts the user for an initial price and three discount amounts

- if there are fewer discounts the user should enter 0.0 as the discount amount,

- if the discount is 30%, the user should enter 0.3 as the discount amount.

The program should then compute the final price after the three discounts are applied.s

Something to consider: does it matter in what order the discounts are applied?