W203 Tuesday 4pm Fall 2018 HW3

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1a) What is the probability that the next customer will request regular gas and fill the tank?

$$\begin{aligned} P(R \cap F) &= P(F|R) * P(R) \\ &= 0.30 * 0.40 \\ &= 0.12 \end{aligned} \tag{1}$$

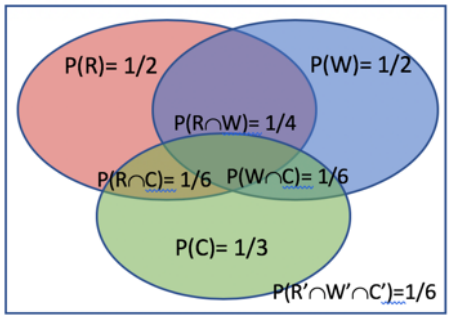
1b) What is the probability that the next customer will fill the tank?

$$\begin{aligned} P(F) &= P(R \cap F) + P(M \cap F) + P(P \cap F) \\ &= P(F|R) * P(R) + P(F|M) * P(M) + P(F|P) * P(P) \\ &= 0.3 * 0.4 + 0.6 * 0.35 + 0.5 * 0.25 \\ &= 0.455 \end{aligned} \tag{2}$$

1c) Given that the next customer fills the tank, what is the conditional probability that they use regular gas?

$$\begin{aligned} P(R|F) &= \frac{P(R \cap F)}{P(F)} \\ &= 0.12/0.455 \\ &= 0.2637 \end{aligned} \tag{3}$$

2a) Draw an area diagram to represent these events



2b) What is the probability of getting a red, waterproof, cool toy?

$$\begin{aligned} P(R \cap W \cap C) &= P(R \cup W \cup C) - P(R) - P(W) - P(C) + P(R \cap W) + P(R \cap C) + P(W \cap C) \\ &= (1 - \frac{1}{6}) - \frac{1}{2} - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{6} \\ &= \frac{1}{12} \end{aligned} \tag{4}$$

2c) You pull out a toy at random and you observe only the color, noting that it is red. Conditional on just this information, what is the probability that the toy is not cool?

$$\begin{aligned} P(\bar{C}|R) &= 1 - P(C|R) \\ &= 1 - \frac{P(C \cap R)}{P(R)} \\ &= 1 - \frac{1/6}{1/2} \\ &= \frac{2}{3} \end{aligned} \tag{5}$$

2d) Given that a randomly selected toy is red or waterproof, what is the probability that it is cool?

$$\begin{aligned} P(C|R \cup W) &= \frac{P(C \cap R \cup W)}{P(R \cup W)} \\ &= \frac{P(C \cap R) + P(C \cap W) - P(C \cap R \cap W)}{P(R) + P(W) - P(R \cap W)} \\ &= \frac{1/6 + 1/6 - 1/12}{1/2 + 1/2 - 1/4} \\ &= 1/3 \end{aligned} \tag{6}$$

3a) What are the maximum and minimum possible values for $P(A \cap B)$?
For P(A) = 1/2, P(B) = 2/3, the maximum occurs when A is a subset of B. The minimum occurs when there is minimum overlap between A and B.
Maximum = 1/2
Minimum = 1/2 + 2/3 - 1 = 1/6

3b) What are the maximum and mimim possible values for $P(A|B)$?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{7}$$

The maximum occurs when $P(A \cap B)$ is the maximum. Likewise for the minimum.

$$\begin{aligned} Maximum &= \frac{1/2}{2/3} = 3/4 \\ Minimum &= \frac{1/6}{2/3} = 1/4 \end{aligned} \tag{8}$$

4) Given that a Berkeley student likes statistics, what is the probability that they have completed w203?
Let C be the event that a student has completed W203, L be the event that the student likes Statistics.
$P(L|C) = 3/4$
$P(L|\bar{C}) = 1/4$
$P(C) = 0.01$

$$\begin{aligned} P(C|L) &= \frac{P(L|C) * P(C)}{P(L)} \\ &= \frac{P(L|C) * P(C)}{P(L|C) * P(C) + P(L|\bar{C}) * P(\bar{C})} \\ &= \frac{3/4 * 0.01}{3/4 * 0.01 + 1/4 * 0.99} \\ &= 0.0294 \end{aligned} \tag{9}$$