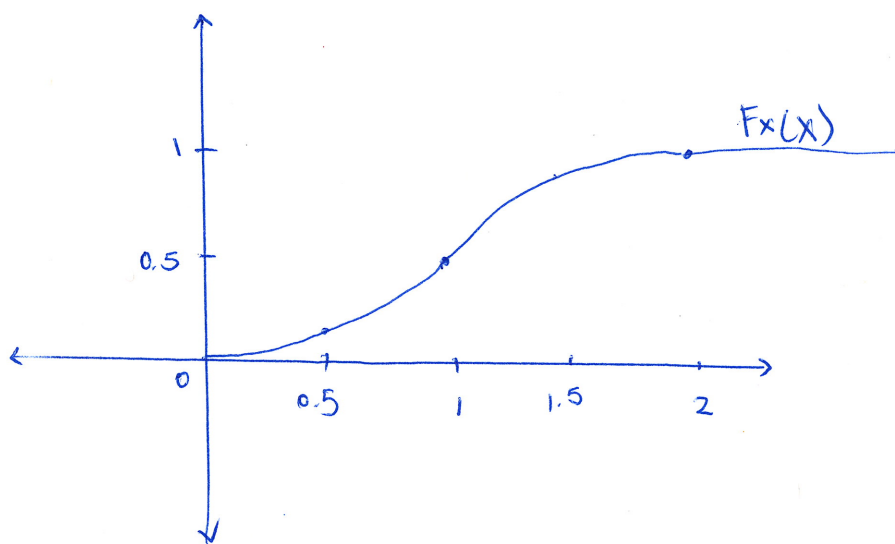


$$f_X(x) = \begin{cases} x & 0 \leq x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$a) F_X(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{2} & 0 \leq x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

$$F_X(0 \leq x < 1) = \int_0^x y \, dy = \frac{1}{2} y^2 \Big|_0^x = \frac{1}{2} x^2$$

$$\begin{aligned} F_X(1 \leq x < 2) &= \frac{1}{2} + \int_1^x (2-y) \, dy \\ &= \frac{1}{2} + \left[2y - \frac{1}{2} y^2 \right]_1^x \\ &= \frac{1}{2} + \left(2x - \frac{1}{2} x^2 \right) - \left(2 - \frac{1}{2} \right) \\ &= 2x - \frac{1}{2} x^2 - 1 \end{aligned}$$



$$b) E(x) \begin{cases} \frac{1}{3} & 0 \leq x < 1 \\ \frac{2}{3} & 1 \leq x < 2 \end{cases}$$

$$\begin{aligned} E(0 \leq x < 1) &= \int_0^1 x f(x) dx \\ &= \int_0^1 x \cdot x \cdot dx \\ &= \int_0^1 x^2 dx \\ &= \frac{1}{3} x^3 \Big|_0^1 \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} E(1 \leq x < 2) &= \int_1^2 x f(x) dx \\ &= \int_1^2 x (2 - x) dx \\ &= \int_1^2 (2x - x^2) dx \\ &= x^2 - \frac{1}{3} x^3 \Big|_1^2 \\ &= 2^2 - \frac{2^3}{3} - \left(1^2 - \frac{1}{3}\right) \\ &= \frac{2}{3} \end{aligned}$$

$$c) \text{var}(X) = \begin{cases} \frac{5}{36} & 0 \leq X < 1 \\ \frac{17}{36} & 1 \leq X < 2 \end{cases}$$

$$\sigma^2 = E(X^2) - \mu^2$$

$$\begin{aligned} \text{For } 0 \leq X < 1, \quad E(X^2) &= \int_0^1 x^2(x) dx \\ &= \int_0^1 x^3 dx \\ &= \left. \frac{x^4}{4} \right|_0^1 \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \frac{1}{4} - \left(\frac{1}{3}\right)^2 \\ &= \frac{5}{36} \end{aligned}$$

$$\begin{aligned} \text{For } 1 \leq X < 2, \quad E(X^2) &= \int_1^2 x^2(2-x) dx \\ &= \int_1^2 (2x^2 - x^3) dx \\ &= \left. \frac{2}{3}x^3 - \frac{1}{4}x^4 \right|_1^2 \\ &= \frac{2}{3}(2)^3 - \frac{1}{4}(2)^4 - \left(\frac{2}{3} - \frac{1}{4}\right) \\ &= \frac{11}{12} \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \frac{11}{12} - \left(\frac{2}{3}\right)^2 \\ &= \frac{17}{36} \end{aligned}$$

d) Y is a random variable because a random variable can be generated by passing another random variable to a function. Both X and Y are random variables.

$$e) E(Y) = E(X^2)$$

$$= \begin{cases} \frac{1}{4} & 0 \leq x < 1 \\ \frac{11}{12} & 1 \leq x < 2 \end{cases}$$