Linked Lists

Yi-Shin Chen
Institute of Information Systems and Applications
Department of Computer Science
National Tsing Hua University
yishin@gmail.com

Array Review

Array

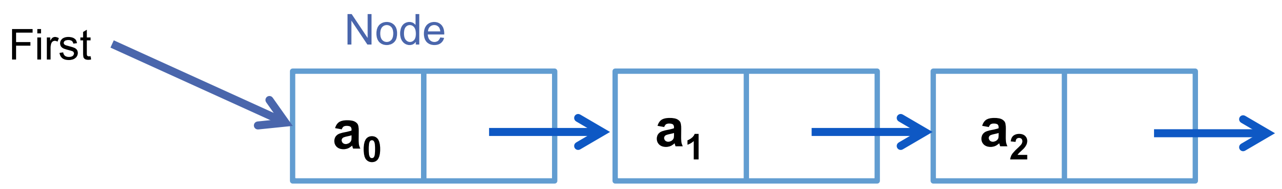
- Store an ordered list
- Using sequential mapping
  - Element(node) a_i is stored in the location L_i of the array
  - Next node is at the location L_i+1
- Pros:
  - Suitable for random access
  - Efficient to insert/delete from the end
  - Adequate for special data structures, Stack and Queue
- Con:
  - Difficult to insert/delete nodes at arbitrary location

Linked List

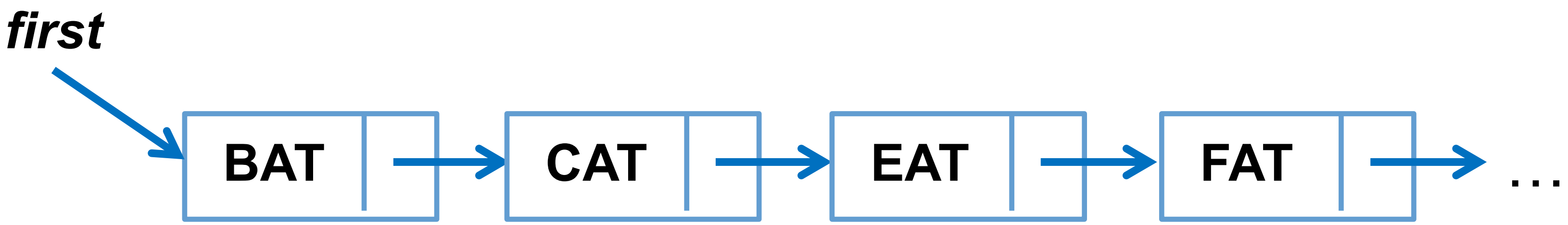
Linked Lists

- Nodes are no longer continue in the memory
- Each node stores the address or location of the next one

- Singly Linked List (SLL)
  - Each node has exactly one pointer field

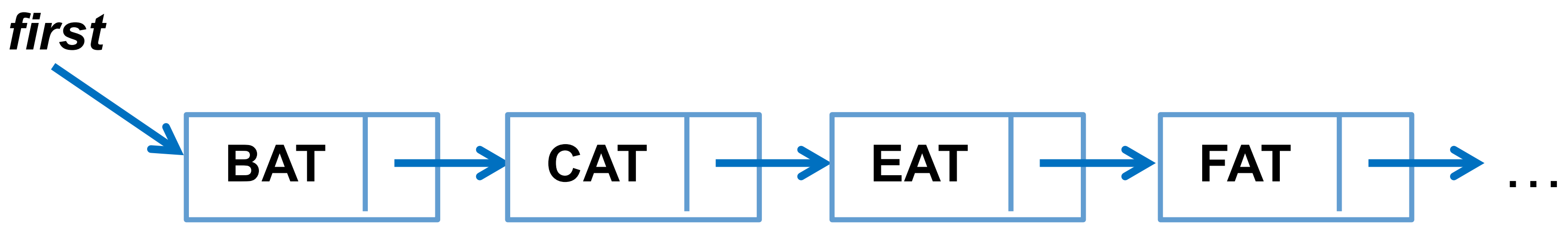


SLL Operation: Insert



- Steps to insert a ”GAT” in between “CAT” and “EAT” nodes
  - Create a new node “newNode” and set data field to “GAT”
  - Set the link field of “newNode” to “EAT” node
  - Set the link field of “CAT” node to “newNode”

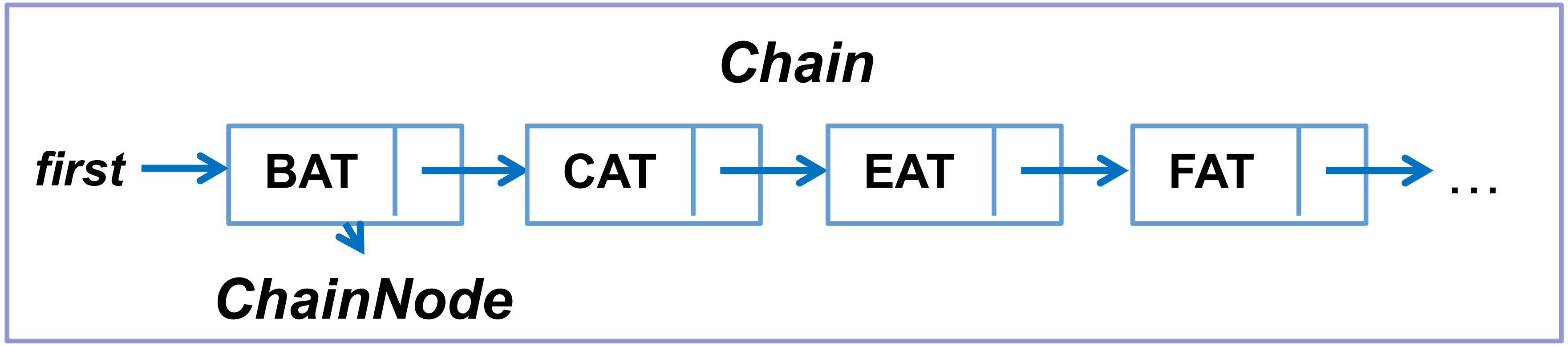
SLL Operation: Delete



- Steps to delete a ”EAT” node from the list
  - Locate the node “temp” precedes the “EAT” node
  - Set the node “delnode” to the node next to “temp”
  - Set the link field of “temp” to node next to “EAT” node
  - Delete the “EAT”(delnode) node

Conceptual Design

- Defining a “ChainNode” class
  - Data field
  - Link field
- Designing a “Chain” class
  - Support various operation on ChainNodes



ChainNode & Chain Classes

```
class ChainNode {
  friend class Chain;
public:
   // Constructor
   ChainNode(int value=0, ChainNode* next=NULL){
      data = value;
      link = next;
    }
private:
  int data;
  ChainNode *link;
};
```

```
class Chain {
  public:
    // Create a chain with two nodes
    void Create2();
    // Insert a node with data=50
    void Insert50(ChainNode *x);
    // Delete a node
    void Delete(ChainNode *x, ChainNode *y);
  private:
    ChainNode *first;
};
```

ChainNode & Chain Classes

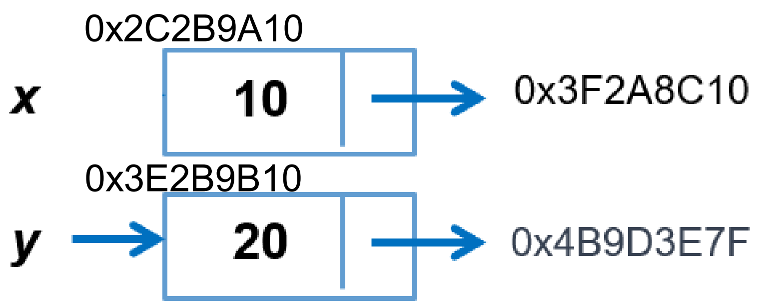
- Nested class

```
class Chain
{
public:
     // Create a chain with two nodes
     void Create2();

     // Insert a node with data=50
     void Insert50(ChainNode *x);

     // Delete a node
     void Delete(ChainNode *x, ChainNode *y);

private:
      class ChainNode{
        public:
          int data;
          ChainNode *link;
     }
     ChainNode *first;
};
```

Review Pointer Manipulation

- **Dereference**
  - NodeA &a1Ref = (*a1);
- **Access members**
  - a1->memData;
  - a1->memFunc();
  - (*a1).memData;
  - (*a1).memFunc();
- **Declaration**
  - NodeA *a1=NULL, *a2=NULL;
- **Allocate memory**
  - a1 = new NodeA;
  - a2 = new NodeA[10];
- **Delete memory**
  - delete a1; a1=NULL;
  - delete [] a2; a2=NULL;

Pointer Assignment

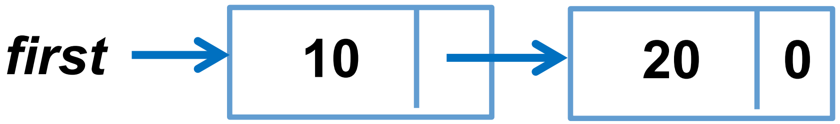



```
ChainNode* x= new ChainNode(10,0x3F2A8C10);
ChainNode* y =new ChainNode(10,0x4B9D3E7F);
```

Chain

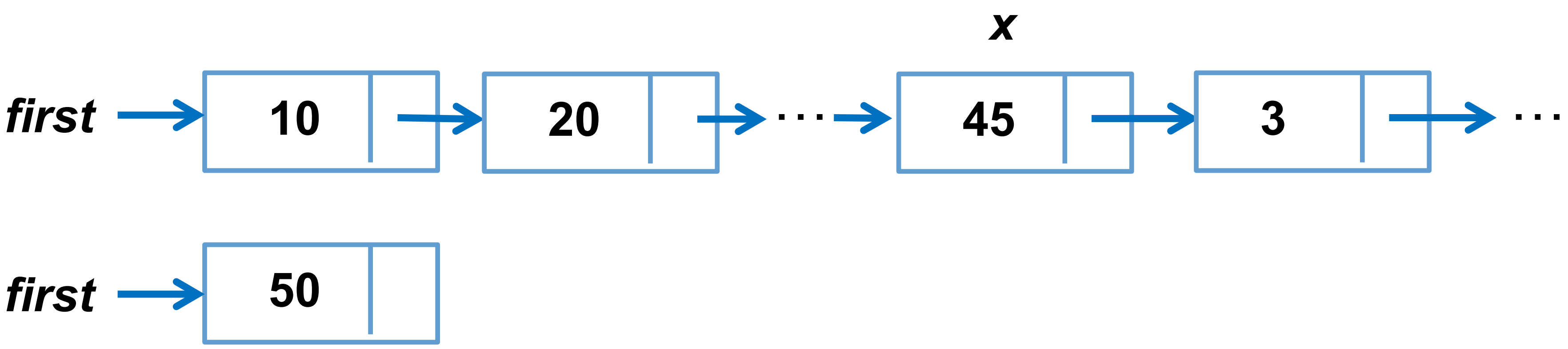
Chain Operations

```
void Chain::Create2()
{
   // Create and set the fields of 2nd node
   ChainNode* second = new ChainNode(20,0);



}
```



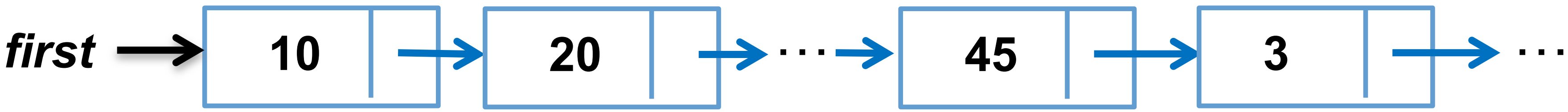
Chain Operations

```
void Chain::Insert50(ChainNode *x)
{
    if(first) // Insert after x
        x→link = new ChainNode(50, x->link);
    else // Insert into empty list
        first = new ChainNode(50);
}
```



Chain Operations

```
void Chain::Delete(ChainNode *x, ChainNode *y)
{   // x is the node to be deleted and
    // y is the node preceding x
    if(x==first) first = first->link;
    else y->link = x->link;
    delete x;
    x=NULL;
}
```



Template Chain Class

```
Template < class T > class Chain;  // Forward declaration

template < class T >
class ChainNode {
friend class Chain <T>;
private:
        T data;
        ChainNode<T>* link;
};

template <class T>
class Chain {
public:
        // Constructor
        Chain(void) {first = last = NULL;}

        // Chain operations…

private:
       ChainNode<T> *first;
       ChainNode<T> *last;
};
```

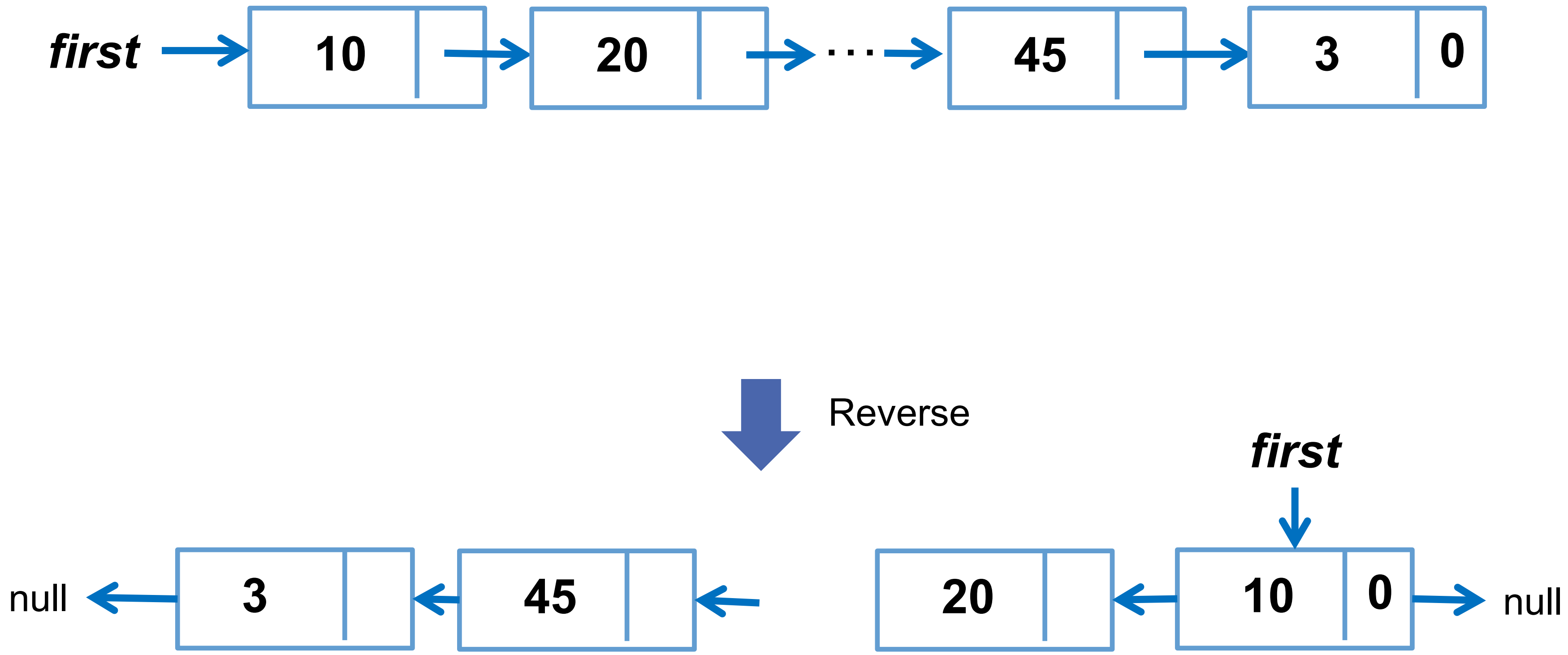
Chain Operations

```
template < class T >
void Chain<T>::InsertBack(const T& e)
{
   if(first) {// Non-empty chain
      last->link = new ChainNode<T>(e);
      last = last->link;
   }
   else // Insert into an empty chain
      first = last = new ChainNode<T>(e);
}
```

```
template < class T >
void Chain<T>::Concatenate(Chain<T>& b)
{  // b is concatenated to the end of *this
   if ( first ) { last->link = b.first; last = b.last; }
   else { first = b.first; last = b.last; }
   b. first = b.last = 0;
}
```

Chain Operations

- Reverse a chain, such that $(a^1, a^2, \cdots, a^n)$ turns into$(a^n, a^{n-1}, \cdots, a^1)$

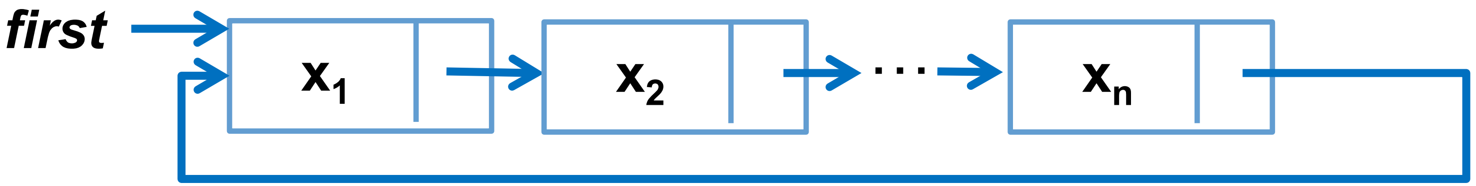


Chain Operations

```
template < class T >
void Chain<T>::Reverse(void)
{  // Turn a chain,(a1, …, an) into (an, …, a1)
   ChainNode<T> *current = first, *previous = NULL;
   while (current) {
      ChainNode<T> *r = previous;
      previous = current;     // r is behind the previous
      current = current->link; // move current to next node
      previous->link = r; // link previous to previous node
   }
   first = previous;
}
```

Circular Lists

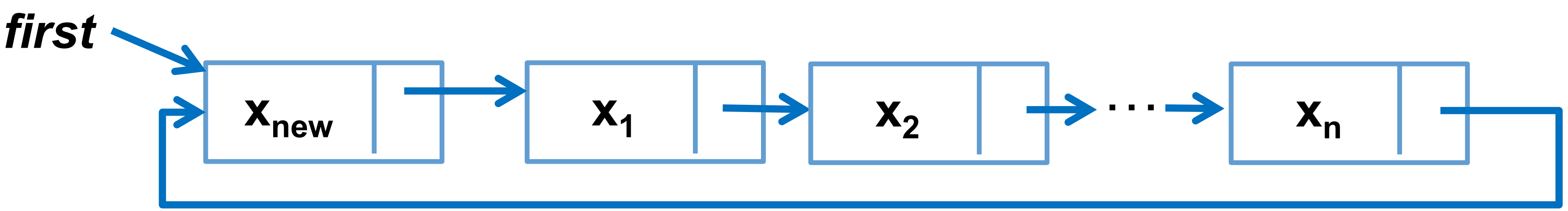
- A singly-linked circular list
- The link field of the last node points to the first node



- Check for the last node
  - if(current->link == ***first***)
- You could visit a node from any position

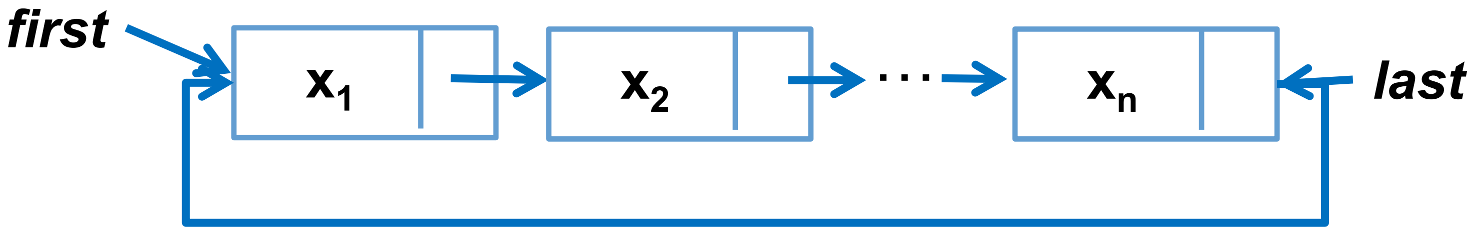
Circular Lists : Insert

- Suppose we want to insert a new node at the front of list
- Set link field of new node to ***first*** and set ***first*** to new node
- Go to the last node and set the link field to new node



Circular Lists

- Computation complexity for finding the last one?
  - O(N)
- If we have the last node pointer, the computation complexity for finding the first one?
  - We could away access the first node via [last->link]
  - O(1)
- It is more convenient to store the last node of a circular list (with the last pointer)

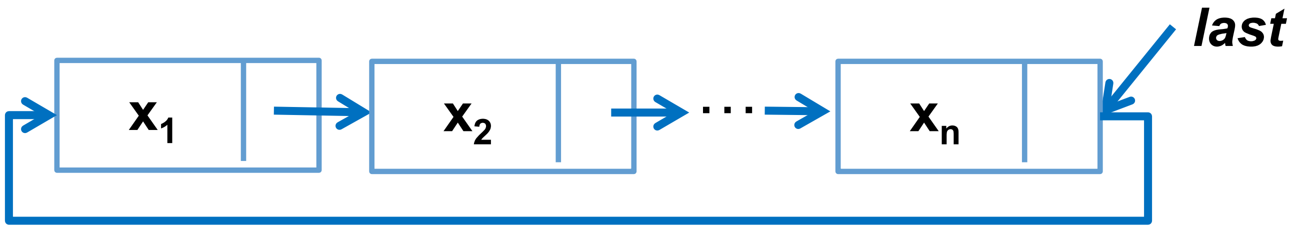


Circular Lists : Insert at Front

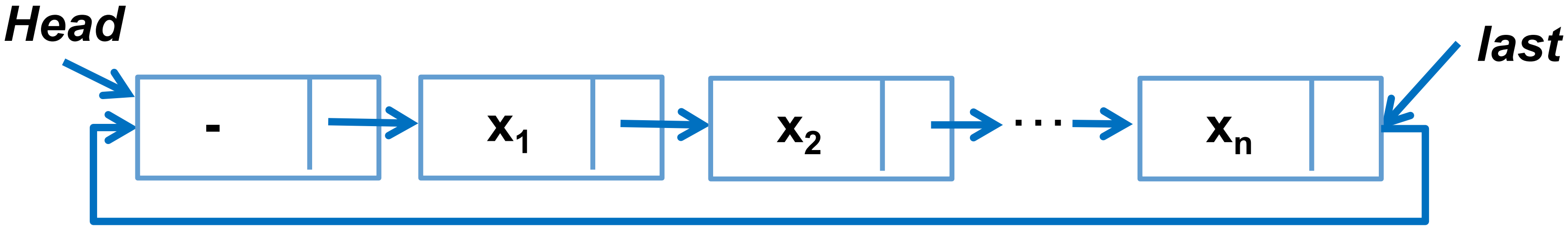
```
Template<class T>
void CircularList<T>::InsertFront(const T& e)
{
   ChainNode<T>* newNode = new ChainNode<T>(e);

   if(last){ // nonempty list
     newNode->link = last->link;
     last->link = newNode;
   }
   else{ // empty list
     last = newNode;
     newNode->link = newNode;
   }
}
```

Circular Lists

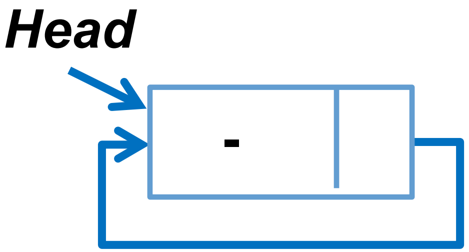
- How to represent an "empty" list?



- Introducing a dummy node "Header"



Empty list



Sparse Matrix

Sparse Matrix

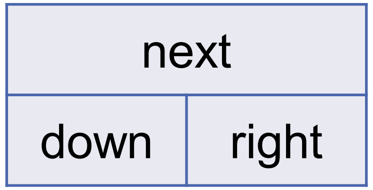
$$a[6][6] = \begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix}$$

A	row	col	value
smArray[0]	0	0	15
smArray[1]	0	3	22
smArray[2]	0	5	-15
smArray[3]	1	1	11
smArray[4]	1	2	3
smArray[5]	2	3	-6
smArray[6]	4	0	91
smArray[7]	5	2	28

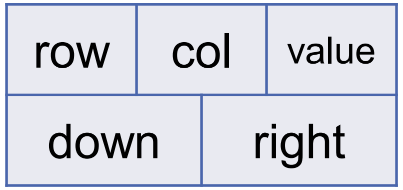
- A matrix has many zero elements
- Devise a sequential array
  - store **non-zero** elements
  - **row-major** order
- Access specific column is difficult
- Using circular lists representation

Linked Structure

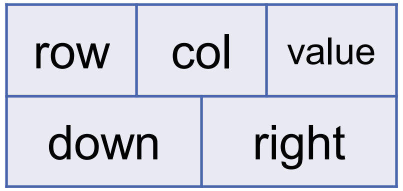
- Header node: for each row or column
  - **Down**: link to the 1st non-zero term in the column
  - **Right**: link to the 1st non-zero term in the row
  - **Next**: link to the next head node
  - The header node for row *i* is also the header node for column *i*



- Element node, each **non-zero** term that stores
  - Data of **row, col, and value**
  - A **down** field to link to the next non-zero term in the same **column**
  - A **right** field to link to the next non-zero term in the same **row**



- The header of header nodes (a circular list)
  - Store dimension of the matrix

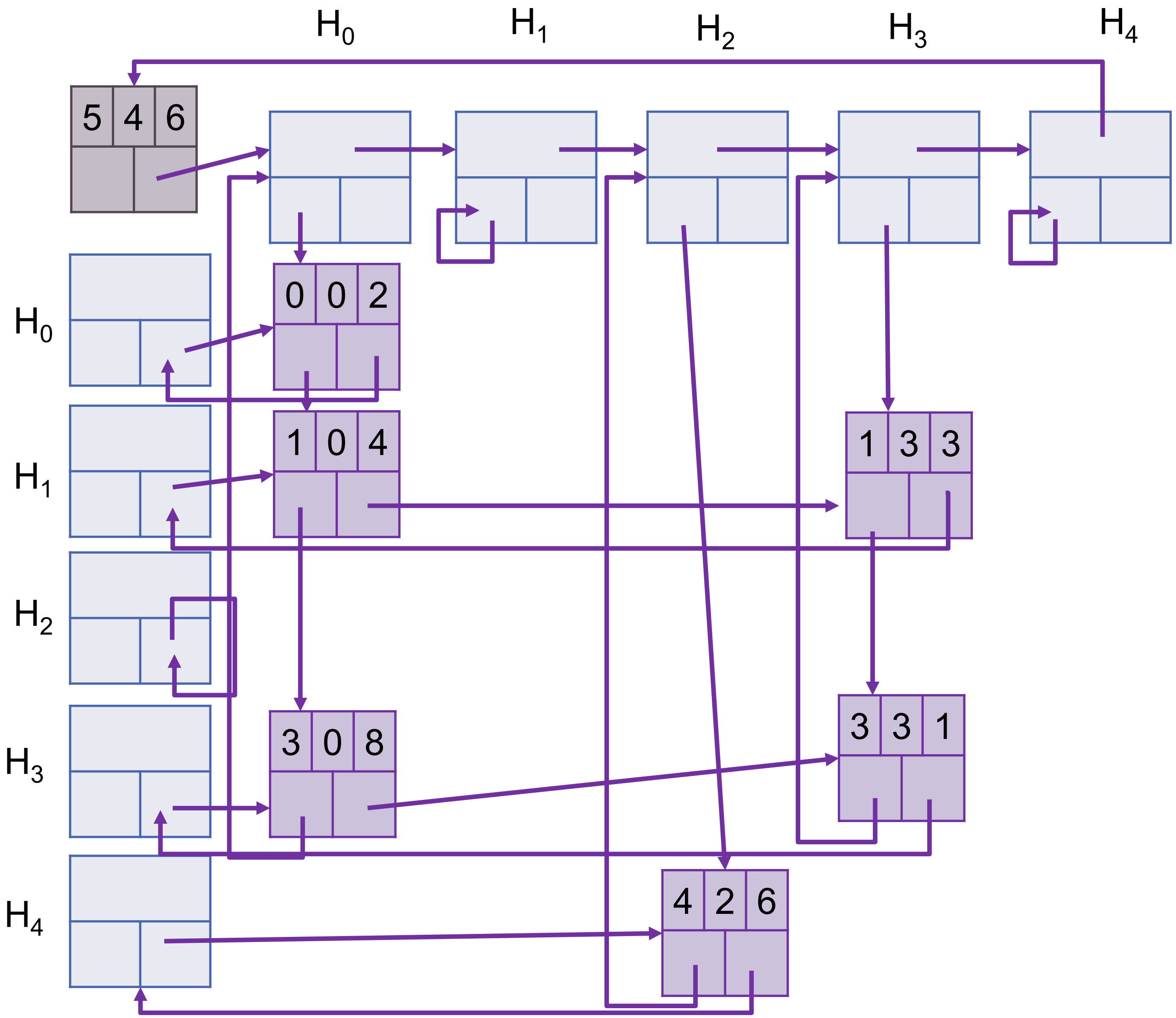


Algorithm

```
struct Triple{int row, col, value;};
class Matrix;   // forward declaration
class MatrixNode {
friend class Matrix;
friend istream& operator>>(istream&, Matrix&); // for reading  in a matrix
private:
    MatrixNode *down , *right;     bool head;
    union {
        MatrixNode *next;
        Triple triple;    };
    MatrixNode(bool, Triple*); } // constructor
 MatrixNode::MatrixNode(bool b, Triple *t) // constructor
{    head = b;
    if (b) {right = down = this;} // row/column header node
    else triple = *t;} // element node or header node of header lists
class Matrix{ friend istream& operator>>(istream&, Matrix&);
public:     ~Matrix(); // destructor
private:     MatrixNode *headnode;};
```

Sparse Matrix in Linked Structure

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$



Create a Sparse Matrix

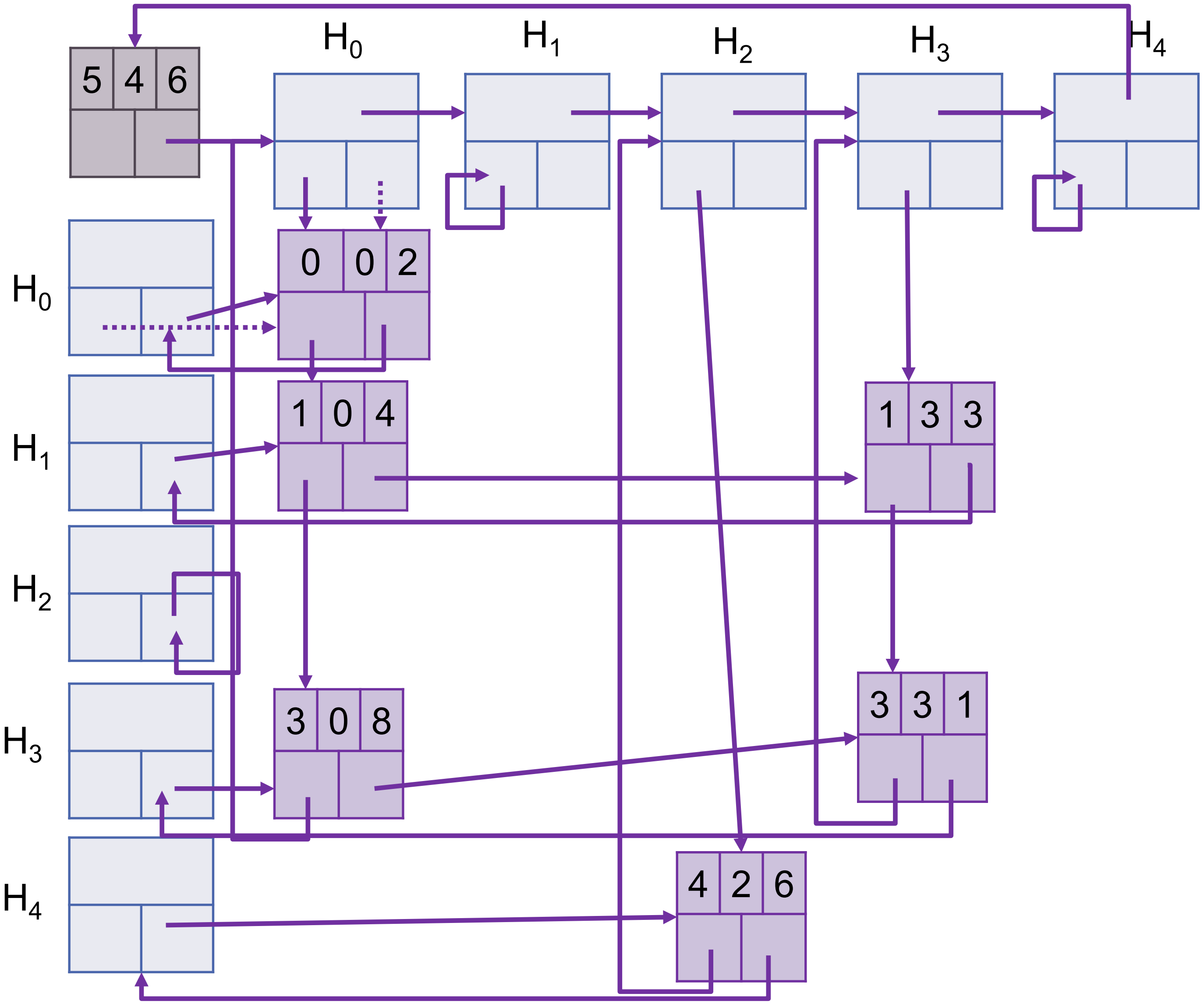
- Given a nxm sparse matrix with r non-zero terms
  - the total number of required nodes are max{n, m} + r + 1
- Input format
  - The 1st line gives the dimension of matric and # of non-zero terms
  - Each subsequent input line is a triple of the form (i, j, a_{ij})
    - Triples are ordered by rows and within rows by columns

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Sparse Matrix in Linked Structure

row	col	value
down	right	

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$



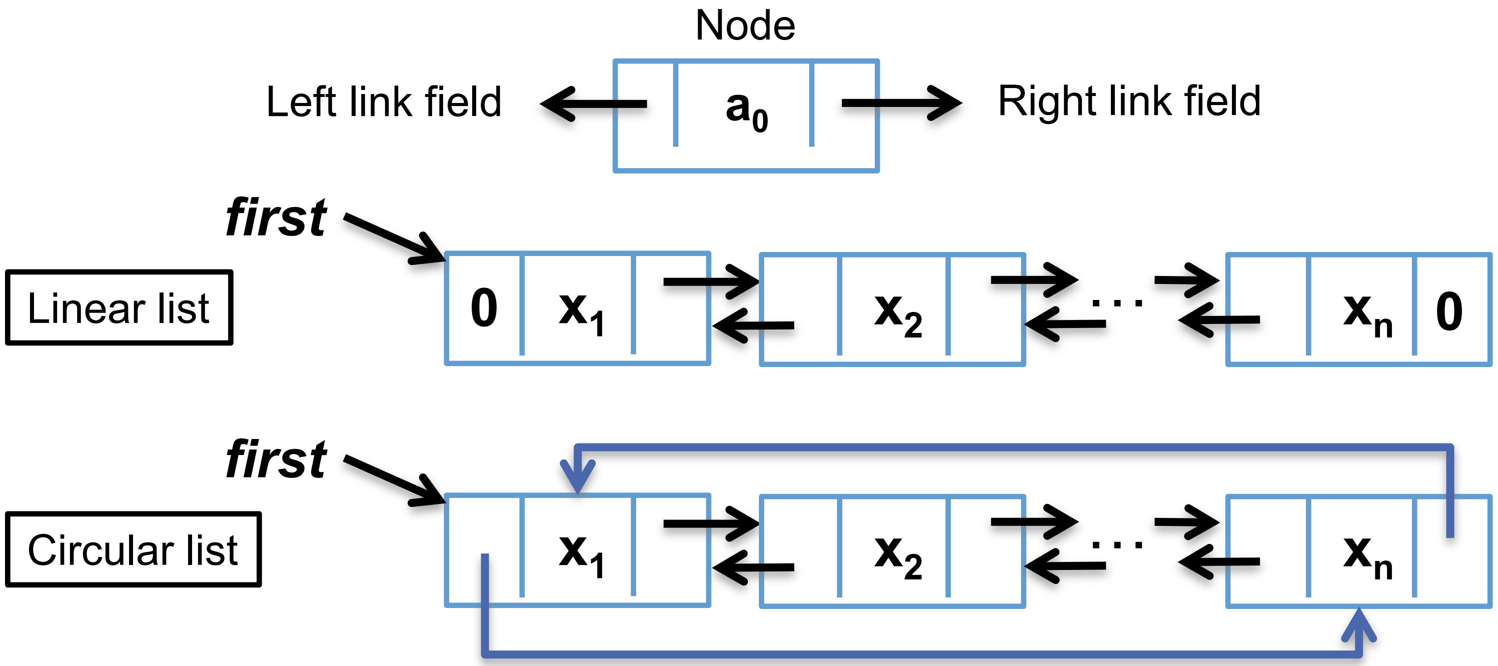
Create a Sparse Matrix

- Performance analysis
  - Set up header nodes, $O(\max\{n,m\})$
  - Set up non-zero nodes, $O(r)$
  - Close column lists, $O(\max\{n,m\})$
  - Link header nodes, $O(\max\{n,m\})$
- Total complexity: $O(\max\{n,m\}+r) = O(n+m+r)$

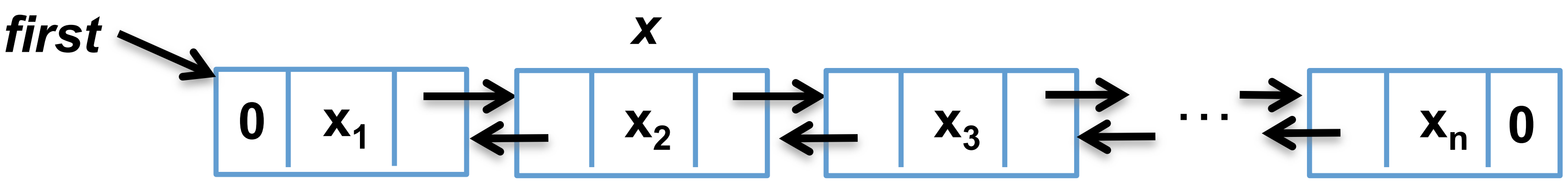
Double Linked Lists

Double Linked Lists

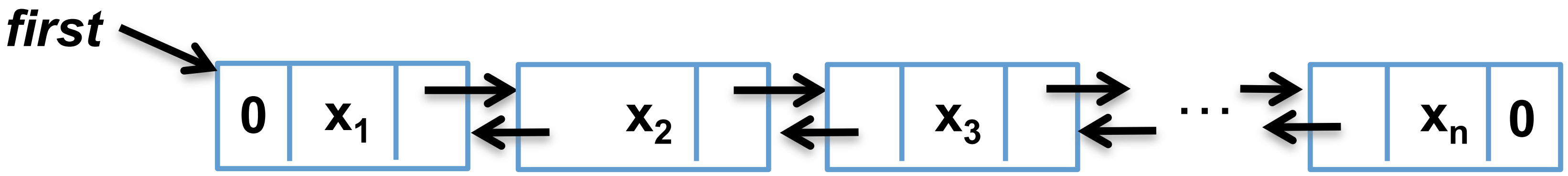
- Each node has two link fields
- Could move in two directions to visit nodes



Double Linked Lists : Delete

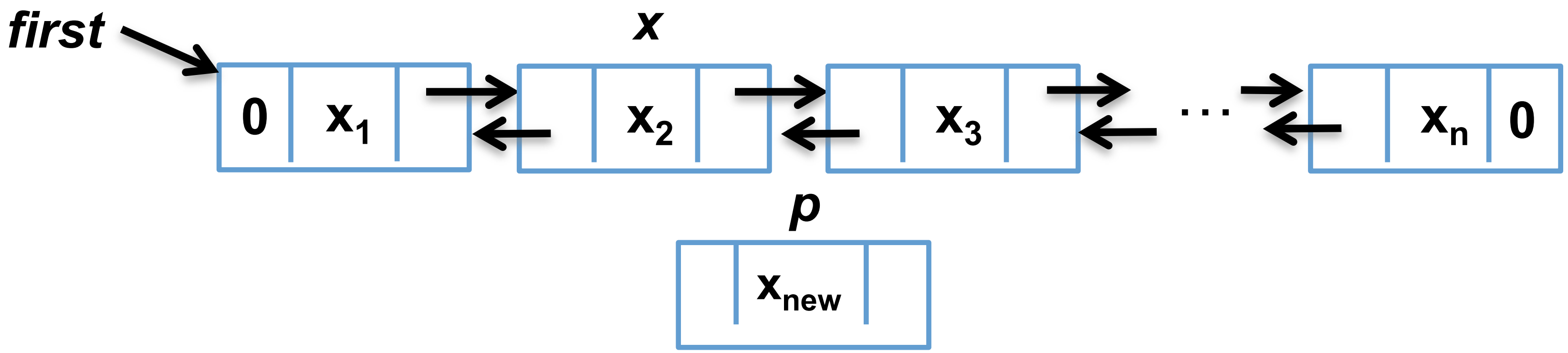



```
x->left->right = x->right;  x->right->left = x->left;
```

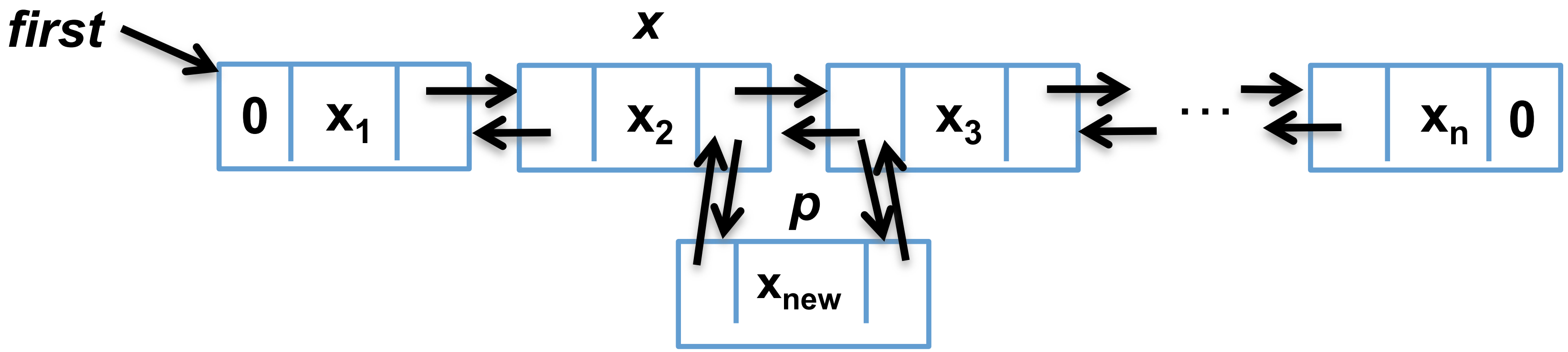



```
delete x;
```

Double Linked Lists : Insert



p->left = x; p->right=x->right



x->right->left = p; ➡ x->right = p;

Self-Study Topics

- Polynomial using linked lists
- Linked stacks and queues

Self Study Topic: Visit Elements in a Container

https://www.youtube.com/watch?v=SgcHcbQ0RCQ

Visit Elements in a Container

- Suppose we have a chain C of datatype Chain<int>.
  - Output all integers in C
  - Obtain the maximum, minimum or mean of all integers in C
  - Obtain the sum, product, or sum of squares of all integers in C

- All operations require to visit every element in the chain C

How to Visit a Container?

```
For each item in C
{
   currentItem = current item in C;
   //do something with currentItem;
}
```

- In an array representation

```
for (int i = 0; i < n; i++)
{
   int currentItem = a[i];
   // do something with currentItem;
}
```

How to Visit a Container?

```
For each item in C
{
   currentItem = current item in C;
   // do something with currentItem;
}
```

- In a linked list representation

```
for (ChainNode<int> *ptr=first; ptr!=0; ptr=ptr->link)
{
   int currentItem = ptr->data;
   // do something with currentItem;
}
```

Visiting a Container using Iterator

- A powerful mechanism to visit a container with arbitrary data type
- Guarantee runtime range safety
- Applicable to all STL algorithms
- Suitable for team development
- Might scarify some amount of performance
-

```
// Possible implementation of STL copy algorithm
template < class Iterator >
void copy(Iterator start, Iterator end, Iterator to)
{ // copy from src[start, end) to dst[to, to+end-start)
   while (start != end)
   { *to = *start ; start++ ; to++; }
}
```

What is an Iterator ?

```
void main()
{
    int x [3] = {0,1,2};
    for (int* y = x; y != x+3; y++)
        cout << *y << endl;
}
```

- An ***iterator*** is a pointer to an element in a container
- Using dereferencing operator (*) to access an element
- Support pre- or post- increment operator (++)

```
void main()
{
    for (Iterator y = start; y != end; y++)
        cout << *y << endl;
}
```

C++ Iterators

- **Input** iterator
  - Read access, pre- and post- “++” operators.
- **Output** iterator
  - Write access, pre- and post- “++” operators.
- **Forward** iterator
  - pre- and post- “++” operators.
- **Bidirectional** iterator
  - pre- and post- “++” and “--” operators.
- **Random access** iterator
  - Permit pointer jumps by arbitrary amounts.
- All iterators supports “==”, “!=” and “*” operators

Forward Iterator for Chain

```
template <class T>
class Chain {
public:
        // Constructor
        Chain(void) {first = last = NULL;}

        // Chain operations…

        class ChainIterator{…};

        // Get the first element
        ChainIterator begin() {return ChainIterator(first);}

        // Get the end of the list
        ChainIterator end() {return ChainIterator(0);}
private:
      ChainNode<T> *first;
      ChainNode<T> *last;
};
```

Forward Iterator for Chain

- General usage

```
void main()
{
   Chain<int> myChain;
   // do operations on myChain here…

   // print out every element in myChain
   Chain<int>::ChainIterator my_it;
   for (my_it = myChain.begin(); myChain!=myChain.end(); ++m_it)
      cout << *m_it << endl;

   // Use STL algorithm to calculate the sum of myChain
   int sum = std::accumulate(myChain.begin(), myChain.end(),0);
}
```

```
Class ChainIterator{ // A nested class within Chain
public:
  // Constructor
  ChainIterator(ChainNode<T>* startNode = 0)
               {current = startNode;}

  // Dereferencing operator
  T& operator*() const {return current->data;}
  T* operator->() const {return &current->data;}

  // Increment operator
  ChainIterator& operator++()  // pre-"++"
  { current = current->link ; return *this; }

  ChainIterator operator++(int)// post- "++"
  {
    ChainIterator old = *this;
    current = current->link;
    return old;
  }

  // Equality operators
  bool operator!=(const ChainIterator right) const
  { return current != right.current; }

  bool operator==(const ChainIterator right) const
  { return current == right.current;}

private:
   ChainNode<T>* current;
};
```