

$$\begin{aligned}
 (\Omega_k(\mathcal{S}))^k &= \min_{\phi(\mathcal{S})=\mathbf{0}} \frac{\phi^\top \mathcal{L}^k \phi}{\phi^\top \phi} \approx \min_{\mathbf{x}} \left(\frac{\mathbf{x}^\top \mathcal{L}^k \mathbf{x}}{\mathbf{x}^\top \mathbf{x}} + \alpha \frac{\mathbf{x}^\top \text{diag}(\mathbf{t}) \mathbf{x}}{\mathbf{x}^\top \mathbf{x}} \right) \bigg|_{\mathbf{t}=\mathbf{1}_\mathcal{S}} = \lambda_k^\alpha(\mathbf{t})|_{\mathbf{t}=\mathbf{1}_\mathcal{S}}
 \end{aligned}$$

The diagram illustrates the derivation of the relaxed eigenvalue problem. The initial constraint $\phi(\mathcal{S})=\mathbf{0}$ is relaxed, leading to the minimization over \mathbf{x} . The term $\text{diag}(\mathbf{t})$ is introduced via binary relaxation, and the final result is expressed as $\lambda_k^\alpha(\mathbf{t})|_{\mathbf{t}=\mathbf{1}_\mathcal{S}}$.