Real bond return parity

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Abstract

We test a set of assumptions that imply the return parity of long-run, real bonds denominated in different currency numeraire. The joint hypothesis is rejected in our post-2009 sample of developing and developed market currencies; however, we document a strong relationship between changes in the log of bilateral, real exchange rate and real holding period bond returns in the direction of parity, contributing to the Meese-Rogoff puzzle on exchange rate determination.

1 Introduction

Exchange rate determination remains an active puzzle (Meese and Rogoff (1983)). The empirical literature has sought to identify significant correlates to changes in exchange rates (Ferraro, Rossi, and Rogoff (2011), Verdelhan (2018), Jiang, Krishnamurthy, Lustig (2018)). The risk-based view of exchange rate determination has helped explain the cross section of currency portfolio returns and individual exchange rate variation (Lustig, Verdelhan (2009), Verdelhan (2018)), generating explainability of monthly, bilateral exchange rate movements as high as 80%.

This paper adopts the risk-based view of exchange rates. Following in the footsteps of Lustig et al. (2019), we make four assumptions: complete markets, the satisfaction of regularity conditions for a decomposition of the stochastic discount factor into a "permanent", martingale component and a "transitory" component related to the inverse of long run real bond prices, a common "permanent", martingale component across currencies, and the validity of the log of ten-year, real bond, monthly holding period returns as an approximation for the log of infinite-maturity, real bond, monthly holding period returns, to derive testable implications for the sign, and magnitude of the relationship between the log of real exchange rate changes and the log of long-maturity (i.e. ten-year) real bond returns. In particular, a positive shock to the log holding period return on the long-run foreign (domestic) real bond should be associated with contemporaneous, and one-to-one depreciation (appreciation) in the log of the foreign real exchange rate; once converted into a common numeraire, log holding period returns on real long-run bonds should be equal across currency baskets. In the style of Lustig et al. (2019) who derived this relation in nominal terms, we call this the "long, real bond return parity."

The joint hypothesis is rejected in our post-2009 sample of developing and developed market currencies; however, we document a strong relationship between changes in the log real exchange rate and real foreign, and domestic holding period bond returns in the direction of parity.

2 Background

To simplify the exposition, we focus on a single currency pair: a domestic and foreign currency. We denote foreign variables by stars.

Suppose we have the stochastic discount factor M_{t+1} with the following representation in terms of the time $t+1$ and t pricing kernels Λ_{t+1}, Λ_t

$$M_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t}. \tag{1}$$

Under regularity conditions, Alvarez and Jermann (2005) present a decomposition of the pricing kernel Λ_t into a martingale, permanent component Λ_t^P and a transitory component Λ_t^T using the price of a k maturity long-term bond $P_t^{(k)}$

$$\Lambda_t = \Lambda_t^P \Lambda_t^T, \text{ where } \Lambda_t^T = \lim_{k\to\infty} \frac{\delta^{t+k}}{P_t^{(k)}}. \tag{2}$$

The holding period return from time t to $t+1$ on the infinite maturity bond, which we denote by $HPR_{t+1}^{(\infty)}$, is the ratio of the transitory components at time t and $t+1$

$$HPR_{t+1}^{(\infty)} = \lim_{k\to\infty} HPR_{t+1}^{(k)} = \lim_{k\to\infty} \frac{P_{t+1}^{(k-1)}}{P_t^{(k)}} = \frac{\Lambda_t^T}{\Lambda_{t+1}^T}. \tag{3}$$

The corresponding log holding period return, which we denote by $hpr_{t+1}^{i,(\infty)}$ is

$$hpr_{t+1}^{i,(\infty)} = -\lambda_{t+1}^{i,T} + \lambda_t^{i,T}. \tag{4}$$

If markets are complete, there exists a unique, real stochastic discount factor. This factor generates a set of unique, currency-specific factors that differ in the real numeraire that they reference, M_{t+1}, M_{t+1}^*. The Alvarez and Jermann representation of M_{t+1} gives us

$$M_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} = \frac{\Lambda_{t+1}^P \Lambda_{t+1}^T}{\Lambda_t^P \Lambda_t^T}, \text{ where } \Lambda_t^T = \lim_{k\to\infty} \frac{\delta^{t+k}}{P_t^{(k)}}. \tag{5}$$

Let Q_{t+1} denote the time $t+1$ real exchange rate, defined as units of the foreign real goods basket per unit of the domestic real goods basket

$$Q_{t+1} = \frac{\text{Units of foreign real goods baskets}_{t+1}}{\text{Domestic real goods basket}}. \tag{6}$$

An accounting identity gives us

$$M_{t+1} = M_{t+1}^* \frac{Q_{t+1}}{Q_t}. \tag{7}$$

This is from the Euler equations of the stand-in domestic and foreign investors: for an asset return R^* expressed in foreign currency basket terms $\mathbb{E}_t[M_{t+1}R^*_{t+1}\frac{Q_t}{Q_{t+1}}] = 1$ and $\mathbb{E}_t[M^*_{t+1}R^*_{t+1}] = 1$. State-by-state equality is an accounting identity that holds under no arbitrage. In logs, this corresponds to

$$\Delta q_{t+1} = m_{t+1} - m^*_{t+1}. \tag{8}$$

The AJ decomposition gives us

$$\Delta q_{t+1} = [(\lambda^T_{t+1} - \lambda^T_t) + (\lambda^P_{t+1} - \lambda^P_t)] - [(\lambda^{T*}_{t+1} - \lambda^{T*}_t) + (\lambda^{P*}_{t+1} - \lambda^{P*}_t)]. \tag{9}$$

Lustig et al. (2019) show that average excess returns on the carry trade using long-run bonds and with a three-month horizon, is zero, suggesting that the entropies of the permanent components are equal.[1] If currency SDFs share a common permanent (martingale) $\lambda^{P*}_t = \lambda^P_t$, this restriction is satisfied and

$$\begin{aligned} \Delta q_{t+1} &= -(\lambda^{T*}_{t+1} - \lambda^{T*}_t) + (\lambda^T_{t+1} - \lambda^T_t) & (10)\\ &= hpr^{*,(\infty)}_{t+1} - hpr^{(\infty)}_{t+1}. & (11) \end{aligned}$$

This is a statement about realizations not expectations. This is not a novel derivation. Lustig et al. (2019) are the first to derive it in nominal terms in their appendix. This paper rewrote it in real terms, and will test it.

Let t index time and i the foreign currency, this directly motivates a regression of the form

$$\Delta q_{t+1} = \alpha + \beta_1\, hpr^{*,(\infty)}_{t+1} + \beta_2\, hpr^{(\infty)}_{t+1} + \epsilon_t \tag{12}$$

where Δq_{t+1} is the log change in the real exchange rate, $hpr^{*,(\infty)}_{t+1}$ is the log holding period return on the foreign, infinite maturity real bond, and $hpr^{(\infty)}_{t+1}$ is the log holding period return on the domestic, infinite maturity real bond.

However, we face a practice problem. There are no infinite maturity real, risk-free bonds. Therefore, we assume that the log of ten-year bond returns

[1] Their result is in nominal terms and on currency portfolios. They argue that the inflation characteristics of portfolios suggest the result holds also in real terms. We find suggestive evidence that this is true when we look at the individual currencies in our sample. Interested readers may find the analysis in the Appendix.

approximatethe log of infinite bond returns; that is, that

$$\Delta q_{t+1} = \alpha + \beta_1 \, hpr^{*,(\infty)}_{t+1} + \beta_2 \, hpr^{(\infty)}_{t+1} + \epsilon_t \tag{13}$$

is closely approximated by

$$\Delta q_{t+1} = \alpha + \beta_1 \, hpr^{*,(10)}_{t+1} + \beta_2 \, hpr^{(10)}_{t+1} + \epsilon_t \tag{14}$$

where $hpr^{*,(10)}_{t+1}$ is the log holding period return on the foreign, ten-year real bond, and $hpr^{(10)}_{t+1}$ is the log holding period return on the domestic, ten-year real bond.

3 Data

In this section, we describe the notation and the data.

3.1 Notation

This section describes notation for exchange rates, bond prices, and currency and bond returns.

3.1.1 Currency and national price levels

Let S_{t+1} denote the time $t+1$ nominal exchange rate, defined as units of foreign currency per unit of the domestic currency

$$S_{t+1} = \frac{\text{Units of foreign currency}_{t+1}}{\text{Unit of domestic currency}}. \tag{15}$$

Let Z_{t+1} denote the time $t+1$ domestic national price level, defined as the domestic nominal price of one unit of the domestic real goods basket

$$Z_{t+1} = \frac{\text{Units of domestic currency}_{t+1}}{\text{Domestic real goods basket}}. \tag{16}$$

Let Π_{t+1} denote gross inflation from time t to time $t+1$ in the domestic currency

$$\Pi_{t+1} = \frac{Z_{t+1}}{Z_t}. \tag{17}$$

National price levels are vis-a-vis currency specific real goods baskets; this is to reflect cross-country differences in what governments consider a basket of real goods when reporting national price levels.

We now define other objects defined vis-a-vis these variables.

Let Q_{t+1} denote the time $t+1$ real exchange rate, defined as units of the foreign real goods basket per unit of the domestic real goods basket

$$Q_{t+1} = \frac{\text{Units of foreign real goods baskets}_{t+1}}{\text{Domestic real goods basket}} = S_{t+1}\frac{Z_{t+1}}{Z^*_{t+1}}. \tag{18}$$

Let Π_{t+1} denote gross inflation from time t to time $t+1$ in the domestic currency

$$\Pi_{t+1} = \frac{Z_{t+1}}{Z_t}, \tag{19}$$

and Π^*_{t+1} denote gross inflation from time t to time $t+1$ in the foreign currency

$$\Pi^*_{t+1} = \frac{Z^*_{t+1}}{Z^*_t}. \tag{20}$$

Following the convention of denoting by the lowercase the logs of variabless, let Δq_{t+1} denote the log change in real exchange rates from time t to time $t+1$

$$\Delta q_{t+1} = \Delta s_{t+1} + \pi_{t+1} - \pi^*_{t+1}. \tag{21}$$

3.1.2 Bonds

Let $P_t^{(N)}$ denote the zero-coupon, N-maturity, risk-free real bond which pays a unit of the real basket at maturity.

Let $HPR_{t+1}^{(N)}$ denote the gross, holding period return on the N-maturity zero coupon real bond from time t to time $t+1$

$$HPR_{t+1}^{(N)} = \frac{P_{t+1}^{(N-1)}}{P_t^{(N)}} \tag{22}$$

and $hpr_{t+1}^{(N)}$ denote its log counterpart

$$hpr_{t+1}^{(N)} = log(HPR_{t+1}^{(N)}). \tag{23}$$

For example, the log, monthly holding period return on a ten-year bond, which we denote by $hpr_{t+1}^{(10)}$, is

$$hpr_{t+1}^{(10)} = log(HPR_{t+1}^{(10)}). \tag{24}$$

$$= log(\frac{P_{t+1}^{(9\frac{11}{12})}}{P_t^{(10)}}). \tag{25}$$

To compute holding period returns, we need real bonds prices at the 10, $9\frac{11}{12}$ -year maturities. While inflation indexed bonds have existed before 2009, limited issuance across the maturity structure makes it difficult to boostrap an entire curve of zero coupon rates. In contrast, inflation swap markets have quotes at multiple and evenly spaced maturities. Furthermore, many have embedded options. In contrast, inflation swaps do not. Thus, we will construct real bonds using a zero coupon, nominal, "risk-free" rate and inflation swap rates. Let M_{t+N} be the nominal stochastic discount factor that prices $t+N$ cash flows in domestic currency.

Consider a zero-coupon, N-maturity risk-free nominal bond which pays a unit of currency at maturity. Its price is $B_t^{(N)}$ and its zero rate is $y_t^{(N)}$

$$B_t^{(N)} = E_t[M_{t+N}] \tag{26}$$

$$\equiv \frac{1}{(1+y_t^{(N)})^N}. \tag{27}$$

Consider a zero-coupon, N-maturity inflation swap. At maturity, the fixed leg investor will pay the fixed rate $i_t^{(N)}$ on the notional; the floating leg investor will pay the realized change in the referenced inflation index series $\frac{Z_{t+N}}{Z_t}$. At time t, the derivative has a price of zero

$$0 = E_t[M_{t+N} \times (\frac{Z_{t+N}}{Z_t} - (1+i_t^{(N)})^N)]. \tag{28}$$

Consider a zero-coupon, N-maturity, risk-free real bond which pays a unit of the real basket at maturity. Its price is $P_t^{(N)}$.

Using the previous expression, we can express $P_t^{(N)}$, which is often not directly observable, as a function of $y_t^{(N)}$, $i_t^{(N)}$, which we often do observe directly:

$$0 = E_t[M_{t+N} \times (\frac{Z_{t+N}}{Z_t} - (1 + i_t^{(N)})^N)] \tag{29}$$

$$E_t[M_{t+N}\frac{Z_{t+N}}{Z_t}] = E_t[M_{t+N} \times (1 + i_t^{(N)})^N)] \tag{30}$$

$$E_t[M_{t+N}\frac{Z_{t+N}}{Z_t}] = E_t[M_{t+N}](1 + i_t^{(N)})^N \tag{31}$$

$$P_t^{(N)} = E_t[M_{t+N}](1 + i_t^{(N)})^N, \text{since } P_t^{(N)} \equiv E_t[M_{t+N}\frac{Z_{t+N}}{Z_t}] \tag{32}$$

$$P_t^{(N)} = \left[\frac{(1 + i_t^{(N)})}{(1 + y_t^{(N)})}\right]^N \tag{33}$$

3.2 Data Sources

We turn to the paper's data. We first describe the dataset, and then discuss the data selection process and features of the data relevant to the interpretation of the results.

3.2.1 Data sources

The data for the study consists of daily closing prices for zero-coupon nominal OIS and zero-coupon inflation swaps, nominal and real bonds, national price levels, nominal exchange rates, and nominal forward exchange rates. The data covers a small, heterogeneous panel of developed and developing market currencies: the Japanese yen, euro, Swedish krona, British pound, U.S. dollar, Australian dollar, South African rand, and Brazilian real.[2]The first six currencies are those issued by the sovereigns of developed market economies; the latter two currencies are those issued by the sovereigns of developing market economies. All data are obtained from Bloomberg at the daily frequency for the period from January 01, 2009 to December 31, 2020. Data were aggregated to the monthly frequency as this corresponds to the most common release frequency of national price level series, which are used in the construction of real exchange rates. The panel and the data series constrained by the availability of data as few currencies have developed, and long-maturity inflation swap, real

[2]We listed the currencies in order of the average level of their three-month, nominal interest rates. We report results in this order as well. This was motivated by the fact that the failure of UIP is a key stylized fact, and so currencies are often thought of in terms of their nominal short interest rate levels.

bond, and OIS markets.[3]

Zero-coupon inflation and overnight index swap rates, and zero-coupon nominal and government bond rates were obtained at the three-month, and one, two, five, nine and ten year maturities. With the exception of inflation swap rates, rates are rates derived by Bloomberg from traded prices. Swap rate quotes are the constant rate on the contract's fixed leg. Intermediate maturities were obtained by local linear approximation.[4]Table 3.2.1 lists the series obtained, and their source.

Table 1: Unbalanced sample data sources and coverage

Data series	Source
Zero-coupon inflation swap rates	Bloomberg
Zero-coupon overnight index swap rates	Bloomberg
Zero-coupon nominal and inflation-indexed government bond rates	Bloomberg
Nominal exchange rates	Bloomberg
Nominal forward exchange rates	Bloomberg, author's calculations
National price levels	Bloomberg

Notes: This table describes the sources and coverage of our data. The data covers a small, heterogeneous panel of developed and developing market currencies: Japanese yen, euro, Swedish krona, British pound, U.S. dollar, Australian dollar, South African rand, and Brazilian real. All data are obtained at the daily frequency for the period from January 01, 2009 to December 31, 2020. Data were aggregated to the monthly frequency as this corresponds to the most common release frequency of national price level series. Nominal forward exchange rates for maturities longer than one year was constructed as in Du et al. (2018).

In Table 3.2.1, we report summary statistics for our main data series by currency. Panel A reports the mean, standard error, and standard deviation of the ten-year nominal rate $y_t^{*,(10)}$, the ten-year real rate $r_t^{*,(10)}$, the annual year-over-year inflation rate π_t, and the ten-year inflation compensation rate $i_t^{*,(10)}$. Panel B reports the same statistics for the one-year real rate $r_t^{*,(1)}$, and the slope of the real rate curve which is the difference between the ten-year and one-year real rates $r_t^{*,(10)} - r_t^{*,(1)}$.

The table paints a picture of a currency sample with appreciable heterogene-

[3]While inflation indexed bonds have existed before 2009, limited issuance across the maturity structure and embedded options make it difficult to reliably bootstrap the real bond curve for our currency set. Long-maturity inflation swap data exist for 2004 onwards, but long-maturity OIS data do not exist until 2009. The choice of OIS as the benchmark is motivated by the TIPS-Treasury puzzle, which indicates "mispricing" in real and/or nominal government cash bond markets in the U.S. (Fleming et al. (2014)), Japan, U.K., Germany (Kita and Tortice, 2019).

[4]Pricing errors, if they mimick classical measurement error, should minimally affect the cross sectional empirics if the variance of the error is small and our sample large enough; for the time series emprirics, they will bias coefficients downwards towards zero.

ity along the recorded dimensions. The currencies with the lowest and highest average nominal rates at the ten-year maturity are, respectively, the Japanese yen at 0.62% and the Brazilian real at 10.4%, a difference of nearly ten percentage points. The range of average real rates is more narrow, but also appreciable. The currencies with lowest and highest real rates at the ten-year maturity is, respectively, the British pound at -1.12% and and the Brazilian real 5.11%. The sharply negative real rate for the British pound, which is below even that for the Japanese yen, underscores the unprecedented nature of the sample's low nominal and real interest rate environment in the context of recorded history. This warrants cautious interpretation and extrapolation of the empirical results.

On the inflation front, developed market currencies tend to have lower and less variable inflation than developing market currencies. Notably, however, the standard deviation of inflation for the British pound is 1.27%, which is similar in magnitude to the corresponding number for the South African rand. This is consistent with the inflationary spike that followed the United Kingdom's decision to leave the European Union in June 2016, which precipitated a sharp exchange rate 10% currency depreciation and a concomittant positive shock to inflation. The average level of inflation swap rates generally increase with the level of inflation. The standard deviation on realized annual inflation is larger than the standard deviation on ten-year inflation swaps, which is suggestive of shocks in the inflation process that are expected to net out when averaged across the ten-year horizon. We emphasize that inflation swap rates are objects that are related to but not equal to either the inflation risk premia, or the holding period return on inflation swaps, the latter of which has a direct relation to the holding period returns on real bonds.

Turning to the yield curve, one-year real yields are negative for all developed market currencies, except the Australian dollar and there is no clear relationship between the level of one-year real yields and the slope of the real yield curve. One possible reason is that statistical relation between these two objects is more obscure when levels of real yields are so low and close across currencies. One-year real yields are positive for the two developing market currencies and are higher the steeper the slope of the real yield curve.

Table 2: Summary statistics for the cross section

(in percentage points)		British pound	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
		Currency							
		Panel A: Summary statistics							
$y_t^{*,(10)}$	Mean	2.13	0.62	1.52	1.88	2.41	3.59	8.17	10.40
	s.e.	[0.09]	[0.04]	[0.10]	[0.09]	[0.07]	[0.13]	[0.05]	[0.24]
	s.d.	1.09	0.48	1.19	1.08	0.81	1.61	0.55	2.46
$r_t^{*,(10)}$	Mean	-1.12	0.32	-0.09	-0.14	0.16	1.09	1.58	5.11
	s.e.	[0.09]	[0.07]	[0.07]	[0.09]	[0.05]	[0.10]	[0.06]	[0.12]
	s.d.	1.09	0.83	0.79	1.02	0.63	1.20	0.66	1.22
π_t	Mean	2.87	0.40	1.15	1.04	1.70	1.99	4.87	6.23
	s.e.	[0.11]	[0.09]	[0.08]	[0.09]	[0.08]	[0.07]	[0.10]	[0.44]
	s.d.	1.27	0.99	0.94	0.98	0.87	0.80	1.13	4.14
$i_t^{*,(10)}$	Mean	3.29	0.31	1.61	2.02	2.25	2.47	6.50	5.01
	s.e.	[0.02]	[0.05]	[0.04]	[0.02]	[0.03]	[0.04]	[0.06]	[0.12]
	s.d.	0.22	0.56	0.44	0.27	0.34	0.45	0.73	1.21
		Panel B: Yield curve characteristics							
$r_t^{*,(1)}$	Mean	-1.93	-0.04	-0.70	-0.72	-0.48	0.42	0.85	3.65
	s.e.	[0.08]	[0.10]	[0.05]	[0.06]	[0.08]	[0.08]	[0.09]	[0.20]
	s.d.	0.91	1.14	0.64	0.67	0.92	0.96	1.03	2.06
$r_t^{*,(10)}$-$r_t^{*,(1)}$	Mean	0.81	0.36	0.61	0.58	0.64	0.67	0.72	1.46
	s.e.	[0.10]	[0.05]	[0.05]	[0.06]	[0.08]	[0.04]	[0.05]	[0.12]
	s.d.	1.21	0.61	0.63	0.76	0.91	0.51	0.63	1.16

Notes: This table displays the mean, standard error of the mean, the standard deviation of the ten-year nominal rate $y_t^{*,(10)}$, the ten-year real rate $r_t^{*,(10)}$, the annual inflation rate π_t, and the ten-year inflation compensation rate $i_t^{*,(10)}$, the one-year real rate based $r_t^{*,(1)}$, and the slope of the real rate curve which is the difference between the ten-year and one-year real rates $r_t^{*,(10)} - r_t^{*,(1)}$.

While we make the best use of available data, we pre-emptively emphasize that features of the dataset warrant cautious interpretation of the results. First, the dataset is limited in size. Our dataset begin in January 2009, and the first return observation is for February 2009. Currencies have monthly observations that cover anywhere from eight to twelve years, which translates to 101 to 143 observations per currency, tenor pair. Observations are all in the post-crisis period during which we have seen historically unprecedented negative rates and central bank intervention in the long maturity bond market. This suggests the results should be cautiously interpreted within the context of the sample period they are based on. Furthermore, we have a small, and heterogeneous cross section, which restricts our analysis to individual currencies, as opposed to currency portfolios in which we might be able to appeal to diversification to isolate currency factors.

3.2.2 Data considerations

To construct the ten-year holding period returns on real bonds, we require zero-coupon real bond prices/rates. We constructed these using zero coupon, risk-free nominal OIS rates and zero-coupon inflation swap rates, except for Brazilian real for which we used the implied zero-coupon inflation swap rate from real bond prices.

Zero coupon nominal rates

As our zero coupon nominal rates we use the discount rates implied by overnight index swaps (OIS). An OIS is a swap with floating payments based on a reference rate for unsecured overnight funding and paid at fixed intervals.[5] Over the lifetime of the contracts counterparties post collateral to keep the net value of the swap contract at zero. In early 2000, the International Swaps and Derivatives Association (ISDA), a global trade organization of participants in the market for over the counter derivatives, conducted a survey that found more than 65% of plain vanilla derivatives, especially interest rate swaps, were collateralized according to the legal standard for collateralization set out in the Credit Support Annex (CSA), a legal document of recommendations in ISDA's master agreement which regulates collateral for derivatives transactions; moreover, they were bilateral in that both counterparties, not just the lower rated counterparty posted collateral (Johannes and Sundaresan (2006)). Thus,

[5] The reference rates are in the Appendix.

the OIS is only exposed to overnight credit risk and is broadly considered "risk free."[6] While OIS contract is approximately risk free, the rate itself is influenced by the credit risk of the refreshed panel of participants in unsecured overnight funding markets.[7] Hull and White (2015) argue that OIS is the best proxy for the risk-free rate; furthermore, post-crisis, practitioners have switched from using LIBOR zero curve to the OIS zero curve for discounting. The use of OIS constrains our sample size as data on long maturity swaps are only available on Bloomberg after 2009. We pivileged the use of OIS rates over government rates because of the TIPS-Treasury puzzle, which indicates "mispricing" in real and/or nominal government cash bond markets in the U.S. (Fleming et al. (2014)), Japan, U.K., Germany (Kita and Tortice, 2019).

In the developed markets, one may be concerned that the "risk-free approximation" error is non-trivial. To alleviate this concern, for the South African rand and the Brazilian real, we construct synthetic OIS rates equal to the OIS rates that would hold if the domestic OIS rate was "risk-free" and covered interest parity held. Let $y_t^{(N)}$ denote the time t, maturity N nominal, domestic OIS rate. Let $\rho_t^{(N)}$ denote the time t, maturity N forward premium for the foreign currency vis-a-vis the domestic currency; a positive premium implies a risk-neutral expected depreciation of the domestic currency. Covered interest rate parity implies that the time t, maturity N, nominal foreign OIS rate, which we denote by $y_t^{*,CIP,(N)}$ is

$$y_t^{*,CIP,(N)} = y_t^{(N)} - \rho. \tag{34}$$

Du, Tepper, Verdelhan document CIP deviations in interbank lending markets that cannot be explained away by credit risk or transaction costs for G10 currencies. They argue that wide deviations on contracts that are on banks' balance sheets at quarter end suggest regulatory capital costs are driving the wedge at quarter end. This would affect developing market currencies as well.

[6] Arora, Gandhi, and Longstaff (2012) study the effect of counterparty credit risk on the pricing of one type of swaps, credit default swaps. They show that differences in the credit risk of dealers selling credit protection have a very small effect on the pricing of CDS contracts; in particular, an increase in the credit spread of a dealer of about 645 basis points maps into only a one-basis-point decline in the price of credit protection (the value of the credit protection being sold is diminished). Their sample, however is based on a set of fourteen well known dealers in the U.S. and U.K. who may not be as active in the OIS markets of emerging market economies. Furthermore, identification is based on variation among private dealers; the paper does not quantify the fixed effect arising from the credit risk of private dealers as a whole.

[7] Measuring the price of credit risk of the refreshed interbank panel, and thus, the "risk-free approximation" error, however, requires the elusive risk-free benchmark.

The authors estimate that for 2010 to 2016 sample, the average annualized absolute value of the basis ranged from -60 to 30 bps at the five-year horizon. For context, the level of LIBOR ranged from around 25 bps to 400 bps. The heterogeneity across currencies points to the cost not being fixed for all currencies, which means we cannot correct our synthetic South African rand, and Brazilian real OIS rates for these deviations by simply subtracting a level factor.

Zero coupon inflation compensation rates

Zero coupon inflation compensation rates are the rates implied by zero-coupon inflation swaps for all currencies, except in the case of the Brazilian real, for which we computed the inflation compensation rates from the rates on zero-coupon nominal, and real bonds; this was possible because Brazilian real bonds do not contain options and their zero rates are published on Bloomberg. For obtaining zero coupon inflation compensation rates, we privileged the use of inflation swaps rates over real bond bond rates because of their standardization across currencies.[8]

Two concerns that arise with the use of inflation swaps data is liquidity and also publication, and contractual lags.

The literature on the liquidity of inflation swap markets tends to focus on those for the United States and the United Kingdom, in which they have found to be reasonably liquid and the prices reliable. For example, Fleming and Sporn (2019) estimate realized bid-ask spreads for customers are on the order of three basis points. As in Fleming, Longstaff, and Lustig (2014), however, to guard against using stale prices in the sample, we only include an inflation swap rate when the rate has changed from the previous day; that we use data aggregated to the monthly frequency in our empirical work should also mitigate concerns of stale prices.

In constructing zero-coupon real rates from nominal, and inflation compensation rates, we implicitly assume that the inflation swap of a given maturity compensats the floating leg investor with the actual, realized inflation of the

[8]Across currencies, real bonds differ along issuance, and the presence and kinds of embedded options. On the first, some currencies only have a few real bonds outstanding, and the heterogeniety in maturities makes it difficult to boostrap zero coupon rates with confidence. This is contrast to inflation swap markets, which typically have liquid quotes at multiple and broadly spaced maturities. On the second, many real bonds feature embedded deflation floors. For example, U.S. real bonds, formally referred to as U.S. Treasury Inflation-Indexed Securities and informally as U.S. Treasury Inflation Protected Securities (TIPS), have deflation floors to protect the principal value in inflation, whereas comparable U.K. real bonds, formally referred to as U.K. Index-Linked Gilts, do not. This is in contrast to inflation swaps for which the floating leg does not have embedded options.

swap's lifetime. This is not strictly true. National price level data take time to collect and construct and are published with a lag, introducing a publication lag. Furthermore, the swaps themselves specify cash flows to be exchanged based on lagged values of the inflation index, introducing a contractual lag. The contractual lag varies across currencies for swaps, but with significantly less variation than for real bonds. For example, U.S. swaps (real bonds) are linked to their inflation index series with a three-month (three-month) lag, whereas U.K. swaps are linked to their inflation index series with a two-month (three to eight-month lag). The publication lag implies that without perfect forecasting abilities, investors cannot know the true real exchange rate today because the national price levels relevant to its computation have yet to be released. This issue is somewhat mitigated by the fact that our data are at the monthly frequency and publication lags are less than a month. The log real bond holding period return will be affected by both the publication and contractual lags. The price impact of these lags is not well studied for our sample of currencies. One study that has some bearing on this issue is Christensen (2018), who exploits variation in the indexation lags of U.K. real bonds and estimates the yield difference between two real bonds identical except in that one has a three-month indexation lag, and the other an eight-month indexation lag, is around two to three basis points for ten-year bonds. While the study does not directly address the effect of contractual lags on the measured versus true log real bond holding period returns, if the pricing effect of the inflation lag is around the same order of magnitude, it should not have a large effect on our empirical results. We do not explicitly account for publication or contractual lags in our study.

Zero coupon real bond rates

By the law of one price, the price of the N-maturity, zero coupon domestic risk-free real bond $P_t^{(N)}$ can be expressed as a function of the corresponding zero-coupon inflation swap $i_t^{(N)}$ and nominal bond rate $y_t^{(N)}$

$$P_t^{(N)} = \frac{(1 + i_t^{(N)})^N}{(1 + y_t^{(N)})^N}. \tag{35}$$

One concern is that this relation may not strictly hold, and therefore, that the real bond price cannot be replicated in this way. In the United States, Fleckstein, Longstaff, and Lustig (2009) find that sovereign bonds are overpriced relative to their inflation indexed counterparts TIPS, by a weighted average of \$2.92, but with significant variation, and a maximum in their sample of nearly \$10. They provide suggestive evidence that this is not due to mispricing in the inflation swaps market: in the corporate bond market, nominal and inflation indexed bond mispricing is minimal, with an average of 1-7 basis points, with standard errors of 60-70 basis points. Even though they do not explicitly account for options, accounting for them would go in the opposite direction–nominal sovereign bonds would be even more overpriced. Kita and Tortorice (2019) document mispricing of similar size and the same direction in the sovereign bond markets for Japan and Germany. Whether the relative mispricing exists because of distortions in the price level of Treasuries, TIPS, or both from their levels in the envisioned, background no arbitrage world is unclear.

National price level series

Countries release multiple national price level series. The national price level series in our sample are those referenced by the relevant inflation swap or real bond contracts. Across currencies, these series differ in the real baskets they reference and the timeliness of their releases. For example, the Japanese index references a core basket of real goods that is meant to capture the average consumer's real consumption basket excluding perishables, a sizeable but relatively volatile component of consumer consumption baskets, whereas the Swedish index references a headline basket of real goods that includes perishables. Meanwhile, the Brazilian general index of market prices (IGP-M) is not simply the consumer price index, but the arithmetic weighted average of the wholesale price index, consumer price index, and national construction cost index. These differences are important in interpreting the empirical results. The empirical results can be validly extrapolated to generate restrictions on models that generate real stochastic discount factors if in the underlying model, the marginal utility of the stand in investor for a given country has utility over the national consumption basket referenced by the inflation index series we use. All series are monthly with the exception of Australia's, which is quarterly and is converted into a monthly series by making month end observation the latest quarterly observation.

Table 3: National price level index series

Currency	National price level index series	Frequency	Publication lag (≈months)	Contract	Contractual lag (months)
British pound	Retail Price Index (RPI)	Monthly	0.5	Inflation swap	2
Japanese yen	Japanese Consumer Price Index ex perishables	Monthly	1	Inflation swap	3
euro	Harmonized Index of Consumer Prices, All items	Monthly	0.5	Inflation swap	3
Swedish krona	Sweden Consumer Price Index	Monthly	0.5	Inflation swap	3
U.S. dollar	Consumer Price Index, All urban consumers (CPI-U)	Monthly	0.5	Inflation swap	3
Australian dollar	Australia CPI, All groups	Quarterly	1	Inflation swap	3
S. African rand	South Africa CPI, Metropolian areas	Monthly	0.5	Inflation swap	4
Brazilian real real	FGV Brazil General Prices IGP-M	Monthly	0.5	Real bond	1

Notes: This table reports the national price level series referenced by the zero coupon inflation swap contracts, or in the case of the Brazilian real, the real bond contract. Additionally, it gives the release frequency, the publication lag of the national price level series, and the contractual lag specified in the relevant securities contract. Information was obtained from Bloomberg and national statistical offices.

3.2.3 Nominal versus real log holding period returns

One may wonder why we bother to compute real log holding period returns. Thus, we consider when the nominal and real regressions are equal. Suppose the null hypothesis of common permanent components holds not just for real stochastic real discount factors, but for nominal ones as well. The nominal and real regressions will be equal when

$$\pi_{t+1} - \pi^*_{t+1} = hpr^{(10),\pi}_{t+1} - hpr^{*(10),\pi}_{t+1} \tag{36}$$

a special case of which is when log inflation is equal to the log holding period return on the ten-year inflation swap

$$\pi_{t+1} = hpr^{(10),\pi}_{t+1} \tag{37}$$

$$\pi^*_{t+1} = hpr^{*(10),\pi}_{t+1}. \tag{38}$$

To see this, note that

$$hpr_{t+1}^{(10)} \equiv log(\frac{\left[\frac{(1+i_t^{(9\frac{11}{12})})}{(1+y_t^{(9\frac{11}{12})})}\right]^{9\frac{11}{12}}}{\left[\frac{(1+i_t^{(10)})}{(1+y_t^{(10)})}\right]^{10}}) \tag{39}$$

$$= log(\frac{\left[\frac{1}{(1+y_t^{(9\frac{11}{12})})}\right]^{9\frac{11}{12}}}{\left[\frac{1}{(1+y_t^{(10)})}\right]^{10}}) - log(\frac{\left[\frac{1}{(1+i_t^{(9\frac{11}{12})})}\right]^{9\frac{11}{12}}}{\left[\frac{1}{(1+i_t^{(10)})}\right]^{10}}) \tag{40}$$

$$= hpr_{t+1}^{(10),nom} - hpr_{t+1}^{(10),\pi} \tag{41}$$

Under the null, the nominal regression will be

$$\Delta s_{t+1} = hpr_{t+1}^{*,nom,(10)} - hpr_{t+1}^{nom,(10)} \tag{42}$$

Under the null, the real regression will be

$$\Delta q_{t+1} = hpr_{t+1}^{*,(10)} + hpr_{t+1}^{(10)} \tag{43}$$

$$\Delta s_{t+1} + (\pi_{t+1} - \pi_{t+1}^*) = hpr_{t+1}^{*,nom,(10)} - hpr_{t+1}^{nom,(10)} + (hpr_{t+1}^{(10),\pi} - hpr_{t+1}^{*(10),\pi}) \tag{44}$$

where $hpr_{t+1}^{(10)}$ is the log holding period return on the real bond, $hpr_{t+1}^{(10),nom}$ is the log holding period return on the nominal bond, $hpr_{t+1}^{(10),\pi}$ is the log holding period return on the inflation swap, and the corresponding variables with stars are the foreign counterparts of the domestic variables just defined.

In the data, inflation differentials and ten-year inflation swap holding period return differentials are weakly correlated. In Table 3.2.3, we report the regression results of regressing the former on the latter; regression coefficients are near zero, and R^2 are near zero.

Table 4: Inflation differentials and ten-year inflation swap holding period return differentials

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$	$(\pi_{t+1} - \pi^*_{t+1})$
$(hpr^{(10),\pi}_{t+1} - hpr^{*(10),\pi}_{t+1})$	0.00	-0.06**	-0.02	-0.02	-0.02	-0.01	-0.02
	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.01)	(0.03)
Constant	0.00***	0.00***	0.00***	0.00***	0.00***	-0.00***	-0.00***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	143	143	143	143	143	143	101
R^2	0.00	0.04	0.00	0.01	0.00	0.01	0.00
	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
Maturity	10y	10y	10y	10y	10y	10y	10y
Freq.	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly

Notes: Panel A reports the results of a regression of inflation differentials on ten-year inflation swap holding period return differentials.

4 Results

4.1 Real

Earlier, we derived the relationship between log changes in the real exchange rate and contemporaneous holding period returns on real, ten-year foreign and domestic bonds under a set of four assumptions. It directly motivates the following test, which, in this section, we implement and report on.

Let t index time and i the foreign currency, this directly motivates a regression of the form

$$\Delta q_{t+1} = \alpha + \beta_1 \, hpr^{*,(10)}_{t+1} + \beta_2 \, hpr^{(10)}_{t+1} + \epsilon_t \tag{45}$$

where Δq_{t+1} is the log change in the real exchange rate, $hpr^{*,(10)}_{t+1}$ is the log holding period return on the foreign, ten-year real bond, and $hpr^{(10)}_{t+1}$ is the log holding period return on the domestic, ten-year real bond. The first testable implication is that the constant term is equal to zero, and coefficients on the foreign and domestic holding period returns equal 1 and -1, respectively.

$$\text{Test I: } H_0 : \alpha = 0; \beta_1 = 1; \beta_2 = -1 \tag{46}$$

$$H_a : \alpha \neq 0; \beta_1 \neq 1; \beta_2 \neq -1. \tag{47}$$

We denote this test the "Coefficient Test" and report a rejection if the parameters are jointly insignificant at the 5% level.

The second testable implication is that the R^2 on the regression is equal to one.

$$\text{Test II: } H_0 : R^2 = 1 \tag{48}$$

$$H_a : R^2 < 1. \tag{49}$$

We denote this test the "R^2 Test" and report a rejection if the parameter under the null is not in the confidence set at the 5% level and an acceptance otherwise. Since the parameter under the null is on the boundary of the parameter space it has a non-standard distribution and traditional bootstrap will generate inconsistent estimators. We do inference using pseudo time series drawn by subsampling (Politis and Romano (1994)). The Appendix contains the algorithm description. The value of this test and of generating confidence intervals based on subsampling in this setting is questionable because of the small sample size; however, it is the only method that has a chance of being asymptotically consistency. [9]

The regression is valid at all frequencies. Since test power increases with sample size, we maximize the number of obesrvations by running the regression at the monthly frequency, which corresponds to the most common release frequency of the national price level series used to construct real exchange rates.

We report the regression point estimates and test outcomes in Table 4.1. We describe the results and then discuss their theoretical implications.

Columns 1-7 report the estimation results from regressing monthly change in the log of real exchange rates denoted by Δq_{t+1} on the contemporaneous holding period return on the real, ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,(10)}$, and the real ten-year domestic bond in domestic currency numeraire $hpr_{t+1}^{(10)}$. Under the null, the coefficient on foreign returns should be one, which implies that a 1% log holding period return on the real, ten-year foreign bond should be associated with a contemporaneous 1% depreciation in the log of real foreign currency, the coefficient on domestic returns should be one, which implies that a 1% log holding period return on the real, ten-year domestic bond should be associated with a contemporaneous 1% appreciation in the log of real foreign currency.

For the set of developed market currencies, the point estimates are of the correct sign: the coefficient on the foreign log holding period return is positive, and the coefficient on the domestic log holding period return is negative. This implies that when there is positive shock to foreign log holding period returns in local currency terms, the foreign currency will tend to contemporaneously

[9] We question the value for two reasons. The first reason is mechanical: the sample is short, and the block length, which was set to twelve to balance the need for a reasonably sized set of subsamples with the need to keep the subsample length large enough to capture data dependence across time, is short; proofs of subsample estimator consistency, however, depend on asymptotics on the sample, and block size. The proof is a result that is outside the practical implementation of subsampling; both the size of subsamples, and the length of a subsample are finite in the practical implementation. The second reason is a repeat of Bayesian criticism of frequentist inference that seems particularly intuitive in this case. The frequentist 95% confidence interval constains the true parameter with probability 95% before one has seen the data. We have seen the data, and the probability that it is in the confidence interval is exactly zero. Our theory is about point-by-point realizations. If our data are all precisely, and correctly measured, a finite sample R^2 less than one is automatic rejection of the hypothesis of point-by-point realizations. Allowing for the possibility that a finite sample R^2 of less than one does not imply automatic rejection requires some idea of why the former might hold, such as measurement error. Suppose the countable, data generating process is such that the null holds true except occasionally. Since the data generating process is countable, in any finite sample, as we take sample size to infinity, the R^2 will approach, but never equal one. Should we accept a hypothesis that makes predictions that will always be rejected in the finite samples?

depreciate; when there is positive shock to domestic log holding period returns, the foreign currency will tend to contemporaneously appreciate. However, while the coefficient on the domestic return is significant across developed market currencies at the 1% level, the coefficient on the foreign return is only significant at the 1% level for the euro, U.S. dollar, and the Australian dollar.

For the set of developing market currencies, the point estimates do not all have the correct sign. In particular, point estimates on foreign log holding period returns are of the wrong sign: they are negative. However, the point estimates on domestic log holding period returns are of the correct sign: they are positive. This implies that when there is positive shock to foreign or domestic log holding period returns, the foreign currency will tend to contemporaneously appreciate. It is possible the sign of the true coefficients are the same as the sign of the estimated coefficients; alternatively, it may be that OIS swaps in developing market economies have sizable and variable risk premia compared to developed market economies and that not controlling for these either introduces an omitted variable bias or introduces noise in our measurement that in our finite sample generates statistically significant coefficients in the wrong direction.

Adversely towards the null, the coefficients vary in magnitude across currencies: the coeffiicent on foreign returns range from 0.16 to 0.76; the coefficient on domestic returns range from -0.49 to -1.00. The R^2 on the regressions range from 8% for the Swedish krona to 33% for the Brazilian real, with standard errors 18% and 14%, respectively. At the 5% level, we reject the null hypotheses that the coefficients on the foreign and domestic holding period returns equal to 1 and -1, respectively, as well as the null hypothesis that the R^2 is one.

Columns 8-9 report the results of a regression of the monthly change in real exchange rates denoted by Δq_{t+1}on the contemporaneous holding period return on the real, CIP implied ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,CIP,(10)}$, and the real ten-year foreign domestic in domestic currency numeraire $hpr_{t+1}^{(10)}$ for the South African rand and the Brazilian real. If the domestic OIS rate is truly risk-free, and covered interest parity held, then $hpr_{t+1}^{*,CIP,(10)}$, not $hpr_{t+1}^{*,(10)}$ is the correct empirical counterpart to the ten-year real bond holding period returns in our derivation. Notably, the sign of the coefficient on the foreign holding period bond return reverses signs, and becomes positive (i.e. in the direction of long real bond return parity) and significant, changing from -0.20 to 0.37 for the South African rand, and -0.76 to 0.28 for the Brazilian real. They are, however, still far from the null of 1. The coefficient on the domestic holding period bond return becomes more negative and closer to the null of -1, changing from -0.36 to -0.69 for the South African rand, and from -0.75 to -1.13 for the British real.

Taken together, the results of the real regression estimation uniformly reject the joint hypothesis.

Table 5: Benchmark regression using private OIS rates (real)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}	Δq_{t+1}
	Panel A: Point estimates								
$hpr_{t+1}^{*,(10)}$	0.16	0.59***	0.21	0.40***	0.61***	-0.20	-0.76***		
	(0.19)	(0.12)	(0.13)	(0.11)	(0.14)	(0.18)	(0.14)		
$hpr_{t+1}^{*,CIP,(10)}$								0.37***	0.28***
								(0.05)	(0.04)
$hpr_{t+1}^{(10)}$	-1.00***	-0.61***	-0.49***	-0.95***	-0.88***	-0.36*	-0.75***	-0.69***	-1.13***
	(0.15)	(0.11)	(0.14)	(0.12)	(0.16)	(0.21)	(0.23)	(0.18)	(0.22)
Constant	0.00*	0.00	0.00	0.00	-0.00	0.00	0.01**	0.00	0.01*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. Obs	143	143	143	143	143	143	101	143	101
R^2	0.24	0.19	0.08	0.29	0.19	0.03	0.33	0.27	0.37
R^2 std. dev.	0.22	0.19	0.18	0.20	0.19	0.16	0.14	0.22	0.24
Currency	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real	South African rand	Brazilian real
Maturity	10y	10y	10y	10y	10y	10y	10y	10y	10y
Freq.	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
	Panel B: Test results								
Coefficient Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
R^2 Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject

Notes: Panel A, Columns 1-7 report the results of a regression of the monthly change in the log of real exchange rates denoted by Δq_{t+1}on the contemporaneous holding period return on the real, ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,(10)}$, and the real ten-year domestic bond in domestic currency numeraire $hpr_{t+1}^{(10)}$. Columns 8-9 report the results of a regression of the monthly change in real exchange rates denoted by Δq_{t+1}on the contemporaneous holding period return on the real, CIP implied ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,CIP,(10)}$, and the real ten-year domestic bond in domestic currency numeraire $hpr_{t+1}^{(10)}$ for the South African rand and the Brazilian real. Newey-standard errors are used with a twelve-month lag and one, two, and three stars denote significance levels at the 10, 5, 1 percent confidence levels. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Reported also are data characteristics: the number of observations, the regression R^2, the standard deviation of pseudo time series generated from subsampling from the data following the Politis, Romano (1994) algorithm with a block length of twelve, the currency, the maturity of the bonds for which holding period returns were constructed, and data frequency. Panel C reports the results of our hypothesis tests. The row Coefficient Test reports on whether we reject or accept the null hypothesis that the coefficient on foreign holding period returns is +1, and the coefficient on domestic holding period returns is -1 at the 5% level. The row R^2 Test reports on whether we reject or accept the null hypothesis that the R^2 is equal to 1 at the 5% level.

4.2 Nominal

If our assumptions hold in nominal terms, we can run the regression in nominal terms as well.

Let t index time and i the foreign currency, this directly motivates a regression of the form

$$\Delta s_{t+1} = \alpha + \beta_1 \, hpr_{t+1}^{*,nom,(10)} + \beta_2 \, hpr_{t+1}^{nom,(10)} + \epsilon_t \tag{50}$$

where Δs_{t+1} is the log change in the nominal exchange rate, $hpr_{t+1}^{*,nom,(10)}$ is the log holding period return on the foreign, ten-year nominal bond, and $hpr_{t+1}^{nom,(10)}$ is the log holding period return on the domestic, ten-year nominal bond. As before, the first testable implication is that the constant term is equal to zero, and coefficients on the foreign and domestic holding period returns equal 1 and -1, respectively; the second testable implication is that the R^2 on the regression is equal to one.

We report the regression results in Table 4.2.

Columns 1-7 report the estimation results from regressing monthly change in the log of nominal exchange rates denoted by Δs_{t+1} on the contemporaneous holding period return on the nominal, ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,nom,(10)}$, and the nominal ten-year foreign domestic in domestic currency numeraire $hpr_{t+1}^{nom,(10)}$. Under the null, the coefficient on foreign returns should be one, which implies that a 1% log holding period return on the nominal, ten-year foreign bond should be associated with a contemporaneous 1% depreciation in the log or nominal foreign currency, the coefficient on domestic returns should be one, which implies that a 1% log holding period return on the nominal, ten-year domestic bond should be associated with a contemporaneous 1% appreciation in the log of nominal foreign currency.

For the set of developed market currencies, the point estimates, with the exception of the Japanese yen, are of the correct sign: the coefficient on the foreign log holding period return is positive and the coefficient on the domestic log holding period return is negative. Comparing the regression estimation results from using nominal variables versus real variables, we find that with the exception of the euro and U.S. dollar, the R^2 are higher in the nominal regressions than in the real regressions for our set of currencies. This makes sense because of the weak time series relation between log inflation differentials and log inflation swap holding period return differentials. The two series look as if generated by a process centered around a mean that is subject to weakly correlated mean zero noise innovations. Thus, innovations to inflation differentials will tend to decrease the R^2 in the real regressions vis-a-vis the nominal regressions since it will add variation to the depedent variable not explained by inflation swap holding period return differentials, and this effect appears to dominate for most currencies.

For the set of developing market currencies, coefficients are of the wrong sign: they are negative for foreign bond returns, and positive for domestic bond returns, implying that a positive shock to foreign (domestic) bond returns are, in domestic currency terms, amplified by foreign currency appreciation (depreciation).

The null hypothesis that the coefficients on the foreign and domestic holding period returns equal to 1 and -1, respectively, is accepted for the Swedish krona and the Australian dollar at the 5% level, and rejected for all other currencies. The null hypothesis that the R^2 is one is rejected for all currencies.

Columns 8-9 report the results of a regression of the monthly change in the log of nominal exchange rates denoted by Δs_{t+1}on the contemporaneous holding period return on the nominal, CIP implied ten-year foreign bond in foreign currency numeraire $hpr_{t+1}^{*,nom,CIP,(10)}$, and the nominal ten-year domestic currency in domestic currency numeraire $hpr_{t+1}^{nom,(10)}$ for the South African rand and the Brazilian real. If the domestic OIS rate is truly risk-free, and covered interest parity held, then $hpr_{t+1}^{*,CIP,(10)}$, not $hpr_{t+1}^{*,(10)}$ is the correct empirical counterpart to the ten-year real bond holding period returns in our derivation. Notably, the sign of the coefficient on the foreign holding period bond return reverses signs, and becomes positive (i.e. in the direction of long real bond return parity), changing from -0.69 to 0.58 for the South African rand, and -0.47 to 0.44 for the Brazilian real. They are, however, still far from the null of 1. The coefficient on the domestic bond return also reverses signs, and becomes negative (i.e. in the direction of long real bond return parity) and closer to the null of -1, changing from 0.08 to -0.99 for the South African rand, and from 0.44 to -1.08 for the British real.

Taken together, the results of the real regression estimation uniformly reject the joint hypothesis.

Table 6: Benchmark regression using private OIS rates (nominal)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}	Δs_{t+1}
	Panel A: Point estimates								
$hpr_{t+1}^{*,nom,(10)}$	-0.96**	0.66***	0.91***	0.20	0.87***	-0.69***	-0.47***		
	(0.39)	(0.16)	(0.17)	(0.14)	(0.14)	(0.10)	(0.06)		
$hpr_{t+1}^{*,nom,CIP,(10)}$								0.58***	0.44***
								(0.10)	(0.07)
$hpr_{t+1}^{nom,(10)}$	-0.88***	-0.59***	-0.83***	-0.79***	-1.03***	0.08	-0.25	-0.99***	-1.08***
	(0.15)	(0.14)	(0.17)	(0.17)	(0.17)	(0.18)	(0.20)	(0.20)	(0.23)
Constant	0.01**	-0.00	-0.00	0.00	-0.00	0.01**	0.01***	0.01**	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
N. Obs	143	143	143	143	143	143	101	143	101
R^2	0.33	0.11	0.18	0.21	0.23	0.27	0.38	0.22	0.30
std. dev. of R^2	0.20	0.16	0.17	0.24	0.17	0.22	0.16	0.22	0.20
Currency	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real	South African rand	Brazilian real
Maturity	10y	10y	10y	10y	10y	10y	10y	10y	10y
Freq.	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
	Panel B: Test results								
Coefficient Test	Reject	Reject	Accept	Reject	Accept	Reject	Reject	Reject	Reject
R^2 Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject

Notes: Panel A Columns 1-7 report the results of a regression of the monthly change in the log of nominal exchange rates denoted byΔs_{t+1} on the contemporaneous log holding period return on the nominal, ten-year foreign bond in foreign currency $hpr_{t+1}^{*,nom,(10)}$ and the nominal, ten-year domestic bond in domestic currency $hpr_{t+1}^{nom,(10)}$. Columns 8-9 report the results of the regression of the monthly log change in nominal exchange rates denoted byΔs_{t+1} on the contemporaneous log holding period return on the nominal, CIP implied ten-year foreign bond in foreign currency $hpr_{t+1}^{*,nom,CIP,(10)}$ and the nominal, ten-year domestic bond in domestic currency $hpr_{t+1}^{nom,(10)}$. Newey-standard errors are used with a twelve-month lag and one, two, and three stars denote significance levels at the 10, 5, 1 percent confidence levels. The domestic currency is the British pound. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Reported also are data characteristics: the number of observations, the regression R^2, the standard deviation of pseudo time series generated from subsampling from the data following the Politis, Romano (1994) algorithm with a block length of twelve, the currency, the maturity of the bonds for which holding period returns were constructed, and data frequency. Panel C reports the results of our hypothesis tests. The row Coefficient Test reports on whether we reject or accept the null hypothesis that the coefficient on foreign holding period returns is $+1$, and the coefficient on domestic holding period returns is -1 at the 5% level. The row R^2 Test reports on whether we reject or accept the null hypothesis that the R^2 is equal to 1 at the 5% level.

5 Conclusion

In this paper, we considered a specification for real stochastic discount factors in which permanent, martingale components are common across currencies, and tested this assumption jointly with the assumptions that the regularity conditions for the decomposition are satisfied, markets are complete, and ten-year bond returns approximate infinite bond returns. Under these four assumptions, we showed that log real exchange rate changes should be positively correlated with contemporaneous log holding period returns on ten-year foreign real bonds, and negatively correlated with contemporaneous log holding period returns on ten-year domestic bonds; moreover, these relationships should be one-to-one. When using OIS zero rates, the sign of the estimated relationship is correct for developed market currencies, they are not for developing market currencies. Adversely to the joint hypothesis, the magnitude of the relationship, does not align with the theory. A test of the one-to-one relationship and of an R^2 of one is rejected for all currencies by the data. Therefore, we reject the joint hypothesis that all four assumptions jointly hold. An open question is the following: the failure of which assumptions or set of assumptions is primarily responsible for the magnitude of the empirical violation? We leave this for future research.

At the same time, in our post-2009 sample, we document a strong relationship between changes in the log real exchange rate and real holding period bond returns in the direction of parity, contributing to the general question of what explains exchange rate changes (Ferraro, Rossi, and Rogoff (2011), Verdelhan (2018), Jiang, Krishnamurthy, Lustig (2018)).

References

Alvarez, F. and U. J. Jermann (2005): "Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth," *Econometrica*, 73, 1977–2016.

Andrews, D. W. K. (2000): "Inconsistency of the Bootstrap when a Parameter is on the Boundary of the Parameter Space," *Econometrica*, 68, 300–405.

Arora, N., P. Gandhi, and F. A. Longstaff (2012): "Counterparty credit risk and the credit default swap market," *Journal of Financial Economics*, 103, 280–293.

Avdjiev, S., W. Du, C. Koch, and H. S. Shin (2019): "The Dollar, Bank Leverage, and Deviations from Covered Interest Parity," *American Economic Review: Insights*, 193–208.

Backus, D. K., S. Foresi, and C. I. Telmer (2001): "Affine Term Structure Models and the Forward Premium," *Journal of Finance*, 279–304.

Christensen, J. H. (2018): "Do Adjustment Lags Matter for Inflation-Indexed Bonds?" Tech. rep., Federal Reserve Bank of San Francisco.

Du, W., J. Im, and J. Schreger (2018a): "The U.S. Treasury Premium," *Journal of International Economics*, 112, 167–181.

Du, W. and J. Schreger (2016): "Local Currency Sovereign Risk," *Journal of Finance*, 71, 1027–1070.

Du, W., A. Verdelhan, and A. Tepper (2018b): "Deviations from Covered Interest Rate Parity," *Journal of Finance*, 73(3), 915–957.

Ferraro, D., K. Rogoff, and B. Rossi (2015): "Can Oil Prices Forecast Exchange Rates?" *Journal of International Money and Finance*, 116–141.

Filipovic, D. and A. B. Trolle (2013): "The term structure of interbank risk," *Journal of Financial Economics*, 109(3), 707–733.

Fleckenstein, M., F. A. Longstaff, and H. Lustig (2013): "Deflation Risk," Working paper, NBER.

Fleckstein, M., F. A. Longstaff, and H. Lustig (2014): "The TIPS-Treasury Bond Puzzle," *The Journal of Finance*, 69, 2151–2197.

FLEMING, M. J. AND J. R. SPORN (2013): "Trading Activity and Price Transparency in the Inflation Swap Market," *FRBNY Economic Policy Review.*

HULL, J. AND A. WHITE (2015): "OIS Discounting, Interest Rate Derivatives, and the Modeling of Stochastic Interest Rate Spreads," *Journal of Investment Management*, 13, 64–83.

JIANG, Z., A. KRISHNAMURTHY, AND H. LUSTIG (2018): "Foreign Safe Asset Demand and the Dollar Exchange Rate," Tech. rep., NBER.

JOHANNES, M. AND S. SUNDARESAN (2007): "The Impact of Collateralization on Swap Rates," *Journal of Finance*, 62(1), 383–410.

LUSTIG, H., A. STATHOPOULOS, AND A. VERDELHAN (2019): "The Term Structure of Currency Carry Trade Risk Premia," *American Economic Review*, 109, 4142–77.

LUSTIG, H. AND A. VERDELHAN (2007): "The Cross Section of Foreign Currency Risk Premia and Consumption Growth Risk," *American Economic Review*, 97, 89–117.

MEESE, R. AND K. ROGOFF (1983): "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample," *Journal of International Economics.*

POLITIS, D. N. AND J. P. ROMANO (1992): "A Circular Block Resampling Procedure for Stationary Data," in *Exploring the Limits of Bootstrap.*

——— (1994): "Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions," *Annals of Statistics*, 22, 2031–2050.

TORTORICE, D. L. AND A. KITA (2018): "Can Risk Models Extract Inflation Expectations from Financial Market Data? Evidence from the Inflation Protected Securities of Six Countries," Working paper, College of the Holy Cross, Department of Economics.

VERDELHAN, A. (2018): "The Share of Systematic Variation in Bilateral Exchange Rates," *Journal of Finance.*

A Alvarez and Jermann decomposition regularity conditions

The regularity conditions in Alvarez and Jermann (2005) are described below.

First, there exists some δ such that

$$0 < \lim_{k\to\infty} \frac{P_t^{(k)}}{\delta^k} < \infty, \forall t \tag{51}$$

which requires the average yield $-1/kP_t^{(k)}$ converges fast enough to a finite, positive value. Second, for each $t+1$ there exist a random variable with finite expected value $E_t(x_{t+1})$ such that $\forall k$ almost surely

$$\frac{\Lambda_{t+1}}{\delta^{t+1}} \frac{P_{t+1}^{(k)}}{\delta^k} \leq x_{t+1} \tag{52}$$

which says the marginal valuation multiplied by $\frac{P_{t+1}^{(k)}}{\delta^k}$ is bounded by a variable that has a finite conditional expectation.

They also maintain the background assumption that stochastic discount factors are strictly stationary. That is, the random vectors

$$\{\frac{\Lambda_{t+1+k}}{\Lambda_{t+k}}\}_{k=i}^{k=i+j}, \{\frac{\Lambda_{t+1+k}}{\Lambda_{t+k}}\}_{k=i}^{k=i+j} \tag{53}$$

have the same joint distribution $\forall i, j$. This implies that all bond returns are stationary.

B Drawing pseudo time series

B.1 Subsampling

Unlike bootstrap, the resample size is smaller than the sample size and resampling is done without replacement. In the simple i.i.d. case, we generate q equals n choose b blocks of size b from the data, where n is data size We generate the bootstrapped statistic with each block of data. We then use the empirical distribution over the q pseudo estimate, where the empirical estimate is treated as if the true mean, to approximate the distribution of the empirical estimate. With time series, one must respect the ordering of the data to preserve time series dependence, so we draw blocks with contiguous observations.

The results about estimates from subsampling come from proofs that take $b, n \to \infty, \frac{b}{n} \to \infty$. In implementation, we are faced with a finite sample, and thus, constrained to a finite b. The asymptotic conditions provide little guidance on how to choose b in this case; as Politis, Romano, and Wolf note, the choice depends on the purpose. We wish in subsampling to preserve time series dependence, and based on autocorrelation charts, choose a block length of fourteen.

Han Hong's primer on subsampling is available here:
`https://web.stanford.edu/~doubleh/lecturenotes/lecture13.pdf`

B.2 Monte-Carlo and stationary block bootstrap

The Politis, Romano (1992) algorithm for the stationary block bootstrap is a way for sampling the data in a way that preserves any time dependence in the data, and stationary. It works as follows. Suppose one has a dataset of length N, and wishes to generate nB pseudo time series. First, one splices the initial data into a series of N blocks. In constructing these blocks, the data are treated as circular so that the next observation after the "last" observation is the "first observation." Two sets of indices of length nB are then randomly generated. The first set of indices is drawn from a uniform distribution. The second set of indices is drawn from a geometric distribution. To generate a pseudo time series, you "choose" and then "stitch together." The choosing is as follows: the first index determines the block number to choose, the second index determines the length of continuous observations to take from that block number.

Both the Monte Carlo algorithm and the Politis, Romano (1992) algorithm can be implemented by installing the R package tseries, and using the function tsbootstrap. The first corresponds to the block type bootstrap where the length of the block is equal to one. The second corresponds to the block type stationary where the length of the mean block is equal to what is desired.

B.3 Cross section

Our motivation for testing the assumption of common permanent components in real currency stochastic discount factors was the finding by Lustig et al. (2019) that average excess returns on the carry trade using long-run bonds and with a three-month horizon, is zero, suggesting that the entropies of the permanent components are equal. Their result is in nominal terms and on currency portfolios. They argue that the inflation characteristics of portfolios suggest the result holds also in real terms. We find suggestive evidence that this is true when we look at the individual currencies in our sample.

B.3.1 Estimated average excess returns by currency

For each currency i, we estimate the average log excess return $\bar{xr}_{t+1}^{i,(10)}$ of going long the ten-year foreign bond and short the ten-year domestic bond expressed in the domestic numeraire (i.e. the British pound basket):

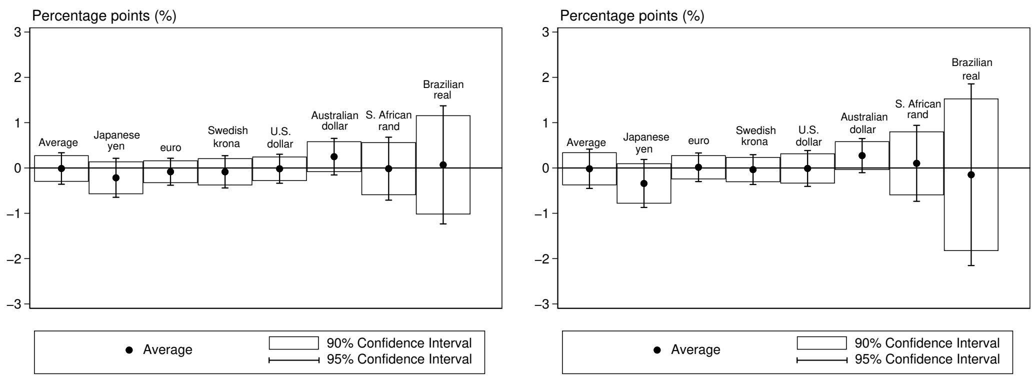
$$\bar{xr}_{t+1}^{i,(10)} \equiv \hat{\mathbb{E}}[hpr_{t+1}^{i,(10)} - \Delta q_{t+1} - hpr_{t+1}^{(10)}]. \tag{54}$$

We do this on both a real and nominal basis, and plot the estimated averaged by currency in Figure B.3.1. The dots denote the point estimatess, the bars denote the 90% confidence interval, and the whiskers denote the 95% confidence interval. Confidence intervals were constructed using standard errors that take into account heteroskedasticity and autocorrelation.

Real, average excess returns range from -22 basis points to 32 basis points, which translates in annualized terms to 2.64 and 3.84 percentage points. Symmetric, 90% and 95% confidence intervals around those point estimates, which are denoted by bars and whiskers respectively, include zero; that is, our estimates are not statistically different from zero. The nominal, average excess returns paint a similar picture: estimates of the average are not statistically different from zero. Confidence intervals on estimates are slightly wider than their real counterparts for the Swedish krona and the Australian dollar.

Figure 1: Average excess returns on long-maturity bonds

(a) Real (b) Nominal



Note: This plot displays the average real and nominal log monthly excess returns on long maturity (i.e. ten-year) bonds. The domestic currency is the British pound, and the real numeraire good is the British pound basket. The dots denote the average, the bars denote the 90% confidence interval, and the whiskers denote the 95% confidence interval. Confidence intervals were constructed using standard errors that take into account heteroskedasticity and autocorrelation. Returns are monthly, holding period returns (i.e. not annualized), reported in percentage points (i.e. $log(1.05) \approx 5\%$ is reported as 5, not as .05), and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

B.3.2 Decomposition

Before we turn to the time series, we present a decomposition of the average real excess return in Table B.3.2. When we decompose the average excess returns on long-maturity bonds, we see that the change in real exchange rates is offsetting the change in the difference in local currency holding period returns. That is, average excess returns are not zero simply because real exchange rate changes, and local currency holding period returns are zero in expectation.

Table 7: Decomposition of the average real excess return on long-maturity bonds

	Currency							
(real returns, in percentage points)	Average	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
Foreign holding period bond return (A)	0.29	0.24	0.25	0.25	0.22	0.34	0.22	0.56
Domestic holding period bond return (B)	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.13
Foreign minus domestic holding period bond return (C=A-B)	0.09	0.04	0.05	0.05	0.02	0.14	0.02	0.43
FX return (D)	0.09	0.26	0.13	0.09	0.03	-0.11	0.03	0.36
Excess returns in domestic currency (E= C − D)	0.00	-0.22	-0.08	-0.05	-0.02	0.25	-0.01	0.07
HAC standard errors (F)	0.18	0.22	0.15	0.18	0.16	0.21	0.36	0.66
No. of monthly return observations	143	143	143	143	143	143	143	101
Sample start	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2012-Jul
Sample end	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec

Notes: This table reports the decomposition of the average real excess return on long-maturity bonds. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Standard errors take into account heteroskedasticity and autocorrelation. The foreign (domestic) holding period bond return is the log holding period return on the ten-year foreign (domestic) bond in local currency numeraire. The foreign minus domestic holding period bond return is the difference between the log holding period return on the ten-year foreign and domestic bonds without adjusting for the fact that the returns being differenced are in different currency numeraires. The FX return is the log return on real exchange rates, where a positive value is a depreciation of the foreign currency numeraire vis-a-vis the domestic currency numeraire. Excess returns in domestic currency is the difference between the log holding period return on the ten-year foreign and domestic bond after having converted the ten-year foreign bond returns into returns that are in terms of the domestic currency numeraire. Apparent discrepencies in the arithmetic sum are due to rounding of the intermediate components. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

B.4 Cross sectional result

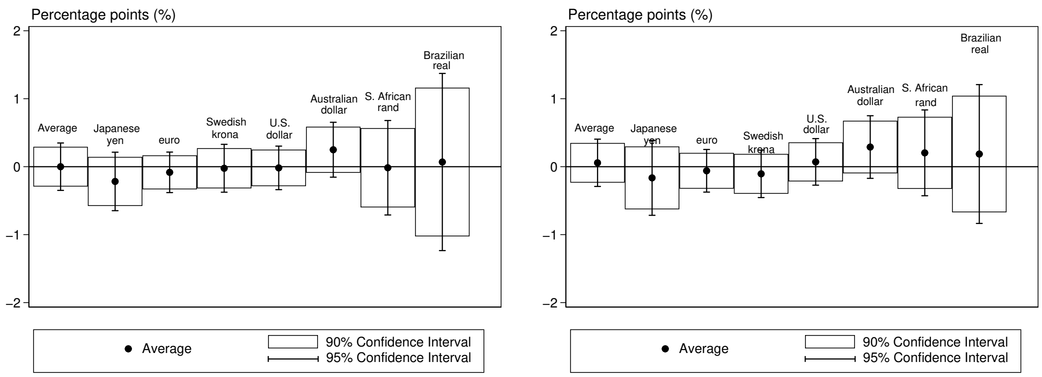
B.4.1 Term structure

To put our long-maturity result in context, we present the corresponding short-maturity result.

Inflation swap data exist only for maturities greater than one year, limiting us to the study of returns on bonds of maturities greater than a year. Therefore, we are unable to estimate the unconditional exchange rate premium as conventionally measured, which is the average monthly bond excess returns on short-run bonds where short-run is the horizon of the trade, in this case one-month. Instead, we estimate the average monthly bond excess returns on short-maturity bonds, defined as two-year bonds, which we plot in Figure B.4.1. We find that excess returns on short-maturity range from -39 basis points, and 36 basis points, but as before, they are not statistically different from zero.

The inability to statistically distinguish form zero is due both to the small magnitude of the point estimates, and large standard errors that reflect the short length of our time series. The point estimates of average excess returns, however, do display a linear relationship vis-a-vis the level of interest rates and the yield curve slopes. Figure B.4.1 plots the average excess return on the long-maturity and short-maturity bonds vis-a-vis the level of real interest rates. This relationship gets slightly stronger for short-maturity bonds, principally because of the observations for the South African rand and Brazilian real.

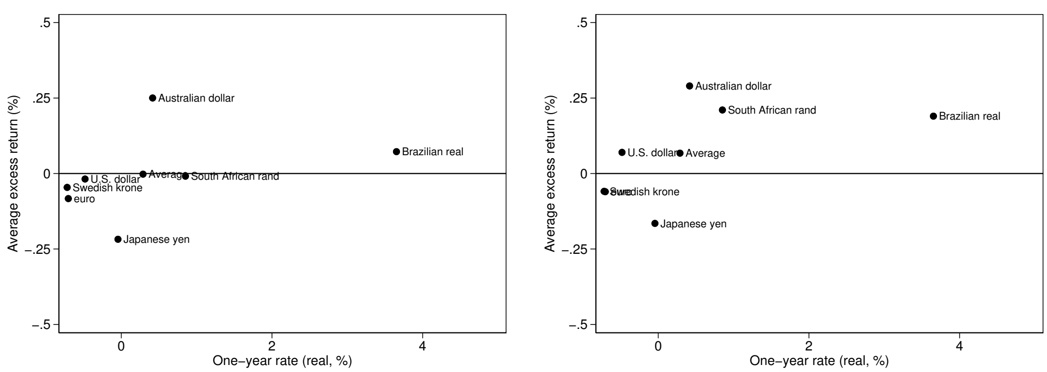
Figure 2: Average real excess returns on long and short-maturity bonds



(a) Long-maturity (b) Short-maturity

Note: This plot displays the average real log monthly excess returns on long maturity (i.e. ten-year) bonds versus short maturity (i.e. two-year) bonds. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

Figure 3: Average excess returns with the long and short-maturity bond versus the one-year rate



(a) Long-maturity bond (b) Short-maturity bond

Note: This plot displays the average real log monthly excess returns on long maturity (i.e. ten-year) bonds versus the one-year rate, and the the yield curve slope of the foreign currency. The one-year rate is the one-year, real rate in local currency. The slope is the ten-year minus the one-year real rate in local currency. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

C Reference interbank rates on plain vanilla overnight index swap contracts by currency

The reference interbank rate on the overnight index swap is typically the average of interest rates at which banks offer to lend unsecured funds to other banks in the interbank market, and is often but not always, an indicative and not market rate; for the Australian dollar the corresponding the rate is the interbank overnight cash rate, and is calculated as the weighted average of the interest rate at which overnight unsecured funds are transacted in the domestic interbank market; for the Brazilian real is the Brazil Interbank Deposit Rate Annualized also known as the CDI, which is the average overnight dollar deposit rate published by the central OTC and private securities and derivatives despoitory CETIP; for the euro is the euro overnight interbank interest rate also known as EONIA; for the British pound is the sterling overnight index average also known as SONIA; for the Japanese yen is the overnight call rate, for the Swedish krona is the tonight next (T/N) Swedish interbank offered rate also known as T/N Stibor; for the U.S. dollar is the interbank interest rate for satisfaction of reserve requirements also known as the fed funds rates.

D Inflation and ten-year inflation swap holding period returns

Table 8: Inflation and ten-year inflaton swap holding period returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	π_{t+1}	π^*_{t+1}	π^*_{t+1}	π^*_{t+1}	π^*_{t+1}	π^*_{t+1}	π^*_{t+1}	π^*_{t+1}
$hpr^{(10),\pi}_{t+1}$	0.00							
	(0.02)							
$hpr^{*(10),\pi}_{t+1}$		-0.05***	-0.07	-0.03	-0.02	0.03	-0.02	-0.03
		(0.02)	(0.05)	(0.02)	(0.02)	(0.03)	(0.01)	(0.03)
Constant	0.00***	0.00	0.00***	0.00***	0.00***	0.00***	0.00***	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	143	143	143	143	143	143	143	101
R^2	0.00	0.06	0.02	0.01	0.01	0.01	0.01	0.01
0.01	British pound	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
Maturity	10y	10y	10y	10y	10y	10y	10y	10y
Freq.	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly

Notes: Panel A reports the results of a regression of inflation on ten-year inflation swap holding period returns.

E Detailed decomposition of holding period returns

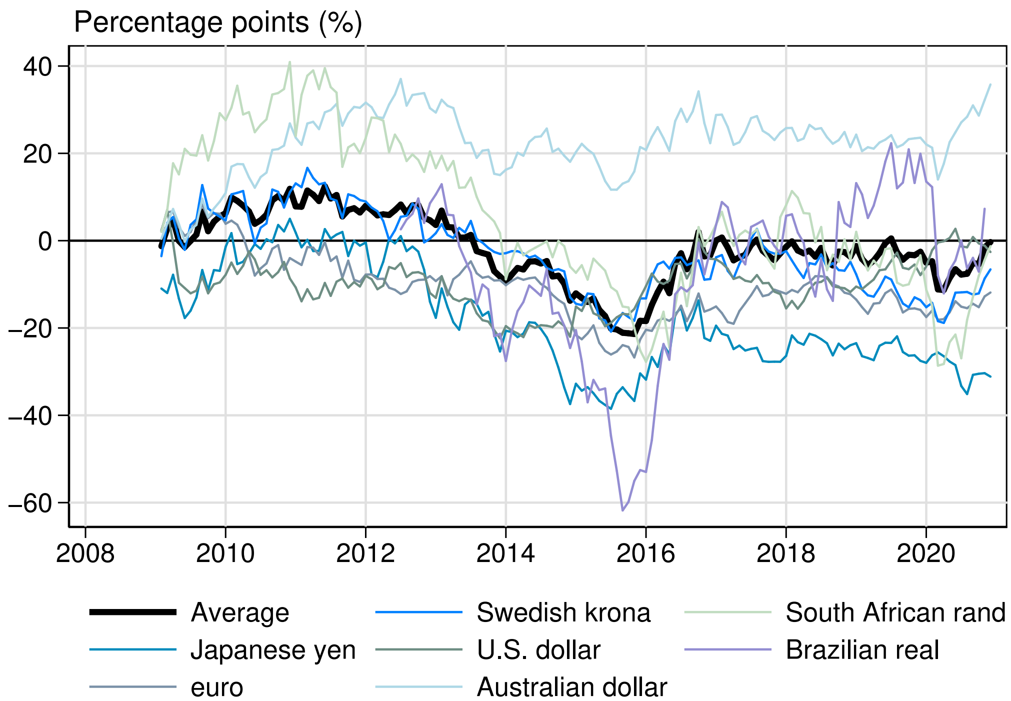
Table 9: Decomposition of the average real excess return on long-maturity bonds (detailed)

	Currency							
in percentage points)	Average	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
Foreign bond return (A = A1 + A2)	0.29	0.24	0.25	0.25	0.22	0.34	0.22	0.56
Nominal bond return (A1)	0.55	0.20	0.49	0.43	0.43	0.58	0.81	1.02
Inflation swap return (A2)	-0.25	0.05	-0.24	-0.19	-0.21	-0.24	-0.59	-0.45
FX return (B = B1 + B2 − B3)	0.09	0.26	0.13	0.09	0.03	-0.11	0.03	0.36
Nominal fx return (B1)	0.07	0.05	-0.01	-0.05	-0.04	-0.17	0.21	0.81
Domestic inflation (B2)	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.19
Foreign inflation (B3)	0.20	0.02	0.09	0.08	0.15	0.16	0.41	0.64
Foreign bond return in domestic currency numeraire (C= A-B)	0.20	-0.01	0.12	0.16	0.19	0.45	0.19	0.21
Domestic bond return (D=D1+D2)	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.13
Nominal bond return (D1)	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.35
Inflation swap return (D2)	-0.28	-0.28	-0.28	-0.28	-0.28	-0.28	-0.28	-0.22
Excess returns in domestic currency (E= C − D)	0.00	-0.22	-0.08	-0.05	-0.02	0.25	-0.01	0.07
HAC standard errors (F)	0.18	0.22	0.15	0.18	0.16	0.21	0.36	0.66
No. of monthly return observations	143	143	143	143	143	143	143	101
Sample start	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2009-Feb	2012-Jul
Sample end	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec	2020-Dec

Notes: This table reports the decomposition of the average real excess return on long-maturity bonds. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Standard errors take into account heteroskedasticity and autocorrelation. The foreign (domestic) bond return is the log holding period return on the ten-year foreign (domestic) bond in local currency numeraire. The foreign (domestic) bond return can be further decomposed into the sum of two components: the nominal bond return and the inflation swap return. The nominal bond return is the log holding period return on the ten-year nominal bond in local currency. The inflation swap return is the holding period return for the holder of the fixed leg of the ten-year inflation swap in local currency. The FX return is the log return on real exchange rates, where a positive value is a depreciation of the foreign currency numeraire vis-a-vis the domestic currency numeraire. The FX return can be further decomposed into the nominal exchange rate return plus domestic inflation minus foreign inflation. The nominal currency return is the log holding period return on the nominal exchange rate. Foreign (domestic) inflation is the log return on the foreign (domestic) national price index referenced by the inflation swap or real bond contract. Ceteris paribus, the foreign currency will appreciate in real terms when there is foreign inflation because for a given nominal exchange rate, units of foreign currency now deliver fewer foreign real baskets. The foreign bond return in domestic currency numeraire is the holding period return on the foreign bond in local currency numeraire converted to be in terms of the domestic currency numeraire. Excess returns in domestic currency is the difference between the log holding period return on the ten-year foreign and domestic bond after having converted the ten-year foreign bond returns into returns that are in terms of the domestic currency numeraire. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

F Cumulative returns of going long the foreign bond and short the domestic bond by currency

Figure 4: Cumulative real excess returns on the foreign long-maturity bond



Notes: This plot displays the cumulative, log monthly real excess returns of going long the long-maturity (i.e. ten-year) foreign bond and short the domestic bond. The domestic currency is the British pound, and the real numeraire good is the British pound basket. Standard errors take into account heteroskedasticity and autocorrelation. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations.

Table 10: Cumulative real excess returns on the foreign long-maturity bond

	Currency							
(real returns, in percentage points)	Average	Japanese yen	euro	Swedish krona	U.S. dollar	Australian dollar	South African rand	Brazilian real
Cumulative returns	-0.64	-30.99	-11.74	-6.21	-2.62	35.90	-1.39	4.26
Standard errors								
Bootstrap scheme I: case resampling	25.12	30.95	20.90	24.13	21.93	28.33	45.08	69.48
Bootstrap scheme II: block bootstrap	25.03	29.35	20.27	26.54	21.76	30.60	54.02	59.07
Horizon in months	143	143	143	143	143	143	143	101

Notes: This plot displays the cumulative, log monthly real excess returns of going long the long-maturity (i.e. ten-year) foreign bond and short the domestic bond.The domestic currency is the British pound, and the real numeraire good is the British pound basket. Standard errors take into account heteroskedasticity and autocorrelation. Returns are monthly, log holding period returns, reported in percentage points, and cover February 2009 to December 2020 with the exception of the Brazilian real, whose series begins July 2012. Returns for the currency Average was constructed by taking the equally weighted return of currencies with return observations. For each currency, we bootstrapped 10,000 samples of non overlapping returns to generate the standard errors on the cumulative returns. We employed two bootstrap schemes. The first was case resampling by currency using the Monte Carlo algorithm in which we resample the data with replacement, and the size of the resample is equal to the size of the original dataset; this kind of algorithm does not take into account potential dependence of observations over time into account. The second was a stationary block bootstrap by currency using the Politis, Romano (1992) algorithm in which data are drawn in blocks of random length and stitched together to form a single draw; the average length of the block was set to fourteen, the choice of which the standard errors are not sensitive to. This kind of algorithm does take into account potential dependence of observations over time into account.