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MEB and Anomaly Detection

Optimization for Data Science

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Abstract

The ABSTRACT is to be in fully-justified italicized text, at the top of the left-hand column, below the author and affiliation information. Use the word "Abstract" as the title, in 12-point Times, boldface type, centered relative to the column, initially capitalized. The abstract is to be in 10-point, single-spaced type. Leave two blank lines after the Abstract, then begin the main text. Abstract should be no longer than 300 words.

1 Introduction

2 Minimum Enclosing Balls

[Here is where I'd do the transition between the formulation of the problem in the introduction and how we intend to solve it using MEB]

2.1 Description of MEB

We can describe the Minimum Enclosing Ball problem in the following way: given a set of points $\mathcal{A} = a_1, ..., a_n$ in d dimensions, the objective is to find the smallest ball $B^n(c, \rho)$ containing all points of $\mathcal{A}$, such that:

$$\mathcal{A} \subseteq B^n(c, \rho) \tag{1}$$

where

$$B^n(c, \rho) = \{x \in \mathbb{R}^n \mid \|x - c\| \leq \rho\} \tag{2}$$

where $c \in \mathbb{R}^n$ and $\rho \in \mathbb{R}$ are the decision variables. We define c as the centre of the minimum enclosing ball and ρ as the radius. A sphere $B^n(c, \rho) \subset \mathbb{R}^n$ is called an enclosing ball of $\mathcal{A}$ if and only if (1) holds.

The problem of calculating an approximation to the minimum enclosing ball of $\mathcal{A}$ can be denoted as MEB($\mathcal{A}$), which can be computed by solving the following optimization problem:

$$\begin{aligned} (\mathcal{P}_1) \quad & \min_{c,\rho} \rho \\ \text{subject to} \quad & a_i - c \leq \rho, \quad i = 1, \ldots, m \end{aligned}$$

We obtain a second optimization problem $\mathcal{P}_2$ with smooth and convex quadratic constraints, by setting $\gamma := \rho^2$ and squaring the constraints of $\mathcal{P}_1$.

$$\begin{aligned} (\mathcal{P}_2) \quad & \min_{c,\gamma} \gamma \\ \text{subject to} \quad & (a_i)^T a_i - 2(a_i)^T c + c^T c \leq \gamma, \quad i = 1, \ldots, m \end{aligned}$$

We can derive the lagrangian dual from $\mathcal{P}_2$ by adding Lagrange multiplier to each constraint.

2.2 References

List and number all bibliographical references in 9-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example [1]. Where appropriate, include the name(s) of editors of referenced books.

References

[1] Authors. The frobnicatable foo filter, 2014. Face and Gesture submission ID 324. Supplied as additional material `fg324.pdf`.