Min-entropy

- In some applications other types of entropy is more appropriate.

- X, p(X)

  Min-entropy of a distribution is defined as:

  $H_{\infty}(X) = -\log \max_x p(x)$

Min-entropy is an important security measure for random number generators.

- Example:
- $p(x_1)=2^{-1}$
- $p(x_2)=p(x_3)\ldots\ldots p(x_{256})= 2^{-8}$

- $H(X)\sim 7.8$ bits
- $H_{\infty}(X) = -\log_2(2^{-1})\sim 1$ bit

- Min-entropy measures the success chance of the best guess.

Min-entropy

- Shannon entropy is expected uncertainty:
  Not a good measure for success chance of best guess

- Example: $\mathcal{X} = \{0,1\}^3$,
- $p(000)=9/16$, $p(001)=\ldots p(111)=1/16= 2^{-4}$
- $H(X) = -(7/16) \log(2^{-4}) - 9/16 \log (9/16)$
  $= 2.22$ bit

- $H_\infty(X) \sim 1$ bit

- Entropy relations for min-entropy are different..

From data to distribution

GPA	Hours Playing Video Games
3.95	1
3.65	7
3.55	5
3.58	3
2.98	10
1.50	17
1.75	18
2.20	9
3.00	6
3.00	5
3.00	5
3.15	4

GPA	Hours Playing Video Games
4.00	0
2.50	15
4.00	3
3.90	5
3.75	5
3.80	0
2.90	13
3.10	8
3.25	7
3.40	8
3.30	9
3.90	12

Distribution from data

Hours / GPA	0-5 hours	6-10 hours	>11 hours
<2.5	0	1/24	3/24
2.51 - 3.2	3/24	2/24	1/24
3.21 - 4	8/24	5/24	1/24

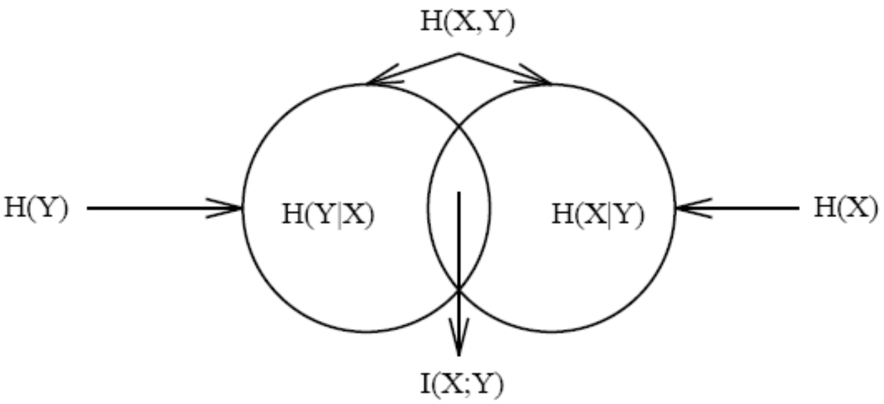
- p(play<6 h)= 11/24
- p(6<play<10 h) = 8/24
- p(play>11 h)= 5/24

- p(GPA<2.5)=3/24
- p(GPA: 2.51-3.2)=6/24
- p(GPA: 3.21-4)= 14/24

- Prob GPA>3.2, for students who play more than 11 h
- p(GPA > 3.2 | play>11h) = p(GPA > 3.2, play>11h) / p(play>11h)
- = (1/24)/ (5/24)= 1/5

Summary

- Measures
  - $H(X)$
  - $H(X,Y)$
  - $H(X|Y)$, $H(X|Y=y)$
  - $I(X;Y)$
  - $D(X \,||\, Y)$



- Relations
- $D(X||Y) \geq 0$

  → $I(X;Y) \geq 0$, $H(X) \geq H(X|Y)$, $\log_2(N) \geq H(X)$
- $H(X) \geq H(f(X))$

- Min-entropy