

Secret Sharing Progress

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Report

Perfect Secrecy:

A ciphertext should not leak any additional information an attacker has prior about the plaintext.

Mathematically this can be expressed as:

An encryption scheme (gen, enc, dec) with message space M and ciphertext space C is perfectly secure iff every probability distribution over M , $\forall m \in M$ and $\forall c \in C$ with $\Pr(C = c) > 0$ then;

$$\Pr(M=m \mid C=c) = \Pr(M = m)$$

A secret sharing scheme where by shares less than the threshold leak information about secret, satisfy correctness but not secrecy (yet to find a good example).

Cheater detection:

(example lifted from previous report)

secret = $6 \in \mathbb{Z}_7$

scheme (3,5)

$P_1(1, 5), P_2(2, 5), P_3(3, 6), P_4(4, 1), P_5(5, 4)$

The first three people put their shares together to reconstruct the secret. Person one decides to cheat and gives an incorrect share. The secret reconstructed is legal ($\in \mathbb{Z}_7$) but incorrect ($s' \neq s$). The cheater goes undetected since the secret is legal. [1] shows how a cheater can successfully deceive $t-1$ participants whereby, they reconstruct the incorrect secret and from that the cheater can learn the correct secret.

If participant one decides to cheat, he interpolation to construct a polynomial $f'(x)$ of degree at most $t-1$ such that $f'(0) = -1$ and $f'(P_2) = \dots = f'(P_t) = 0$.

The participant therefore announces instead their share + $f'(P_1)$. Interpolation guarantees that t participants will reconstruct the polynomial $f(x) + f'(x)$ which has constant term $f(0) + f'(0) = \text{secret} - 1$. Cheating will go undetected unless the secret happened to be 0. (Tried this with the above example but still don't get the correct answer. Maybe I am making mistakes somewhere in calculations or missing a concept.)