

Discrete Optimization

The Knapsack Problem:
Search Strategies

Goals of the Lecture

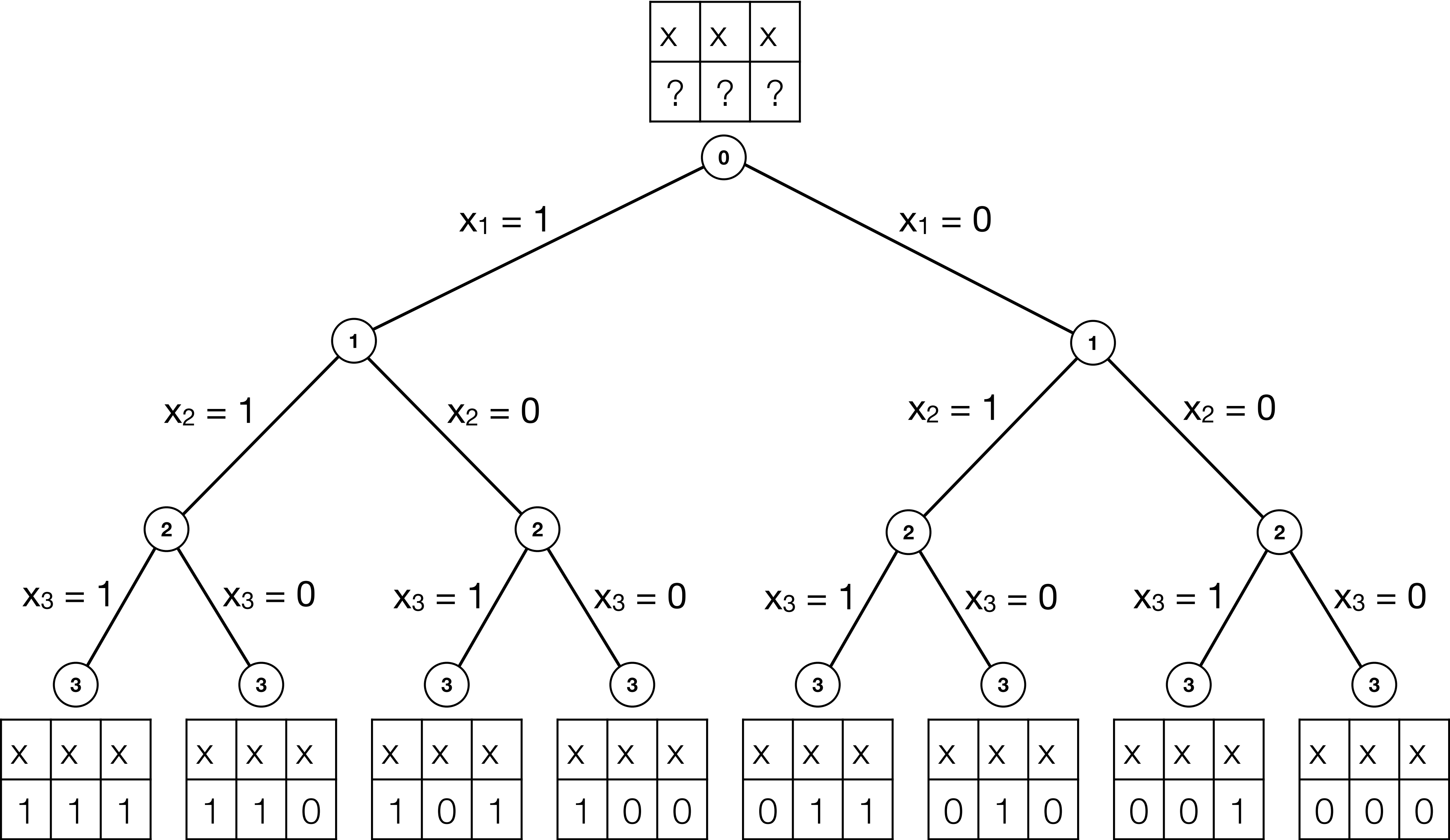
- ▶ Introduce search strategies for branch and bound

A Knapsack Model

- Of course, if the items are archeological artifacts...



Exhaustive Search



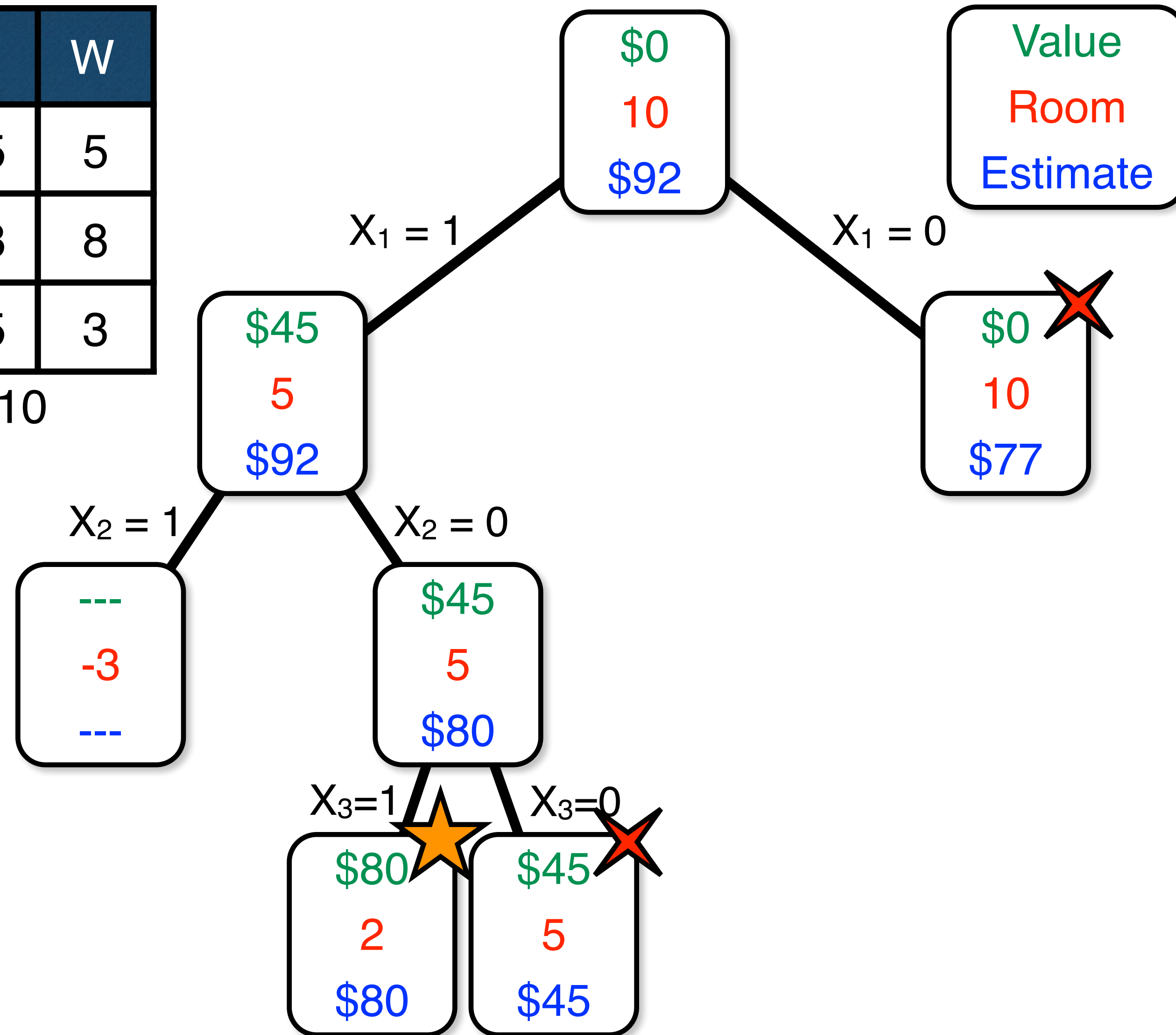
Branch and Bound

- ▶ Search Strategies
 - depth-first, best-first, least-discrepancy
 - many others
- ▶ Depth-first
 - prunes when a node estimation is worse than the best found solution
 - memory efficient
- ▶ Best-First
 - select the node with the best estimation
- ▶ Least-Discrepancy
 - trust a greedy heuristic

Depth-First Branch and Bound

i	V	W
1	45	5
2	48	8
3	35	3

$K = 10$

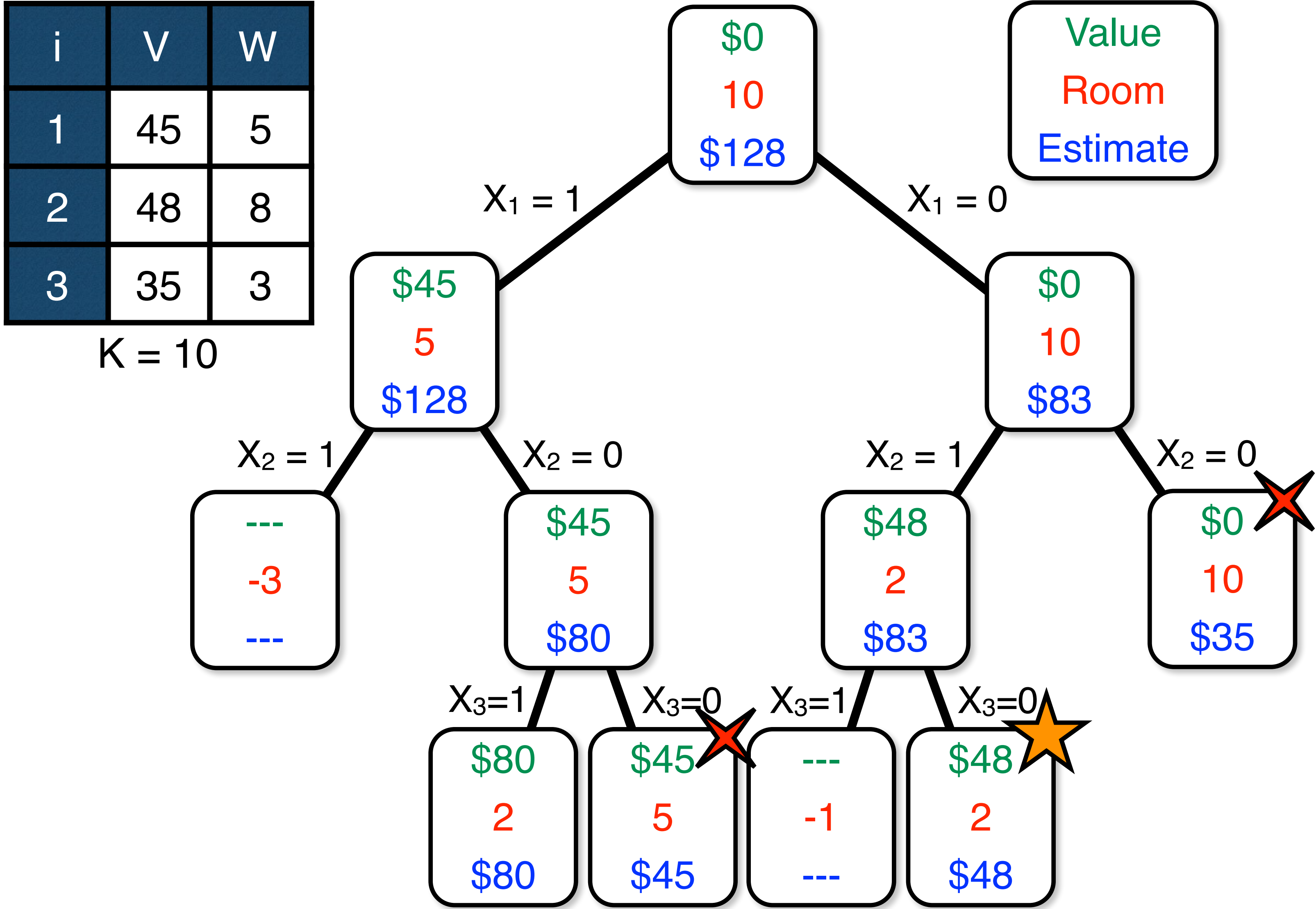


Branch and Bound

► Depth-First

- select the node to the left
- when does it prune?
 - when it finds a new node worse than the found solution
- is it memory efficient?
 - exaggerate!

Best-First Branch and Bound



Branch and Bound

► Best-First

- select the node with the best estimation
- when does it prune?
 - when all the nodes are worse than a found solution
- is it memory efficient?
 - exaggerate!

Limited Discrepancy Search

- ▶ Assume that a good heuristic is available
 - it makes very few mistakes
 - the search tree is binary
 - following the heuristic means branching **left**
 - branching **right** means the heuristic was wrong
- ▶ Limited Discrepancy Search (LDS)
 - explore the search space in increasing order of mistakes
 - trusting the heuristic less and less

Limited Discrepancy Search

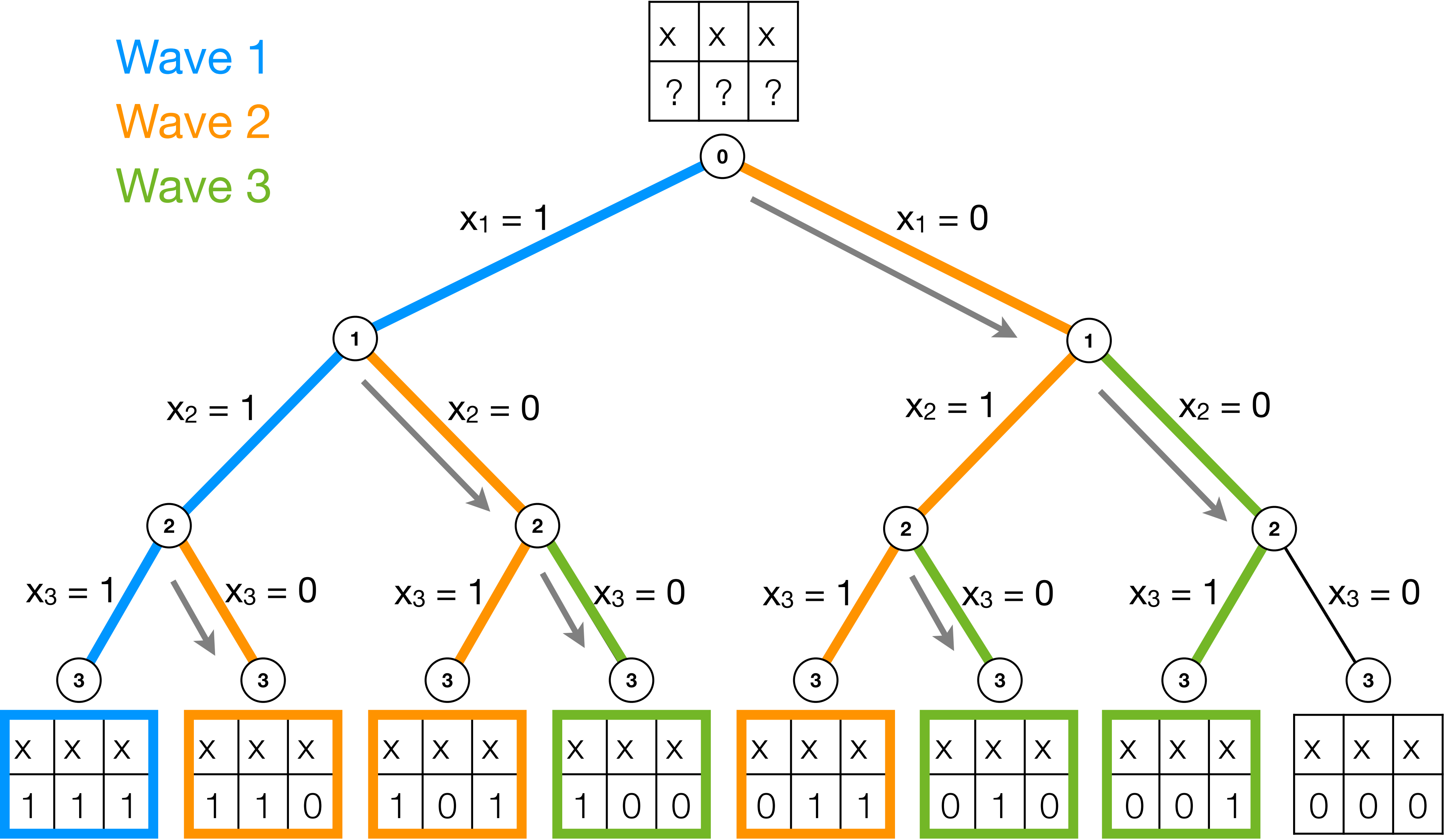
- ▶ Limited Discrepancy Search (LDS)
 - explore the search space in increasing order of mistakes
 - trusting the heuristic less and less
- ▶ Explores the search space in waves
 - no mistake
 - one mistake
 - two mistakes
 -

LDS Waves

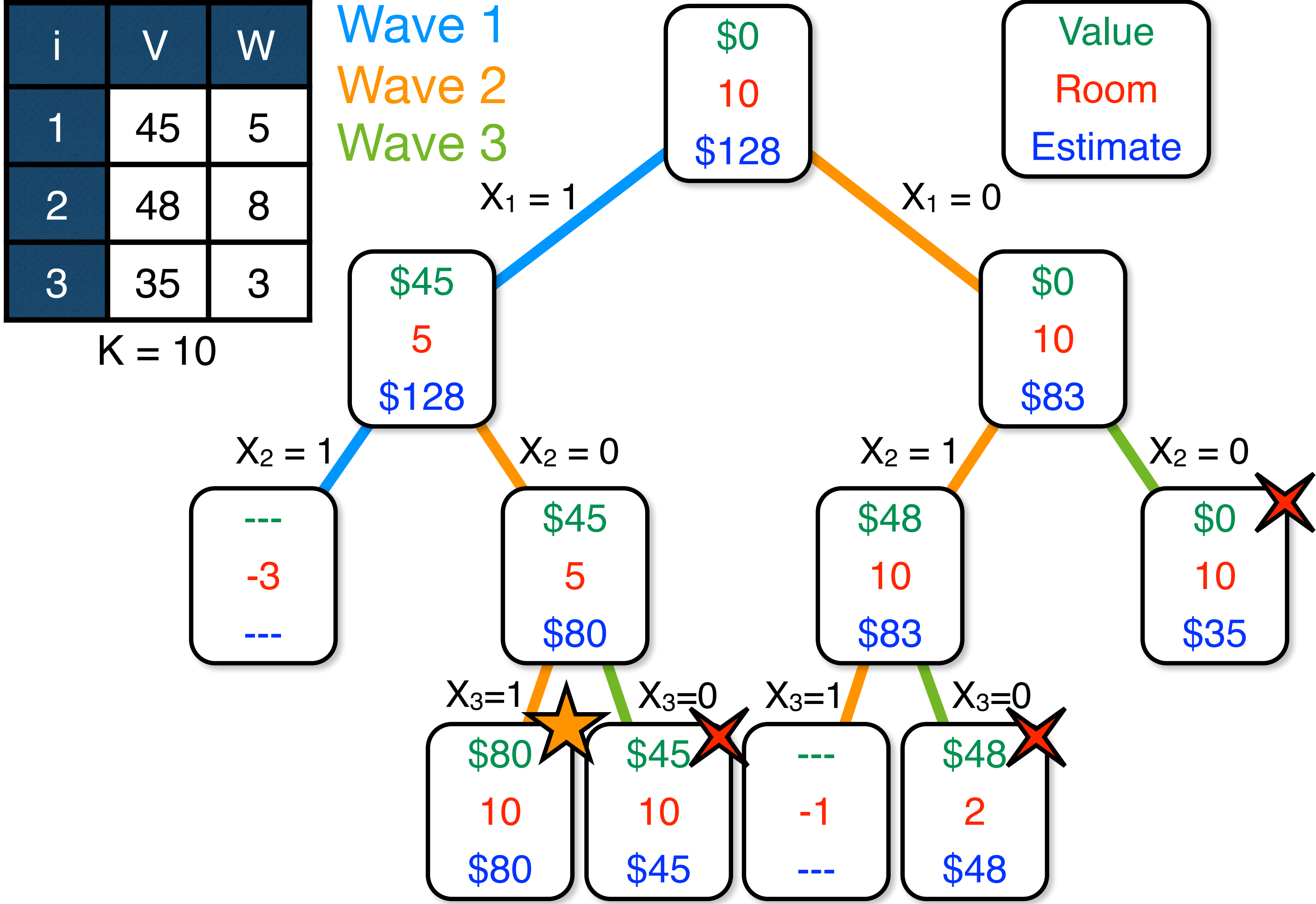
Wave 1

Wave 2

Wave 3



LDS Branch and Bound



Branch and Bound

- ▶ Least Discrepancy Search
 - trust the greedy heuristic
 - when does it prune?
 - is it memory efficient?
 - compared to depth-first and best-first?

Search Strategies

- ▶ Relaxation and Search
 - a match made in discrete optimization heaven
- ▶ Can be problem specific
- ▶ Can you think of a new one?

Until Next Time

Citations

Stone Foundation Tablet with Inscription of Gudea - 41221 (http://commons.wikimedia.org/wiki/File:Sumerian_-_Stone_Foundation_Tablet_with_Inscription_of_Gudea_-_Walters_41221_-_View_A.jpg). Artist Unknown. Walters Art Museum [Public domain, CC-BY-SA-3.0 (<http://creativecommons.org/licenses/by-sa/3.0>)], via Wikimedia Commons

Stone Foundation Tablet with Inscription of Gudea - 41220 (http://commons.wikimedia.org/wiki/File:Sumerian_-_Stone_Foundation_Tablet_with_an_Inscription_of_Gudea_-_Walters_41220_-_View_A.jpg). Artist Unknown. Walters Art Museum [Public domain, CC-BY-SA-3.0 (<http://creativecommons.org/licenses/by-sa/3.0>)] via Wikimedia Commons

Buddha at the Moment of Victory (http://commons.wikimedia.org/wiki/File:Thai_-_Buddha_at_the_Moment_of_Victory_-_Walters_542775.jpg). Artist Unknown. Walters Art Museum [Public domain, CC-BY-SA-3.0 (<http://creativecommons.org/licenses/by-sa/3.0>)], via Wikimedia Commons

Hoxne Hoard two gold bracelets (http://commons.wikimedia.org/wiki/File:Hoxne_Hoard_two_gold_bracelets_side.JPG) by Fæ (<http://commons.wikimedia.org/wiki/User:F%C3%A6>) [CC-BY-SA-3.0 (<http://creativecommons.org/licenses/by-sa/3.0>)], via Wikimedia Commons

Ring with the engraved portrait of Ptolemy VI Philometor ([http://commons.wikimedia.org/wiki/File:Ring_with_engraved_portrait_of_Ptolemy_VI_Philometor_\(3rd_%E2%80%93_2nd_century_BCE\)_-_2009.jpg](http://commons.wikimedia.org/wiki/File:Ring_with_engraved_portrait_of_Ptolemy_VI_Philometor_(3rd_%E2%80%93_2nd_century_BCE)_-_2009.jpg)<[http://commons.wikimedia.org/wiki/File:Ring_with_engraved_portrait_of_Ptolemy_VI_Philometor_\(3rd-2nd_century_BCE\)_-_2009.jpg](http://commons.wikimedia.org/wiki/File:Ring_with_engraved_portrait_of_Ptolemy_VI_Philometor_(3rd-2nd_century_BCE)_-_2009.jpg)>) By Unknown. (Photographed by PHGCOM in 2009.) [Public domain], via Wikimedia Commons

Calice du sacre Tau (http://commons.wikimedia.org/wiki/File:Calice_du_sacre_Tau.jpg) By Vassil (<http://commons.wikimedia.org/wiki/User:Vassil>) (Own work) [Public domain], via Wikimedia Commons

Citations

the mask of agamemnon (<http://www.flickr.com/photos/rosemania/5705122218/>) by Xuan Che (<http://www.flickr.com/people/rosemania/>) CC BY-2.0 (<http://creativecommons.org/licenses/by/2.0/deed.en>)

Terracotta Warrior (<http://www.flickr.com/photos/59627558@N00/4677378806/>) by fixermark (<http://www.flickr.com/photos/59627558@N00/>) CC BY 2.0 (<http://creativecommons.org/licenses/by/2.0/deed.en>)